Symbolic Verification of Complexity-Theoretic Properties of Cryptographic Protocols and Attack Discovery Using First Order Logic

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with

Hubert Comon-Lundh (ENS Cachan), Mitsuhiro Okada (Keio University)

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- Participants:
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  - Gergei Bana (University of Luxembourg)
- “Computationally Complete Symbolic Attacker”
  - Project for automated verification/attack finding of complexity theoretic properties (provable security) of security protocols
  - First Order Logic
  - “Unconditionally” computationally sound

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more precise but more complex

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- Computationally complete symbolic attacker: covers all PPT adversaries symbolically

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  - Many good results, but for either tools:
    - No attack is found, incomplete proof may mean weak tool, weak user, or insecure protocol
    - Hard to use effectively for others than developers
    - Various hidden assumptions

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- Algebraic operations, Standard hardness assumptions (DDH, etc) and security notions of primitives (CPA, CCA, etc) are easy to formalize

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- Attack model: Negation of security property consistent with axioms

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- First order language on the terms produced by the protocol execution. (Actions in the protocol execution(s) are not part of the first order language)

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  - $t_1,...,t_n \sim u_1,...,u_n$; semantics: computational indistinguishability of PPT algorithms $t_1,...,t_n$ from $u_1,...,u_n$

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- At the end: A list of properties that if satisfied by the implementation, then secure.

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- I.e. implementation should satisfy the properties in S

Core Axioms 1

Axioms for indistinguishability.

REFL: $\vec{x} \sim \vec{x}$

SYM: $\vec{x} \sim \vec{y} \longrightarrow \vec{y} \sim \vec{x}$

TRANS: $\vec{x} \sim \vec{y} \wedge \vec{y} \sim \vec{z} \longrightarrow \vec{x} \sim \vec{z}$

RESTR: If p projects and permutes onto a sublist,
$\vec{x} \sim \vec{y} \longrightarrow p(\vec{x}) \sim p(\vec{y})$

FUNCAPP: for any $\vec{f}$: $\mathsf{message}^n \rightarrow \mathsf{message}^m$, $\vec{f} \in \mathcal{F} \cup \mathcal{G}$,
$\vec{x} \sim \vec{y} \longrightarrow \vec{x}, \vec{f}(\vec{x}) \sim \vec{y}, \vec{f}(\vec{y})$

TFDIST: $\neg\,(\boldsymbol{true} \sim \boldsymbol{false})$

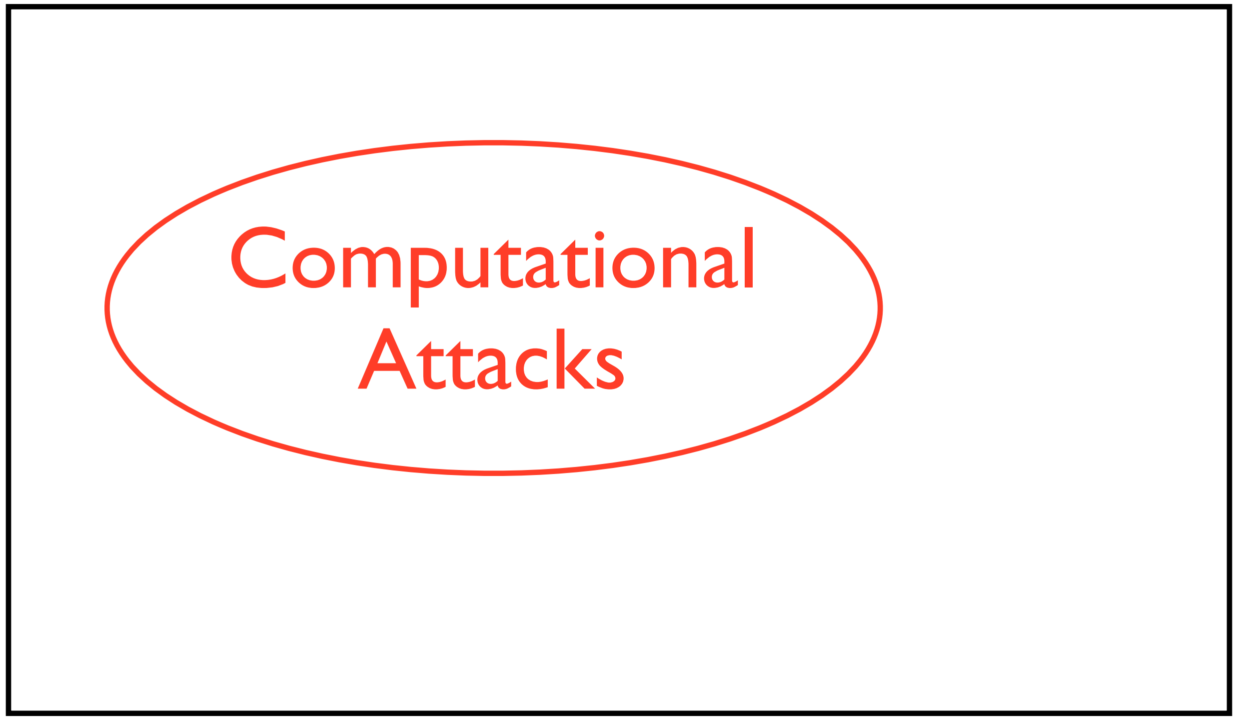
Axioms for equality.

