Query Rewriting Under Existential Rules

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based on joint work with Pablo Barceló, Gerald Berger, Andrea Calì, Georg Gottlob, Marco Manna, Giorgio Orsi and Pierfrancesco Veltri

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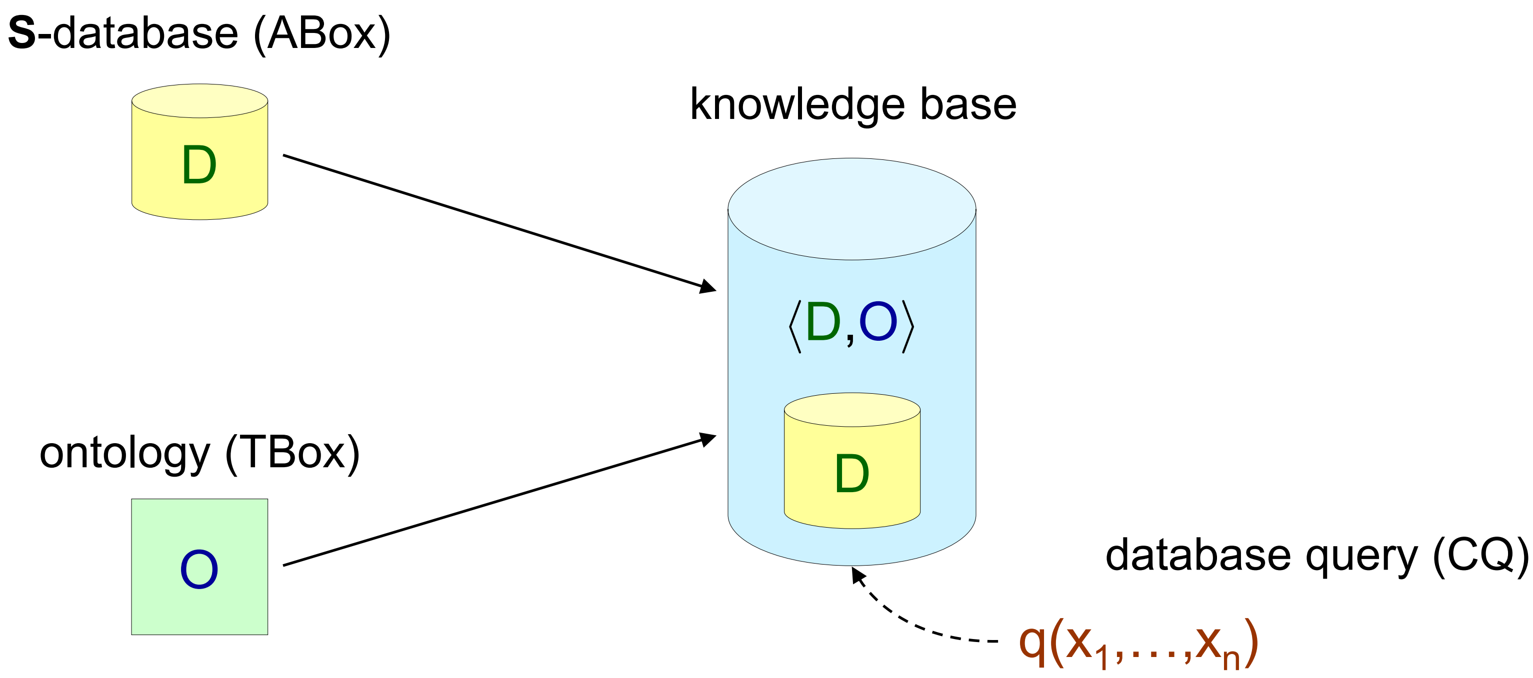
this talk is about first-order rewritability under the

basic decidable classes of existential rules

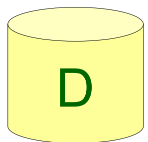
Ontology-Based Query Answering



$$\text{Certain-Answers}(q, D, O) \;=\; \{\, (c_1,\ldots,c_n) \in \text{dom}(D)^n \;\mid\; D \wedge O \models q(c_1,\ldots,c_n) \,\}$$

Ontology-Mediated Queries

S-database (ABox)



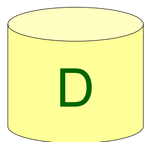
ontology-mediated query (OMQ)

$Q = (\mathbf{S}, O, q(x_1,\ldots,x_n))$

$Q(D) \;=\; \text{Certain-Answers}(q, D, O)$

Scalability in OMQ Evaluation

S-database (ABox)



ontology-mediated query (OMQ)

$Q = (\mathbf{S}, O, q(x_1,\ldots,x_n))$

?

Exploit standard RDBMSs - efficient technology for answering queries

Query Rewriting

$$Q = (\mathbf{S}, O, q(x_1,\ldots,x_n))$$

$$\Big\downarrow \text{rewrite}$$

$$Q_{rew}(x_1,\ldots,x_n)$$

a query that can be executed by a standard DBMS - **first-order query**

for every **S**-database D : $Q(D) = Q_{rew}(D)$

[Calvanese, De Giacomo, Lembo, Lenzerini & Rosati, AAAI 2005, J. Autom. Reasoning 2007]

Query Rewriting: An Example

$\{\ \forall x\ (Person(x) \rightarrow \exists y\ HasFather(x,y) \wedge Person(y)) \quad \equiv \quad Person \sqsubseteq \exists HasFather.Person\ \}$

$\{\ Person(\cdot),\ HasFather(\cdot,\cdot)\ \}$

$\exists x\ Person(x) \wedge HasFather(John,x)$

$Q = (\mathbf{S}, O, q())$

$\Big\downarrow$ rewrite

$Q_{rew} = \exists x\ Person(x) \wedge HasFather(John,x) \ \vee\ Person(John)$

First-Order Rewritability (FO-Rewritability)

(OL,QL)

an ontology language

(fragment of first-order logic)

a database query language

(sublanguage of first-order queries)

Definition: An OMQ language O is FO-Rewritable if every Q $\in$ O is FO-Rewritable

FO-Rewritability: The Main Questions

1. **Can we isolate meaningful OMQ languages that are FO-Rewritable?**

2. **For non-FO-Rewritable languages, can we decide FO-Rewritability?**

3. **What is the size of the FO rewritings? Can we do better?**

...have been extensively studied for DL- and rule-based OMQ languages

Existential Rules

(a.k.a. tuple-generating dependencies)

$$\forall \mathbf{x} \forall \mathbf{y} \, (\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z} \, \psi(\mathbf{x},\mathbf{z}))$$

$\forall x \, (\text{Person}(x) \rightarrow \exists y \, \text{HasFather}(x,y) \wedge \text{Person}(y)) \quad \equiv \quad \text{Person} \sqsubseteq \exists \text{HasFather}.\text{Person}$

$\forall x \forall y \, (\text{HasChild}(x,y) \wedge \text{Human}(y) \rightarrow \text{Human}(x)) \quad \equiv \quad \exists \text{HasChild}.\text{Human} \sqsubseteq \text{Human}$

Existential Rules

(a.k.a. tuple-generating dependencies)

$$\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\ \psi(\mathbf{x},\mathbf{z})$$

