Reading Group on Deep Learning: Session 3

Introduction to Convolutional Neural Networks

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1 July 2016

What are CNNs ?

CNN = Neural Network with a convolution operation instead of matrix multiplication in at least one of the layers

Neural Networks





Output example : one class

airplane dog automobile frog bird horse cat ship deer truck

A typical CNN architecture



Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions



Biological neuron & mathematical model



Convolution

The convolution operation



The convolution operation



3 reasons why convolution is cool

Reason 1 : Sparse Connectivity





Reason 2 : Parameter sharing



Reason 3 : Equivariant Representations

When the input changes -> output changes in the same way

Eg. Let I be a function giving images brightness at integer coordinates Let g be a function mapping one image function to another image function, such that I' = g(I) is the image function with I'(x,y) = I(x - 1,y). This shifts every pixel of I one unit to the right. If we apply this transformation to I, then apply convolution, the result will be the same as if we applied convolution to I', then applied the transformation g to the output.



32x32x3 image



5x5x3 filter

Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

Filters always extend the full depth of the input volume



5x5x3 filter

Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"







For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Stride









7x7 input (spatially) assume 3x3 filter



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7x7 input (spatially) assume 3x3 filter



7x7 input (spatially) assume 3x3 filter

=> 5x5 output



7x7 input (spatially) assume 3x3 filter applied **with stride 2**



7x7 input (spatially) assume 3x3 filter applied **with stride 2**



7x7 input (spatially) assume 3x3 filter applied with stride 2 => 3x3 output!



7x7 input (spatially) assume 3x3 filter applied **with stride 3?**



7x7 input (spatially) assume 3x3 filter applied **with stride 3?**

doesn't fit! cannot apply 3x3 filter on 7x7 input with stride 3.



Ν

Output size: (N - F) / stride + 1
Zero-Padding





Zero-Padding: common to the border



e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

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7x7 output!

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e.g. input 7x7 **3x3** filter, applied with stride 1
pad with 1 pixel border => what is the output?

7x7 output!

in general, common to see CONV layers with
stride 1, filters of size FxF, and zero-padding with
(F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Examples time:

Output volume size: ?



Examples time:





Examples time:



Number of parameters in this layer?

Examples time:





Summary

- Accepts a volume of size $W_1 imes H_1 imes D_1$
- Requires four hyperparameters:
 - Number of filters K,
 - $\circ\;$ their spatial extent F,
 - $\circ\;$ the stride S ,
 - the amount of zero padding P.
- Produces a volume of size $W_2 imes H_2 imes D_2$ where:
 - $\circ W_2 = (W_1 F + 2P)/S + 1$
 - $\circ~~H_2 = (H_1 F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $\circ D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 imes H_1 imes D_1$
- Requires four hyperparameters:
 - Number of filters K,
 - \circ their spatial extent F,
 - $\circ\;$ the stride S ,
 - $\circ\;\;$ the amount of zero padding $P.\;$
- Produces a volume of size $W_2 imes H_2 imes D_2$ where:

$$W_2 = (W_1 - F + 2P)/S + 1$$

Common settings:

K = (powers of 2, e.g. 32, 64, 128, 512)

- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 5, S = 2, P = ? (whatever fits)
- F = 1, S = 1, P = 0

- $H_2 = (H_1 F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry) • $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
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Local connectivity & tiled convolution

Local connectivity

 s_2

С

 x_2

d

*s*3

e

 x_3

 s_5

 x_5

 s_4

g

 x_4

h

 s_1

 x_1

Б

 \mathbf{a}

Locally connected layer

Convolutional layer

Fully connected layer





Tiled convolution



Locally connected layer

Tiled convolution

Convolutional layer







Effect = invariance to small translations of the input





- makes the representations smaller and more manageable
- operates over each activation map independently



Max Pooling

Single depth slice



У

4

Χ

max pool with 2x2 filters and stride 2



Summary

- Accepts a volume of size $W_1 imes H_1 imes D_1$
- Requires three hyperparameters:
 - $\circ\;$ their spatial extent F ,
 - $\circ\;$ the stride S ,
- Produces a volume of size $W_2 imes H_2 imes D_2$ where:
 - $W_2 = (W_1 F)/S + 1$
 - $\circ H_2 = (H_1 F)/S + 1$
 - $\circ D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- · Note that it is not common to use zero-padding for Pooling layers

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F = 2, S = 2 F = 3, S = 2

Back propagation



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

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e.g. x = -2, y = 5, z = -4

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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

$$Chain rule: \qquad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

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$$\frac{\partial f}{\partial x}$$

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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

$$\overset{x - 2}{-4}$$

$$\frac{y - 5}{-4}$$

$$y = \frac{1}{-4}$$

$$\frac{y - 5}{-4}$$

$$\frac{y - 5}{-4}$$

$$\frac{y - 5}{-4}$$

$$\frac{y - 12}{-4}$$

$$\frac{y - 1}{-4}$$

$$\frac{y - 1}$$












Patterns in backward flow

add gate: gradient distributor
max gate: gradient router
mul gate: gradient... "switcher"?









- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

Sigmoid $\sigma(x) = 1/(1+e^{-x})$



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- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

- 1. Saturated neurons "kill" the gradients
- 2. Sigmoid outputs are not zerocentered
- 3. exp() is a bit compute expensive



- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

tanh(x)

[LeCun et al., 1991]



Computes f(x) = max(0,x)

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

ReLU (Rectified Linear Unit)

[Krizhevsky et al., 2012]



ReLU (Rectified Linear Unit) Computes f(x) = max(0,x)

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- ReLU units can "die"



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
 will not "die".

Leaky ReLU

 $f(x) = \max(0.01x, x)$

[Mass et al., 2013] [He et al., 2015]

In practice

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU
- Try out tanh but don't expect much
- Don't use sigmoid

Preprocessing data

Preprocessing data



Preprocessing data



In practice: for images

e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet) (mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet) (mean along each channel = 3 numbers)

Not common to normalize variance, to do PCA or whitening

Weights initialization

Weights initialization

- If the weights in a network start too small, then the signal shrinks as it passes through each layer until it's too tiny to be useful.
- If the weights in a network start too large, then the signal grows as it passes through each layer until it's too massive to be useful.

Weights initialization

• All zero initialization

• Small random numbers

 Draw weights from a Gaussian distribution with standard deviation of sqrt(2/n), where n is the number of outputs to the neuron





Initialization of NNs by explicitly forcing the activations throughout the network to take on a unit Gaussian distribution at the beginning of the training.

Normalization is a simple differentiable operation

[loffe and Szegedy, 2015]



Usually inserted after Fully Connected and/or Convolutional layers, and before nonlinearity.

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout

Thank you for your attention

Deep Learning



What society thinks I do



What my friends think I do



What other computer scientists think I do



What mathematicians think I do



What I think I do



AlexNet example



Input: 227x227x3 images

[Krizhevsky et al. 2012]

First layer (CONV1): 96 11x11 filters applied at stride 4 => Q: what is the output volume size? Hint: (227-11)/4+1 = 55



pooling

Input: 227x227x3 images

[Krizhevsky et al. 2012]

First layer (CONV1): 96 11x11 filters applied at stride 4 => Output volume [55x55x96]

Q: What is the total number of parameters in this layer?

slide from: Fei-Fei Li & Andrej Karpathy & Justin Johnson

pooling

dense

1000

2048

2048

densé

2048

dense



[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4 =>

Output volume **[55x55x96]** Parameters: (11*11*3)*96 = **35K**

[Krizhevsky et al. 2012]



Input: 227x227x3 images After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2

Q: what is the output volume size? Hint: (55-3)/2+1 = 27

[Krizhevsky et al. 2012]



Input: 227x227x3 images After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2 Output volume: 27x27x96

Q: what is the number of parameters in this layer?

[Krizhevsky et al. 2012]



Input: 227x227x3 images After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2 Output volume: 27x27x96 Parameters: 0!

[Krizhevsky et al. 2012]

. . .



Input: 227x227x3 images After CONV1: 55x55x96 After POOL1: 27x27x96

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture: [227x227x3] INPUT [55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0 [27x27x96] MAX POOL1: 3x3 filters at stride 2 [27x27x96] NORM1: Normalization layer [27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2 [13x13x256] MAX POOL2: 3x3 filters at stride 2 [13x13x256] NORM2: Normalization layer [13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1 [13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1 [13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1 [6x6x256] MAX POOL3: 3x3 filters at stride 2 [4096] FC6: 4096 neurons [4096] FC7: 4096 neurons [1000] FC8: 1000 neurons (class scores)


Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture: [227x227x3] INPUT [55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0 [27x27x96] MAX POOL1: 3x3 filters at stride 2 [27x27x96] NORM1: Normalization layer [27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2 [13x13x256] MAX POOL2: 3x3 filters at stride 2 [13x13x256] NORM2: Normalization layer [13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1 [13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1 [13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1 [6x6x256] MAX POOL3: 3x3 filters at stride 2 [4096] FC6: 4096 neurons [4096] FC7: 4096 neurons [1000] FC8: 1000 neurons (class scores)



Details/Retrospectives: -first use of ReLU

- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- -Learning rate 1e-2, reduced by 10

manually when val accuracy plateaus

- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

slide from: Fei-Fei Li & Andrej Karpathy & Justin Johnson