EQREFL: $x = x$

EQCONG: $=$ is a congruence relation with respect to the current syntax.

EQTHEO: $=$ preserves the equational theory of functions

Axioms Limit the Attacker



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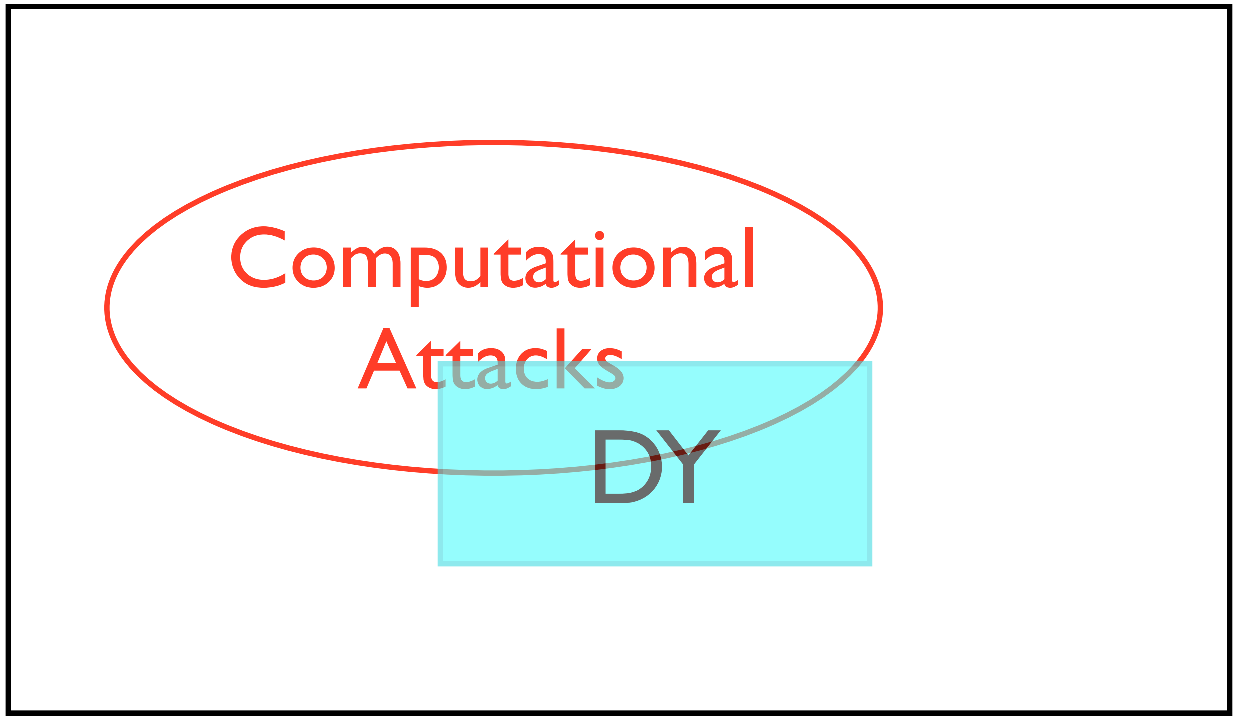