Person(x) $\rightarrow \exists$y HasFather(x,y), Person(y) ≡ Person $\sqsubseteq \exists$HasFather.Person

HasChild(x,y), Human(y) $\rightarrow$ Human(x) ≡ $\exists$HasChild.Human $\sqsubseteq$ Human

Existential Rules

(a.k.a. tuple-generating dependencies)

$$\varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\ \psi(\mathbf{x},\mathbf{z})$$

($\exists$Rules,CQ)

Guardedness

Frontier-Guarded

one body-atom contains all the $\forall$-variables in the head

$$R(\mathbf{x}), \varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\, \psi(\mathbf{x},\mathbf{z})$$

[Baget, Leclère, Mugnier & Salvat, IJCAI 2009, Artif. Intell. 2011]

↑

Guarded

one body-atom contains all the $\forall$-variables

$$R(\mathbf{x},\mathbf{y}), \varphi(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\, \psi(\mathbf{x},\mathbf{z})$$

[Calì, Gottlob & Kifer, KR 2008, J. Artif. Intell. Res. 2013]

↑

Linear

one body-atom

$$R(\mathbf{x},\mathbf{y}) \rightarrow \exists \mathbf{z}\, \psi(\mathbf{x},\mathbf{z})$$

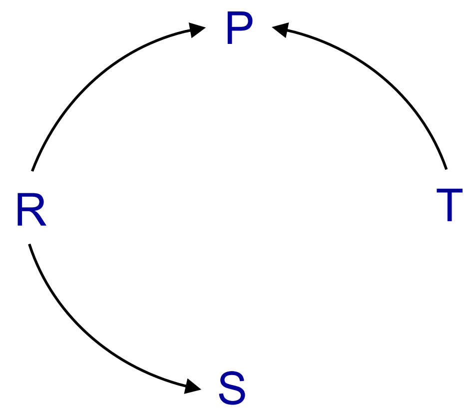
[Calì, Gottlob & Lukasiewicz, PODS 2009, J. Web Sem. 2012]

Acyclicity

(…or, non-recursive - the predicate graph is acyclic)

$$R(x,y), R(y,z) \rightarrow \exists w\, P(x), S(x,w)$$

$$T(x) \rightarrow P(x)$$



Stickiness

(…or, do not forget the joins)

$R(x,y), P(y,z) \rightarrow \exists w\, T(x,y,w)$

$T(x,y,z) \rightarrow \exists w\, S(y,w)$ ✓

$R(x,y), P(y,z) \rightarrow \exists w\, T(x,y,w)$

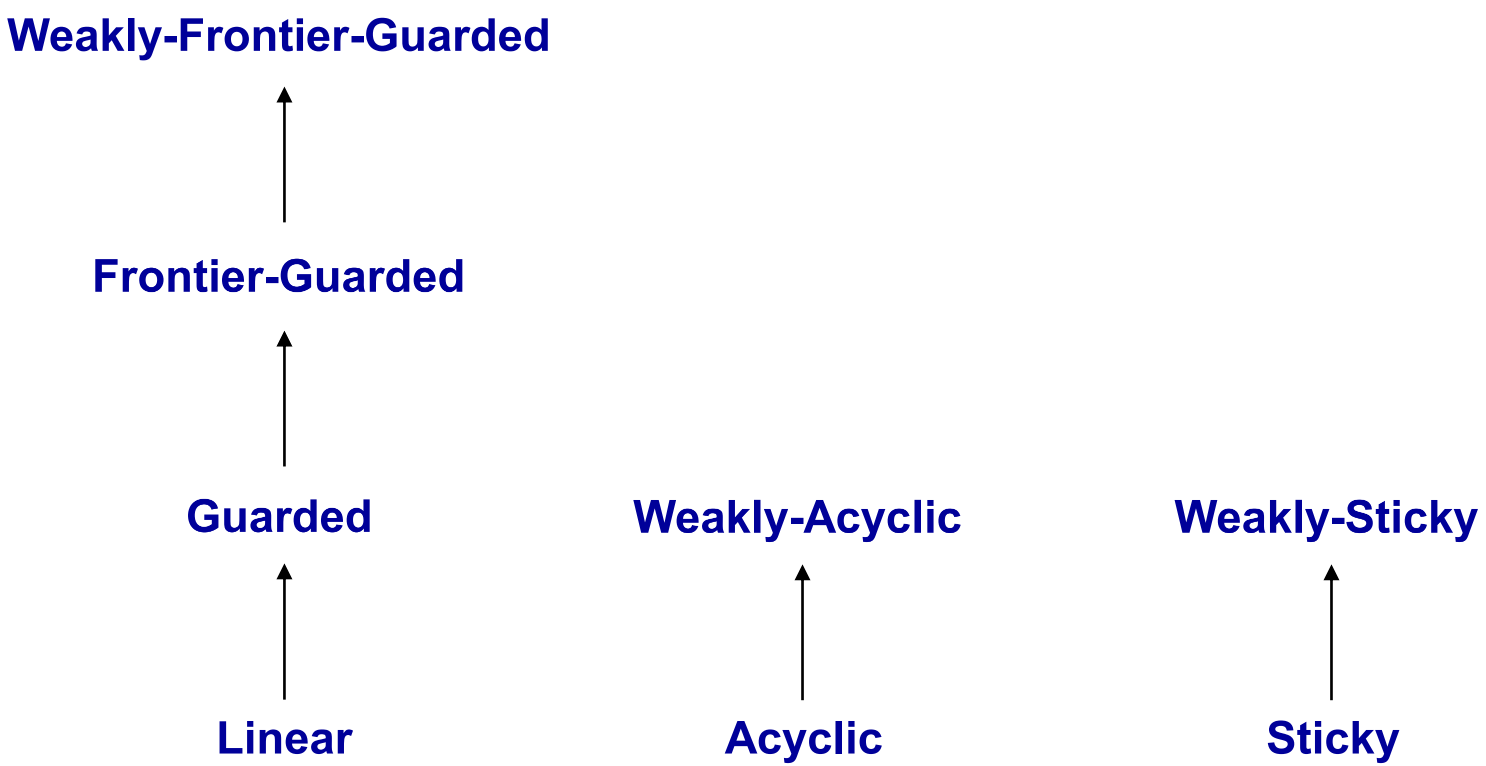
$T(x,y,z) \rightarrow \exists w\, S(x,w)$ ✗

$$R(x_1,\ldots,x_n), P(y_1,\ldots,y_m) \rightarrow T(x_1,\ldots,x_n,y_1,\ldots,y_m)$$ ✓

[Calì, Gottlob & P., PVLDB 2010, Artif. Intell. 2012]

Classes of Existential Rules

(a.k.a. Datalog$^{\pm}$ languages)



Classes of Existential Rules

(a.k.a. Datalog$^{\pm}$ languages)

What about FO-Rewritability?