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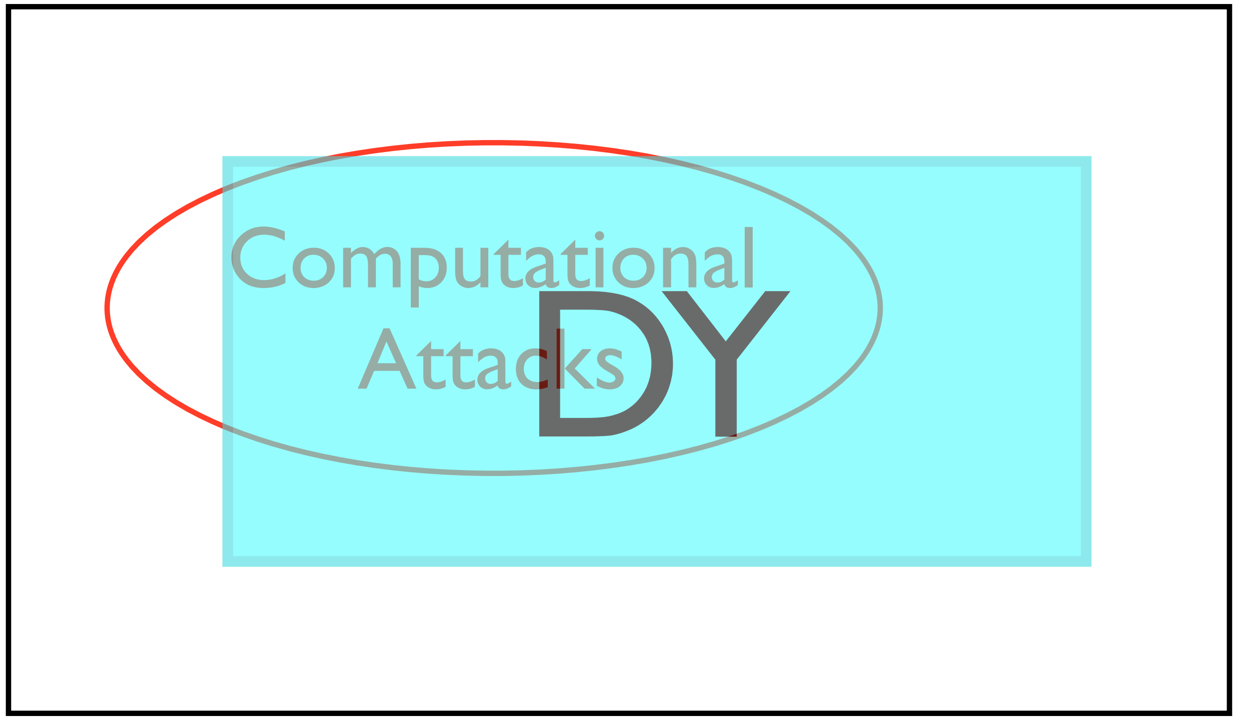