Weakly-Frontier-Guarded

Frontier-Guarded

Guarded

Linear

Weakly-Acyclic

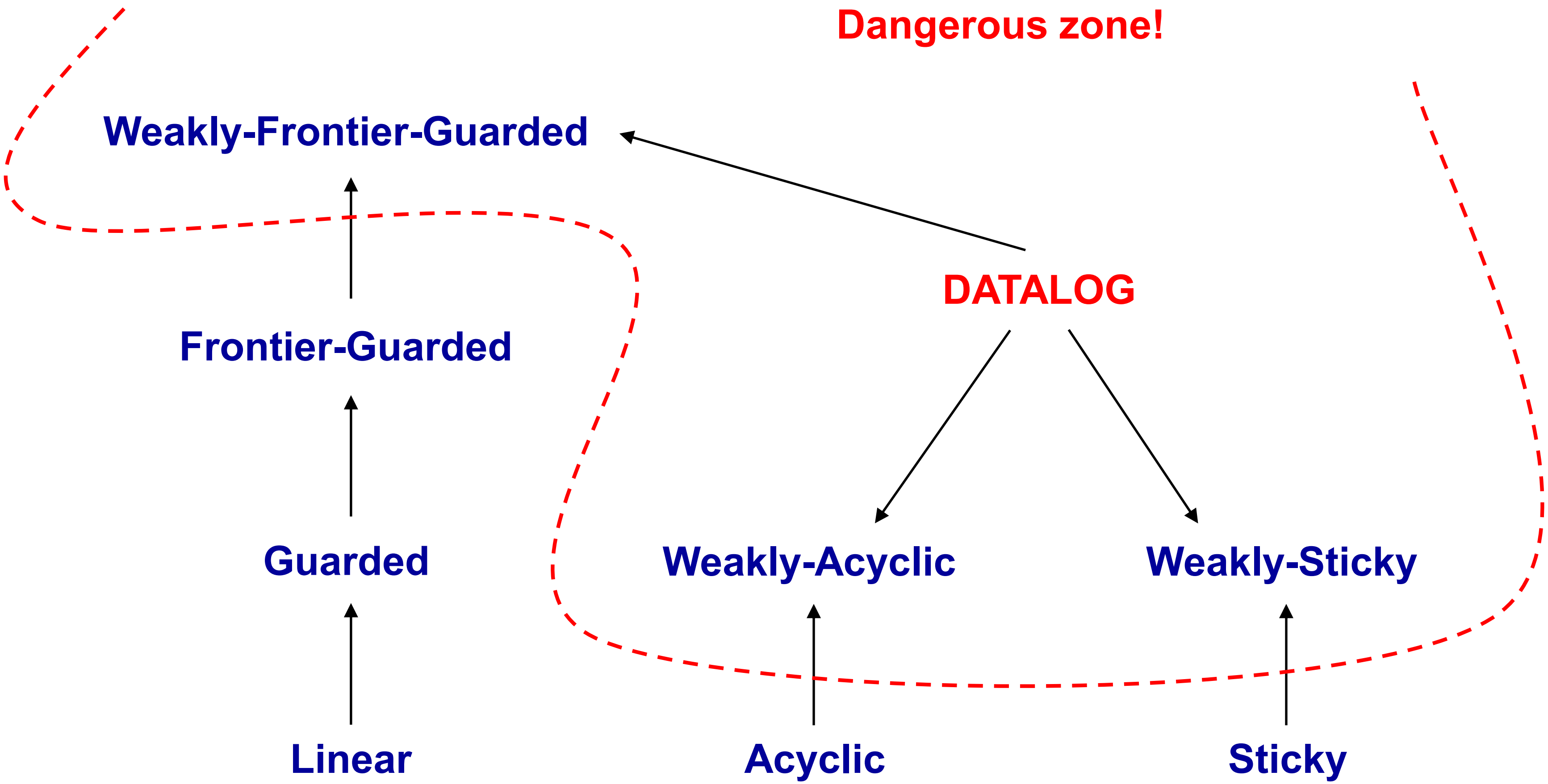
Acyclic

Weakly-Sticky

Sticky

Classes of Existential Rules

(a.k.a. Datalog$^{\pm}$ languages)



Guardedness and FO-Rewritability

Theorem: (**Guarded**,**CQ**) is not FO-Rewritable

$Q = (\{P, R\},\ \{R(x,y), P(y) \rightarrow P(x)\},\ P(c_n))$

$D \supseteq \{P(c_1)\}$, and contains no other P-atom

Q_{rew} has to check for the existence of an R-path in D of unbounded length

$$c_n \xrightarrow{R} \#_{n-1} \xrightarrow{R} \#_{n-2} \xrightarrow{R} \cdots \xrightarrow{R} \#_2 \xrightarrow{R} c_1$$

compute the transitive closure of R - not possible via a first-order query

FO-Rewritable OMQ Languages

Theorem: (**L**,**CQ**), where **L** $\in$ { **Linear**, **Acyclic**, **Sticky** }, is FO-Rewritable

Via the Bounded Derivation Depth Property (BDDP)

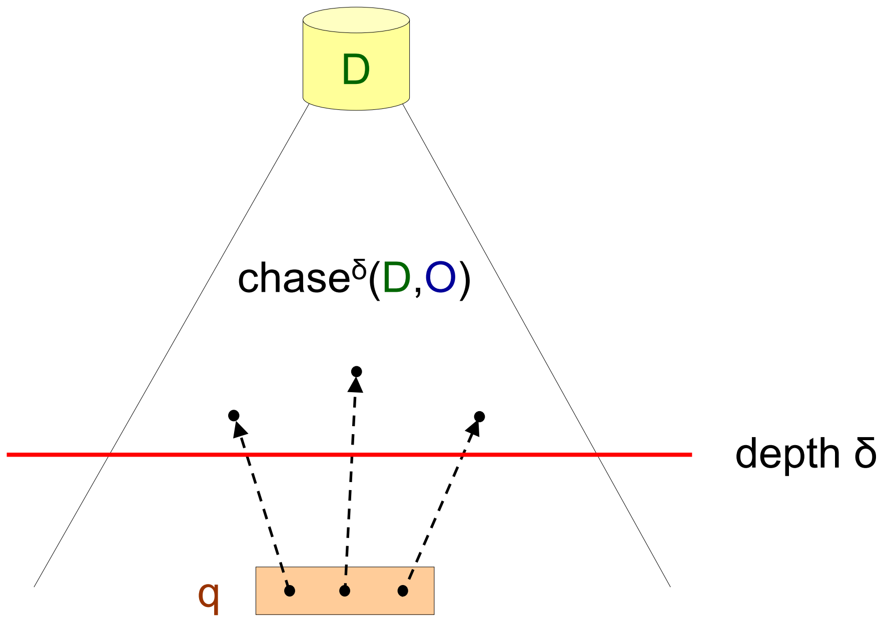
[Calì, Gottlob & Lukasiewicz, PODS 2009, J. Web Sem. 2012] + [Calì, Gottlob & P., PVLDB 2010, Artif. Intell. 2012]

Bounded Derivation Depth Property (BDDP)

Definition: (**L**,**CQ**) enjoys the BDDP if:

for every Q = (**S**, O, q) ∈ (**L**,**CQ**), there exists $\delta \geq 0$ such that,