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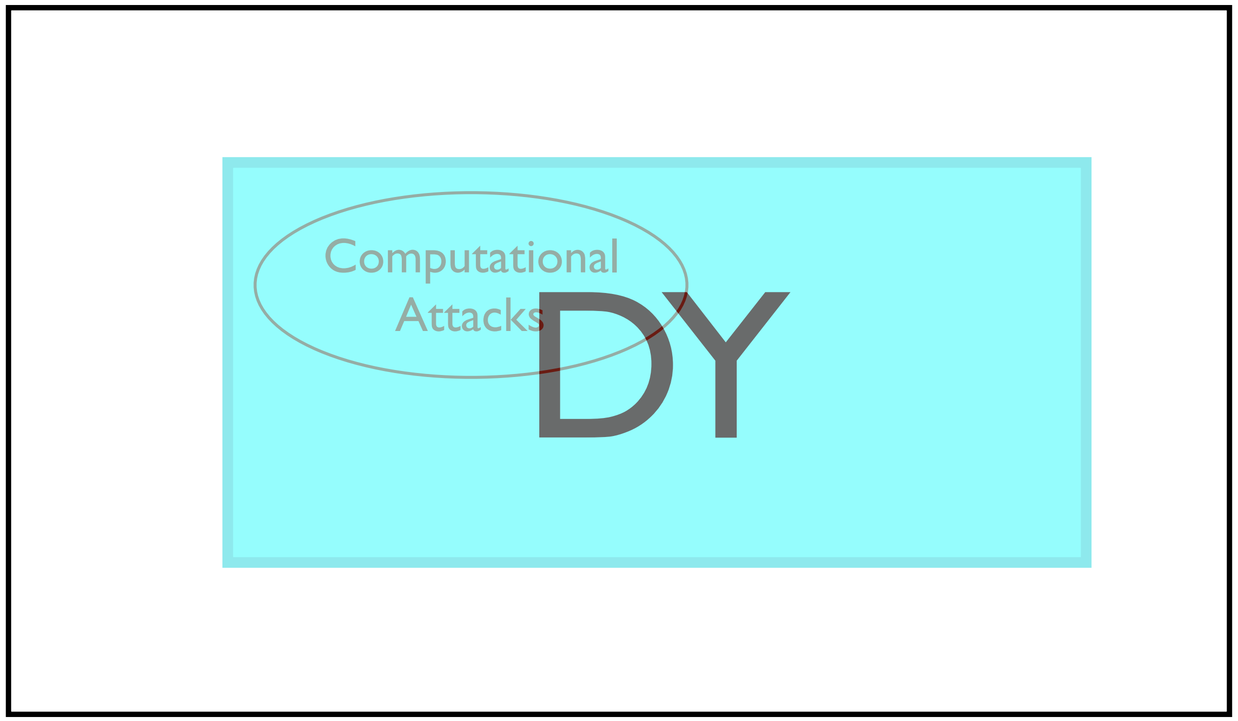

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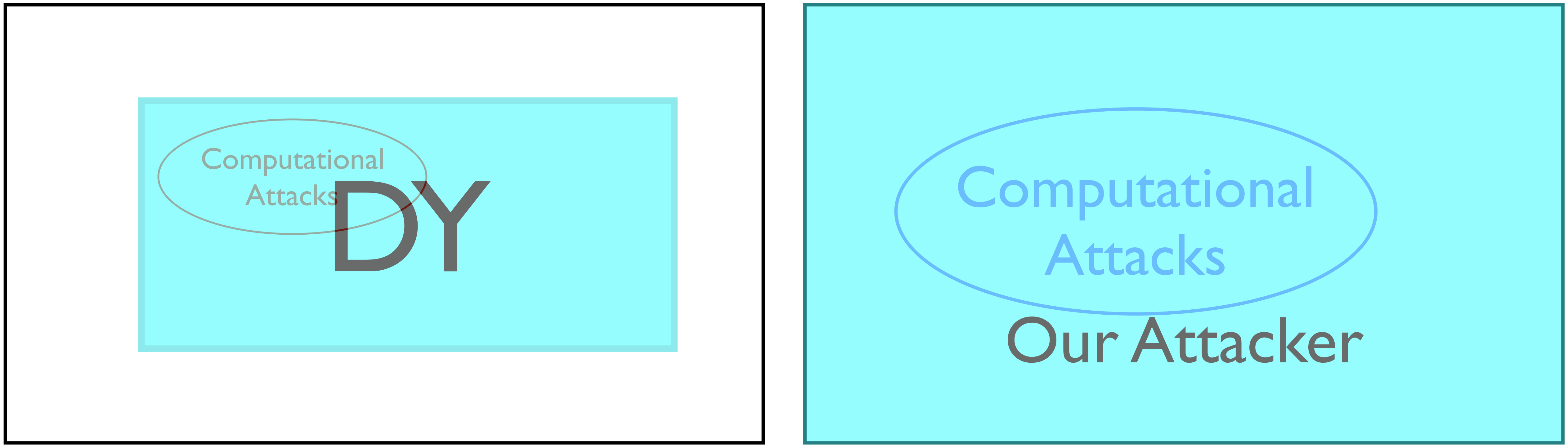