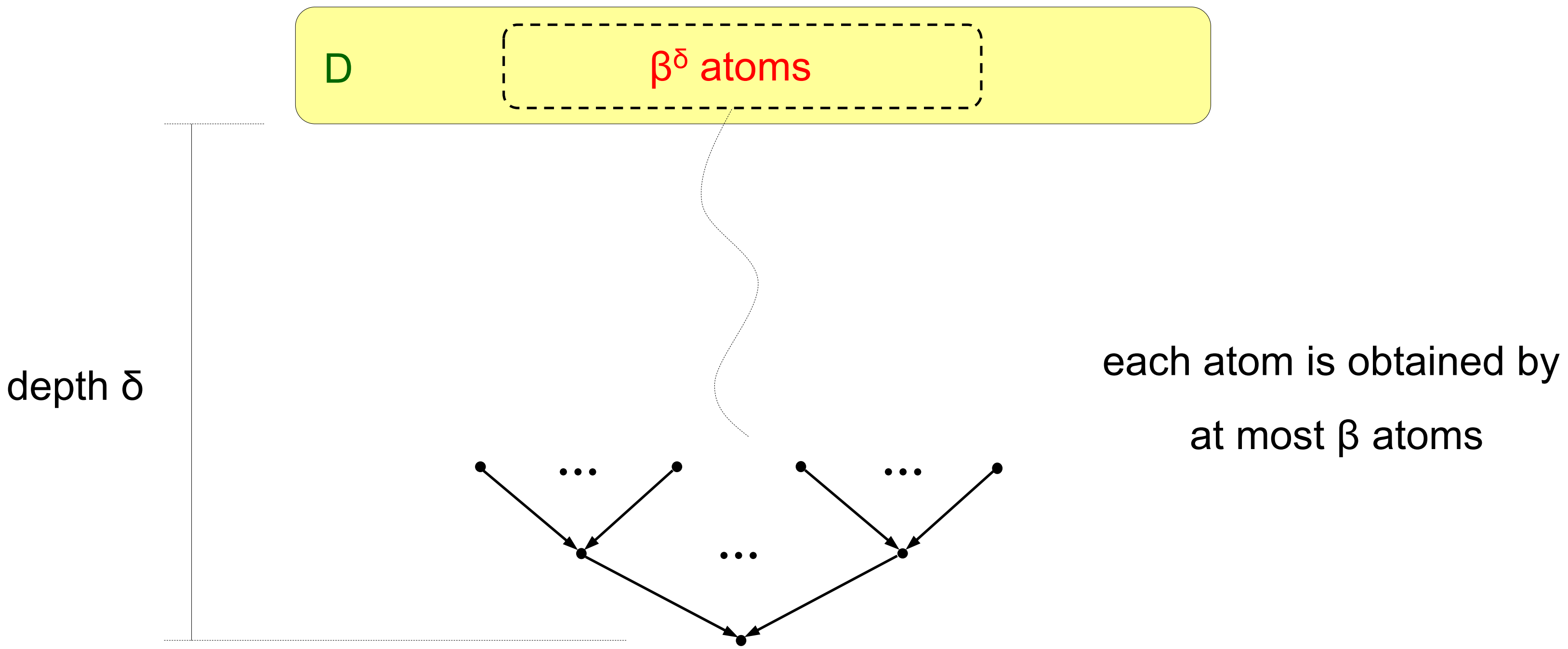
for every **S**-database D, Q(D) = q(chase$^{\delta}$(D,O))



[Calì, Gottlob & Lukasiewicz, PODS 2009, J. Web Sem. 2012]

Bounded Derivation Depth Property (BDDP)

Proposition: BDDP $\Rightarrow$ FO-Rewritability



$\Rightarrow$ to entail a CQ q we need at most $|q| \cdot \beta^{\delta}$ database atoms

Bounded Derivation Depth Property (BDDP)

Proposition: BDDP $\Rightarrow$ FO-Rewritability

Given an OMQ (**S**, O, q):

- $\mathbf{D}_{\beta,\delta,q}$ be the set of all possible **S**-databases of size at most $|q| \cdot \beta^{\delta}$
- $\mathbf{C} = \{ D \in \mathbf{D}_{\beta,\delta,q} \mid q(\mathrm{chase}(D,O)) \text{ is non-empty} \}$
- Convert **C** into a UCQ

…in fact, the other direction also holds - FO-Rewritability $\Leftrightarrow$ BDDP

FO-Rewritable OMQ Languages

Theorem: (**L**,**CQ**), where **L** $\in$ { **Linear**, **Acyclic**, **Sticky** }, is FO-Rewritable

Via the Bounded Derivation Depth Property (BDDP)

but, the BDDP-based algorithm is very expensive

can we do better?

[Calì, Gottlob & Lukasiewicz, PODS 2009, J. Web Sem. 2012] + [Calì, Gottlob & P., PVLDB 2010, Artif. Intell. 2012]

Perfect Reformulation

Fig. 2 The algorithm PerfectRef

Algorithm PerfectRef $(q, \mathcal{T})$
Input: conjunctive query q, TBox $\mathcal{T}$
Output: union of conjunctive queries PR
$PR := \{q\}$;
repeat
  $PR' := PR$;
  for each $q \in PR'$ **do**
  (a) **for each** g in q **do**
    for each PI I in $\mathcal{T}$ **do**
      if I is applicable to g
      then $PR := PR \cup \{q[g/gr(g, I)]\}$
  (b) **for each** g_1, g_2 in q **do**
      if g_1 and g_2 unify
      then $PR := PR \cup \{\tau(reduce(q, g_1, g_2))\}$;
until $PR' = PR$;
return PR

rewriting step

reduction step

Applicability → Soundness

Reduction → Completeness

[Calvanese, De Giacomo, Lembo, Lenzerini & Rosati, AAAI 2005, J. Autom. Reasoning 2007]

Perfect Reformulation for Existential Rules

$$\begin{tikzcd}
R(y,x), P(y) \rightarrow \exists z\, T(z,x,x) \arrow[dr] & & \exists u \exists v \exists w\, T(u,v,w), P(w) \arrow[dl, "{g = \{u \rightarrow z,\ v \rightarrow x,\ w \rightarrow x\}}"] \\
& T(z,x,x) &
\end{tikzcd}$$

thus, we can simulate a chase step by applying a backward resolution step

$$\exists u \exists v \exists w\, T(u,v,w), P(w) \ \vee\ \exists x \exists y\, R(y,x), P(y), P(x)$$

Perfect Reformulation for Existential Rules

$R(y,x), P(y) \rightarrow \exists z\, T(z,x,x)$ $\qquad\qquad$ $\exists u \exists v \exists w\, T(u,v,w), \mathbf{P(u)}$

$g = \{u \rightarrow z,\ v \rightarrow x,\ w \rightarrow x\}$

$T(z,x,x)$

thus, we can simulate a chase step ~~by~~ applying a backward resolution step

$\exists u \exists v \exists w\, T(u,v,w), P(u) \;\vee\; \exists x \exists y \exists u\, R(x,y), P(x), \mathbf{P(u)}$

unsound rewriting

Perfect Reformulation for Existential Rules

$$\begin{tikzcd}
R(y,x), P(y) \rightarrow \exists z\, T(z,x,x) \arrow[dr] & & \exists u \exists v \exists w\, T(u,v,w), \mathbf{P(u)} \arrow[dl, "g = \{u \rightarrow z,\ v \rightarrow x,\ w \rightarrow x\}"] \\
& T(z,x,x) &
\end{tikzcd}$$

Applicability condition: constants, join variables and free variables in the query do NOT unify with ∃-variables

…but, it may destroy completeness

Perfect Reformulation for Existential Rules

$R(y,x), P(y) \rightarrow \exists z\ T(z,x,x)$ $\exists u \exists v \exists w\ T(u,v,w), P(u)$

$T(x,y,z) \rightarrow P(x)$

$\exists u \exists v \exists w\ T(u,v,w), P(u)\ \vee$

$\exists u \exists v \exists w \exists y \exists z\ T(u,v,w), T(u,y,z)\ \vee$

(by the reduction step) $\exists u \exists v \exists w\ T(u,v,w)\ \vee$

(by the rewriting step) $\exists x \exists y\ R(x,y), P(x)$

XRewrite

ALGORITHM 1: The algorithm XRewrite

Input: a CQ q over a schema $\mathcal{R}$ and a set Σ of TGDs over $\mathcal{R}$
Output: the perfect rewriting of q w.r.t. Σ

$i := 0;$
$Q_{\text{REW}} := \{\langle q, \mathsf{r}, \mathsf{u}\rangle\};$
repeat
  $Q_{\text{TEMP}} := Q_{\text{REW}};$
  foreach $\langle q, x, \mathsf{u}\rangle \in Q_{\text{TEMP}}$, where $x \in \{\mathsf{r}, \mathsf{f}\}$ **do**
    foreach $\sigma \in \Sigma$ **do**
      /* rewriting step
      foreach $S \subseteq body(q)$ such that σ is $\boxed{\text{applicable}}$ to S **do**
        $i := i + 1;$
        $q' := \gamma_{S,\sigma^i}(q[S/body(\sigma^i)]);$
        if there is no $\langle q'', \mathsf{r}, \star\rangle \in Q_{\text{REW}}$ such that $q' \simeq q''$ **then**
          $Q_{\text{REW}} := Q_{\text{REW}} \cup \{\langle q', \mathsf{r}, \mathsf{u}\rangle\};$
        end
      end
      /* factorization step
      foreach $S \subseteq body(q)$ which is $\boxed{\text{factorizable}}$ w.r.t. σ **do**
        $q' := \gamma_S(q);$
        if there is no $\langle q'', \star, \star\rangle \in Q_{\text{REW}}$ such that $q' \simeq q''$ **then**
          $Q_{\text{REW}} := Q_{\text{REW}} \cup \{\langle q', \mathsf{f}, \mathsf{u}\rangle\};$
        end
      end
    end
    /* query q is now explored
    $Q_{\text{REW}} := (Q_{\text{REW}} \setminus \{\langle q, x, \mathsf{u}\rangle\}) \cup \{\langle q, x, \mathsf{e}\rangle\};$
  end
until $Q_{\text{TEMP}} = Q_{\text{REW}};$
$Q_{\text{FIN}} := \{q \mid \langle q, \mathsf{r}, \mathsf{e}\rangle \in Q_{\text{REW}}\};$
return Q_{FIN}

applicability condition for existential rules

apply only useful reduction steps

[Gottlob, Orsi & P., ICDE 2011, ACM Trans. Database Syst. 2014]

FO-Rewritable OMQ Languages

Theorem: (**L**,**CQ**), where **L** $\in$ { **Linear**, **Acyclic**, **Sticky** }, is FO-Rewritable

Via the Bounded Derivation Depth Property (BDDP)

but, the BDDP-based algorithm is very expensive

can we do better?

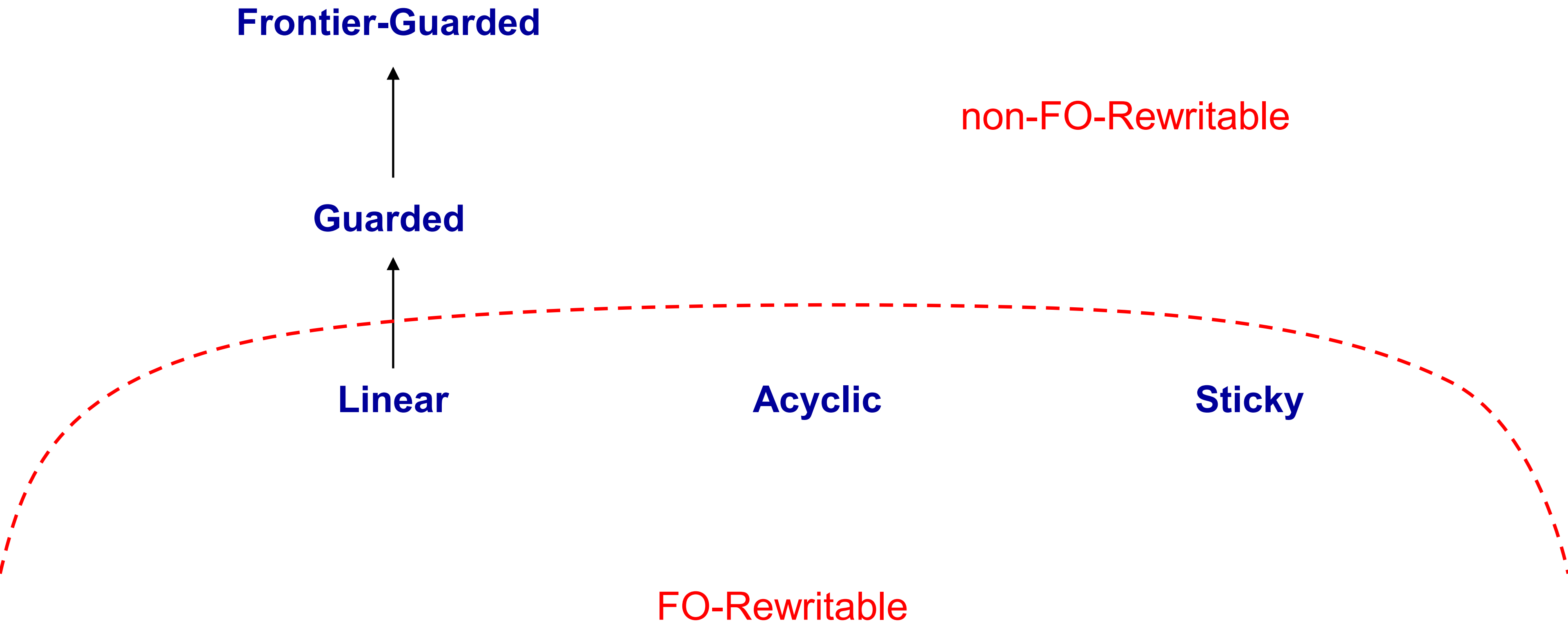
use the XRewrite algorithm

Piece-based rewriting - based on a refined notion of unification

[König, Leclère, Mugnier & Thomazo, RR 2012, Semantic Web 2015]

Recap

What about deciding FO-Rewritability?