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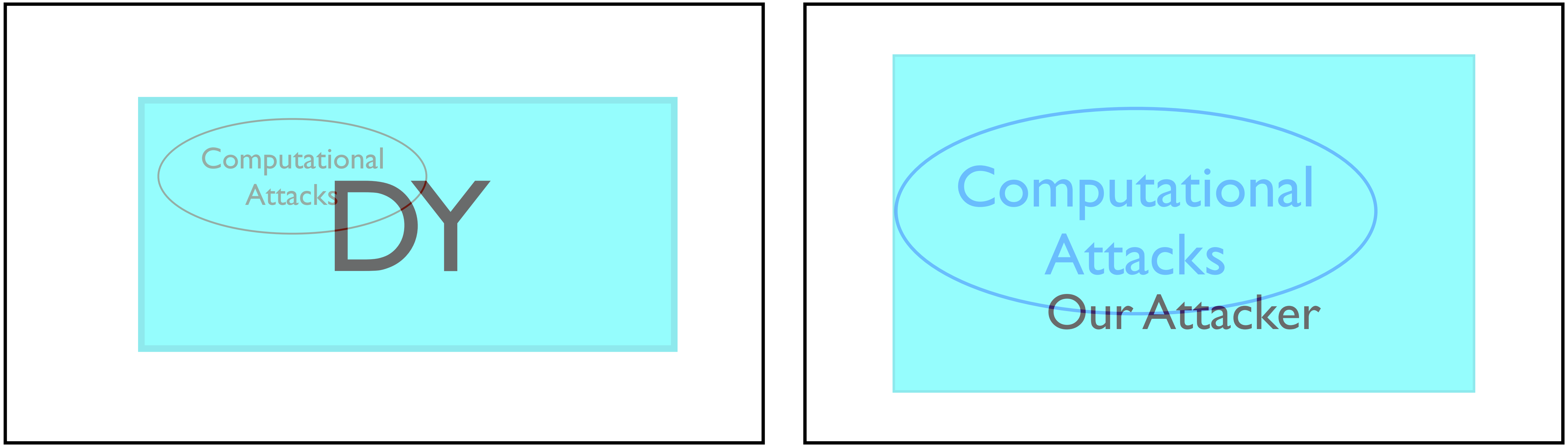