Deciding FO-Rewritability

$\{\, R(x,y),\ S(y) \rightarrow S(x),\quad R(x,y),\ P(x) \rightarrow S(y) \,\}$

$\{\, P(\cdot),\ R(\cdot,\cdot),\ S(\cdot) \,\}$

$P(x) \wedge R(x,y) \wedge S(y)$

$Q = (\mathbf{S},\ O,\ q(x,y))$

rewrite

$P(x) \wedge R(x,y)$

Deciding FO-Rewritability

$\{ R(x,y), S(y) \rightarrow S(x), \quad R(x,y), P(x) \rightarrow S(y) \}$

$\{ P(\cdot), R(\cdot,\cdot), S(\cdot) \}$

$S(y)$

$Q = (\mathbf{S}, O, q(y))$

rewrite

✘

Deciding FO-Rewritability

FORew(L,QL)

Input: an OMQ Q $\in$ (**L**,**QL**)

Question: is Q FO-Rewritable?

What is the complexity of **FORew(Guarded,CQ)** and **FORew(Frontier-Guarded,CQ)**?

☹

Deciding FO-Rewritability

FORew(L,QL)

Input: an OMQ Q $\in$ (**L**,**QL**)

Question: is Q FO-Rewritable?

Theorem: FORew(L,CQ), where **L** $\in$ { **Guarded**, **Frontier-Guarded** } is in 3EXPTIME, and 2EXPTIME-hard even for bounded arity

[Barceló, Berger & P., 2017]

Deciding FO-Rewritability

Theorem: FORew(Guarded,BCQ) is in 3EXPTIME and 2EXPTIME-hard even for bounded arity

Upper Bound:

- Characterize FO-Rewritability via the finiteness of a set of certain "tree-like" databases

- Construct an alternating tree automaton **A**, with double-exponentially many states, such that the OMQ is FO-Rewritable iff the language of **A** is finite

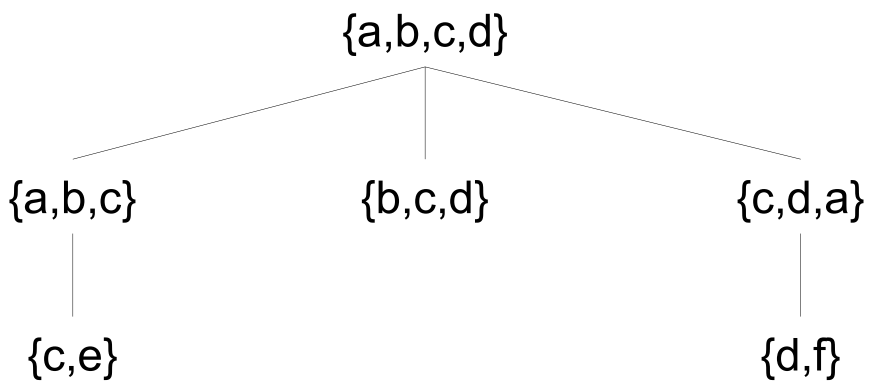
Lower Bound:

- Inherited from **FORew(ELI,CQ)**
  [Bienvenu, Hansen, Lutz & Wolter, IJCAI 2016]

[Barceló, Berger & P., 2017]

Tree Decomposition

D = { R(a,b,c), T(c,e), R(b,c,d), S(c,d,a), P(d,f), T(f,f) }



Tree Decomposition

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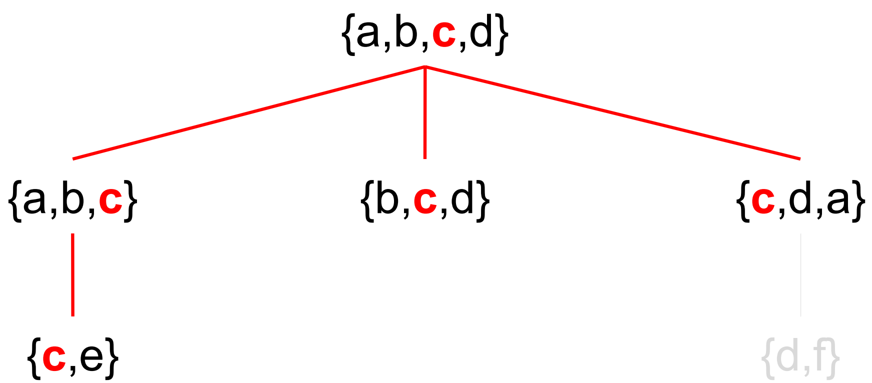

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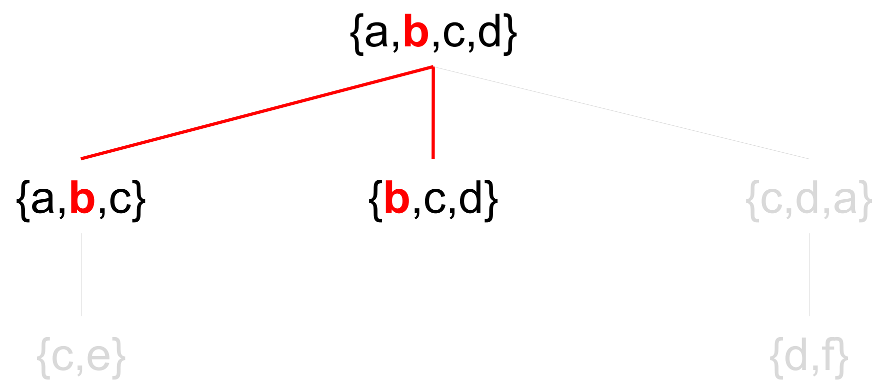

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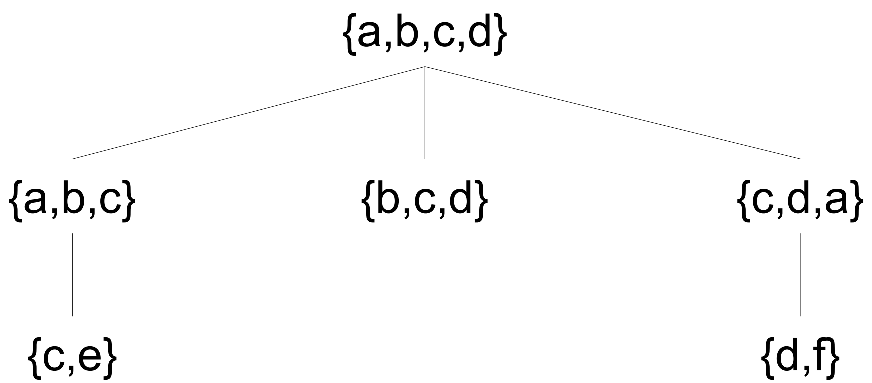