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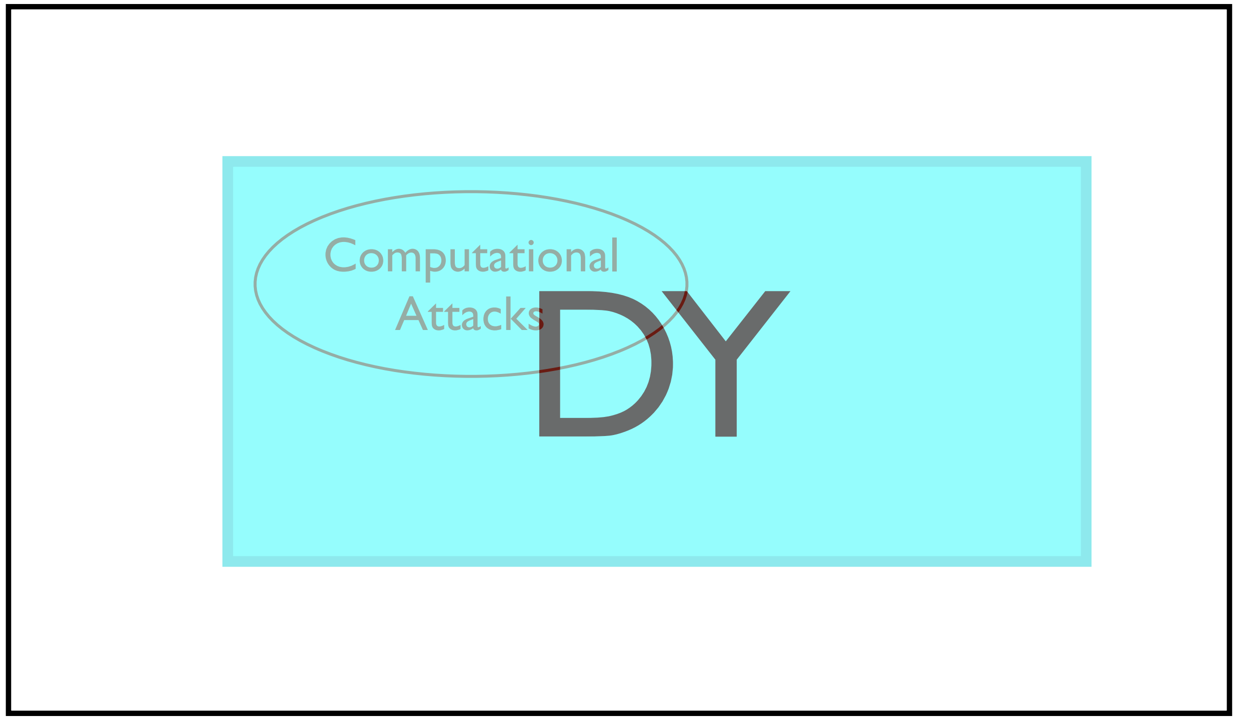