Tree Decomposition

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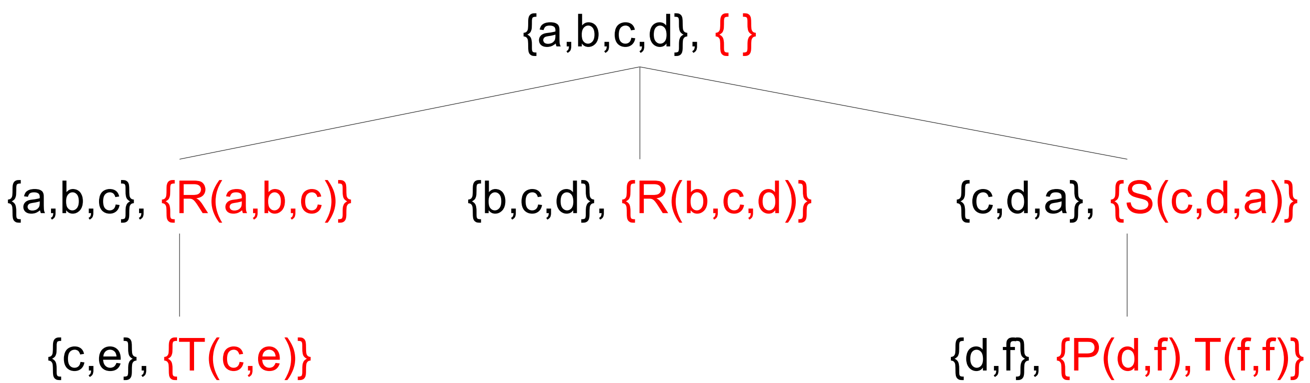
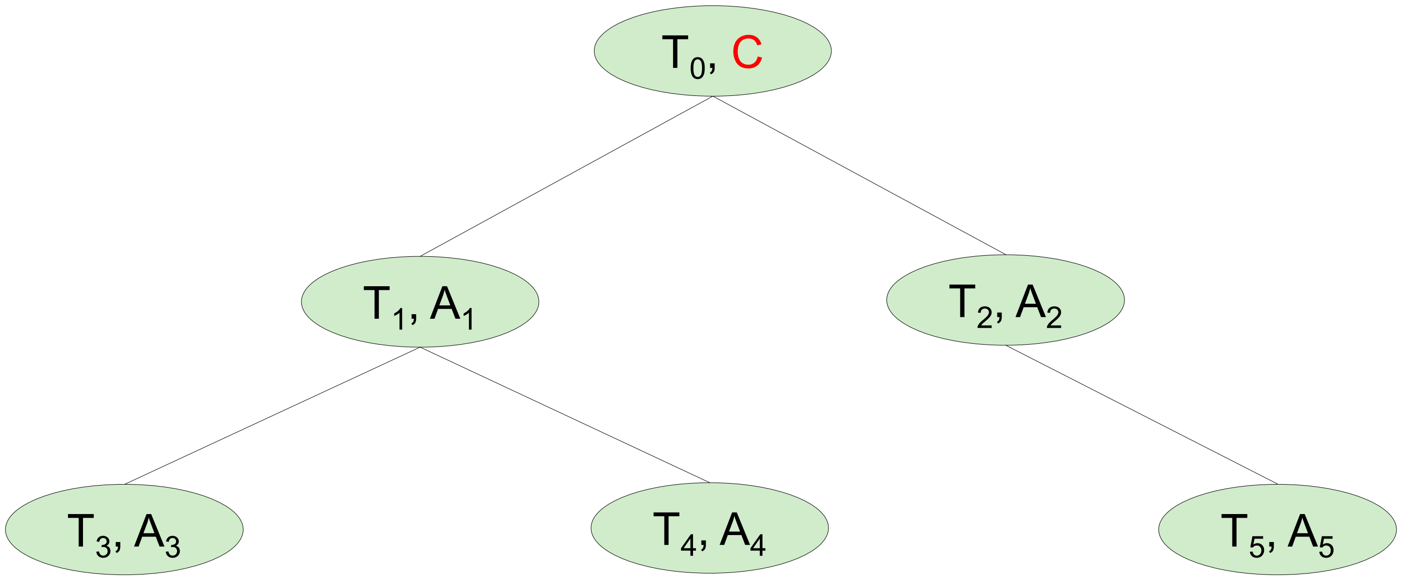


C-Tree Databases

(…or, almost "tree-like" databases)

Definition: An **S**-database D is a C-tree, where C $\subseteq$ D, if it has the form:



for each i > 0, $|T_i| \leq$ arity(**S**)

Characterizing FO-Rewritability

Proposition: Let Q = (**S**, O, q) $\in$ (**Guarded**,**BCQ**):

Q is FO-Rewritable

unravelling and compactness $\uparrow$ $\Updownarrow$ $\downarrow$ Q is UCQ-Rewritable

there exists k ≥ 0 such that, for every C-tree D over **S**,

with |dom(C)| ≤ (arity(**S**,O) $\cdot$ |q|), it holds that:

D $\models$ Q $\Rightarrow$ there exists D' $\subseteq$ D with |D'| ≤ k such that D' $\models$ Q

Characterizing FO-Rewritability

Proposition: Let Q = (**S**, O, q) $\in$ (**Guarded**,**BCQ**):

Q is FO-Rewritable

$\Updownarrow$

there exist finitely many (non-isomorphic)

C-trees D over **S**, with |dom(C)| ≤ (arity(**S**,O) $\cdot$ |q|), such that:

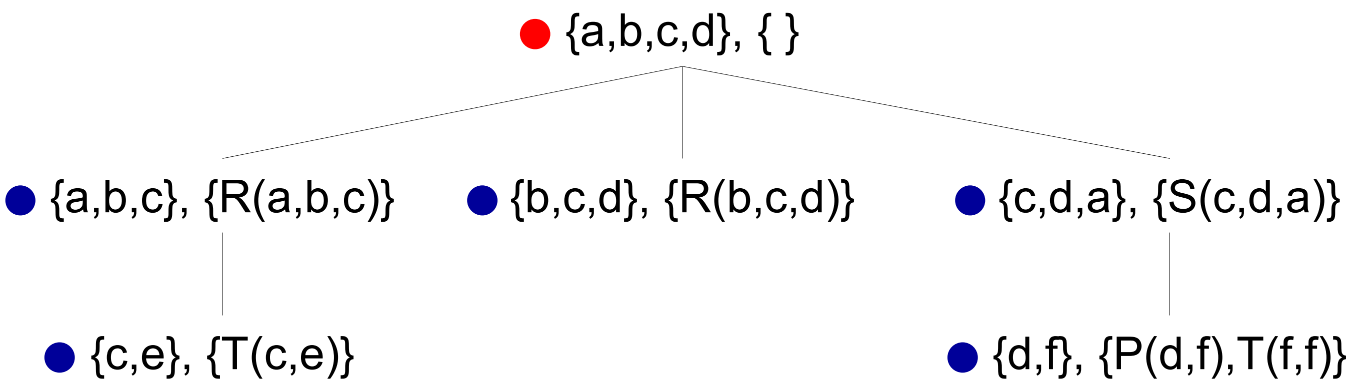
(i) D $\models$ Q

(ii) remove an atom from D $\Rightarrow$ Q is violated

(iii) D is non-redundant

Well-Colored Tree Decomposition

D = { R(a,b,c), T(c,e), R(b,c,d), S(c,d,a), P(d,f), T(f,f) }



node v is red $\Rightarrow$ v is the least common ancestor of a non-empty set of blue nodes

Characterizing FO-Rewritability

Proposition: Let Q = (**S**, O, q) $\in$ (**Guarded**,**BCQ**):