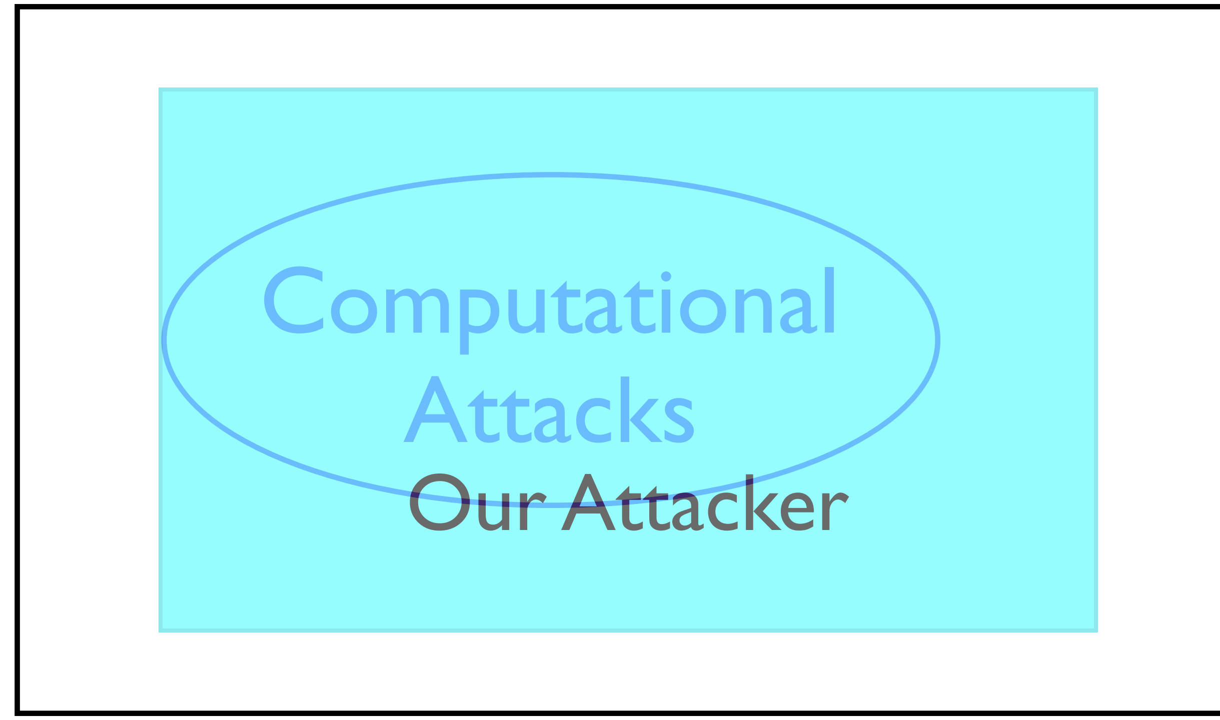

- Computationally sound axioms
- Idea works for other than computational
- Why computational:
  - Lot of work on computational model
  - We want to compare

Decisional Diffie-Hellman Assumption

$$(\mathsf{G}(n), \mathsf{g}(n), \mathsf{g}(n)^{\mathtt{r}(n_1)}, \mathsf{g}(n)^{\mathtt{r}(n_2)}, \mathsf{g}(n)^{\mathtt{r}(n_1)\mathtt{r}(n_2)})$$
$$\sim$$
$$(\mathsf{G}(n), \mathsf{g}(n), \mathsf{g}(n)^{\mathtt{r}(n_1)}, \mathsf{g}(n)^{\mathtt{r}(n_2)}, \mathsf{g}(n)^{\mathtt{r}(n_3)})$$

Encryptions - CPA, CCA1, CCA2

$$\vec{t}[\text{if } \mathbf{EQ}(\mathtt{L}(u), \mathtt{L}(u')) \text{ then } \{u\}_{\mathsf{pk}(n_1)}^{\mathsf{r}(n_2)} \text{ else } \mathbf{0}\,]$$

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$$\vec{t}[\text{if } \mathbf{EQ}(\mathtt{L}(u), \mathtt{L}(u')) \text{ then } \{u'\}_{\mathsf{pk}(n_1)}^{\mathsf{r}(n_3)} \text{ else } \mathbf{0}\,]$$

- L is length.
- Depending on conditions on **u**, **u'**, **t'**, it covers various standard notions of security:
  - Secure against Chosen Plaintext Attack
  - Secure against Chosen Ciphertext Attack 1
  - Secure against Chosen Ciphertext Attack 2

History

- Initial version G. Bana - K. Hasebe - M. Okada 2008 (Franco-Japanese): Derive security with **FOL**
- New momentum G. Bana - H. Comon POST'12: Computationally complete symbolic attacker (**trace properties**)
    - Followup: G. Bana - P. Adao - H. Sakurada FSTTCS'12, Bana-Hasebe-Okada CCS'13
        - Library of axioms, analyzed several protocols for **agreement**, **authentication** for arbitrary number of sessions by hand, found **new attacks**
    - Hubert Comon student Guillaume Scerri's PhD thesis - simple verification **tool** Scary
- **Indistinguishability** G. Bana - H. Comon CCS'14: - basics with simple **anonymity**
    - Followup: G. Bana - R. Chadha (Univ. of Missouri) eprint'15
        - Library of axioms **digital signatures**, **CPA**, **CCA security**, **exponentiation**, **DDH assumption**, various versions of **Diffie-Hellman key exchange** for arbitrary number of sessions by hand, **NSL** protocol, **real-or random secrecy**, **anonymity**, **agreement**, **authentication**
- G. Scerri - R. Stanley-Oakes (Bristol) CCS'16: Security of **Key Wrapping API's**
- H. Comon - A. Koutsos CSF'17: proving **unlinkability and authentication of RFID protocols** (XOR)
- Recently completed with student of R. Chadha: Various versions of DH key exchange, Station-to-Station protocol proofs in **COQ**

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  - e-voting