Q is FO-Rewritable

$\Updownarrow$

there exist finitely many (non-isomorphic)

C-trees D over **S**, with |dom(C)| $\leq$ (arity(**S**,O) $\cdot$ |q|), such that:

(i) D $\models$ Q

(ii) remove an atom from D $\Rightarrow$ Q is violated

(iii) D is well-colored

the language of an alternating tree automaton **A**

with double-exponentially many states

Characterizing FO-Rewritability

Proposition: Let Q = (**S**, O, q) $\in$ (**Guarded**,**BCQ**):

Q is FO-Rewritable

$\Updownarrow$

the language of **A** is finite

(which is feasible in exponential time in the number of states)

Deciding FO-Rewritability

Theorem: FORew(Frontier-Guarded,BCQ) is in 3EXPTIME

Q $\in$ (**Frontier-Guarded**,**BCQ**)

a BCQ is a frontier-guarded rule

Q' $\in$ (**Frontier-Guarded**,**BAQ**)

by treeifying the rule-bodies

[Bárány, ten Cate & Segoufin, ICALP 2011, J. ACM 2015]

Q'' $\in$ (**Guarded**,**BAQ**)

Q is FO-Rewritable $\Leftrightarrow$ Q'' is FO-Rewritable

[Barceló, Berger & P., 2017]

Deciding FO-Rewritability: Next Steps

- Practical rewriting algorithms for (**Frontier-Guarded**,**CQ**)

- Such a practical algorithm exists for (**EL**,**AQ**)

  [Hansen, Lutz, Seylan & Wolter, IJCAI 2015]

- …and it has been recently extended to (**EL**,**CQ**)

  [Hansen & Lutz, DL 2017]

Recap

What about the size of the FO rewritings?

Frontier-Guarded

can be checked in 3EXPTIME

Guarded

Linear **Acyclic** **Sticky**

FO-Rewritable

Height/Size of XRewrite(Q)

Given an OMQ $Q = (\mathbf{S}, O, q) \in (\mathbf{L}, \mathbf{CQ})$

worst-case optimal

L	Height	Size
Linear	$\lvert q \rvert$	$\lvert \mathbf{S} \rvert^{\lvert q \rvert} \cdot (\mathrm{arity}(\mathbf{S}) \cdot \lvert q \rvert)^{\mathrm{arity}(\mathbf{S}) \cdot \lvert q \rvert}$
Acyclic	$\lvert q \rvert \cdot \mathrm{body}(O)^{\#\mathrm{pred}(O)}$	$2\hat{}(\lvert \mathbf{S} \rvert \cdot (\lvert q \rvert \cdot \mathrm{body}(O)^{\#\mathrm{pred}(O)} \cdot \mathrm{arity}(\mathbf{S}))^{\mathrm{arity}(\mathbf{S})})$
Sticky	$\lvert \mathbf{S} \rvert \cdot (\#\mathrm{terms}(q) + 1)^{\mathrm{arity}(\mathbf{S})}$	$2\hat{}(\lvert \mathbf{S} \rvert \cdot (\#\mathrm{terms}(q) + 1)^{\mathrm{arity}(\mathbf{S})})$

- **Linear**: the rewriting step replaces an atom with one atom
- **Acyclic**: the rewriting can be seen as a tree of depth at most #pred(O)
- **Sticky**: only variables of q occur more than once in a disjunct

Upper/Lower Bound for **Frontier-Guarded**

- The automata-based approach provides a UCQ-rewriting - disjunction of the trees accepted by the automaton (very large - 5EXP)

- Triple-exponential lower bound for the size of UCQ-rewritings for (**EL**,**CQ**)

  [Bienvenu, Lutz & Wolter, IJCAI 2013]

Target More Succinct Query Languages

In particular, what about

- Positive existential queries (PE)
- Non-recursive Datalog queries (NDL)
- First-order queries (FO)

Even for (**DL-Lite$_R$**,**CQ**)

- No PE/NDL-rewriting of polynomial size
- No FO-rewriting of polynomial size (unless the PH collapses)

…it holds even for (**Acyclic**,**CQ**)

[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014]

FO-Rewritability: Pure Approach

Two crucial limitations:

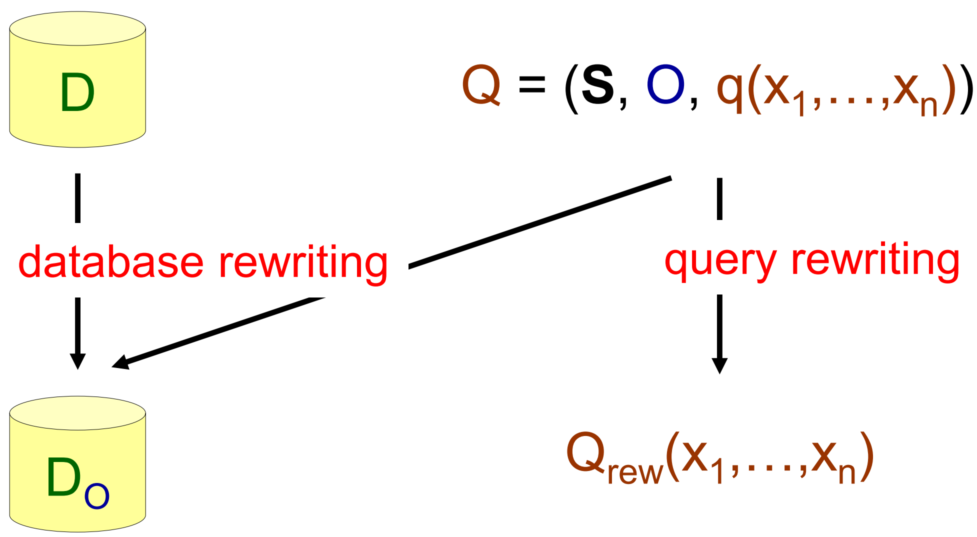
- No small rewritings - even for lightweight languages like **Linear** or **DL-Lite$_R$**

- Simple OMQs are immediately excluded, e.g.,

({HasChild, Human}, {HasChild(x,y), Human(y) $\rightarrow$ Human(x)}, Human(x))

a more refined approach is needed

FO-Rewritability: Combined Approach



both steps in polynomial time!!!

for every **S**-database D : Q(D) = $Q_{rew}(D_O)$

[Lutz, Toman & Wolter, IJCAI 2009]

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	**Linear**	**Acyclic**	**Sticky**	**Guarded**	**Fr-Guarded**
∞	∞	✓	[×]	[[×]]	×	×
∞	$\leq k$	✓	[×]	✓	[[×]]	×
$\leq k$	∞	✓	✓	[[×]]	×	×
$\leq k$	$\leq k$	✓	✓	✓	✓	?

[×] - assuming PSPACE $\neq$ NEXPTIME

[[×]] - assuming PSPACE $\neq$ EXPTIME

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	Linear	Acyclic	Sticky	Guarded	Fr-Guarded
∞	∞	✓	[×]	[[×]]	×	×
∞	≤ k	✓	[×]	✓	[[×]]	×
≤ k	∞	✓	✓	[[×]]	×	×
≤ k	≤ k	✓	✓	✓	✓	?

via the Polynomial Witness Property

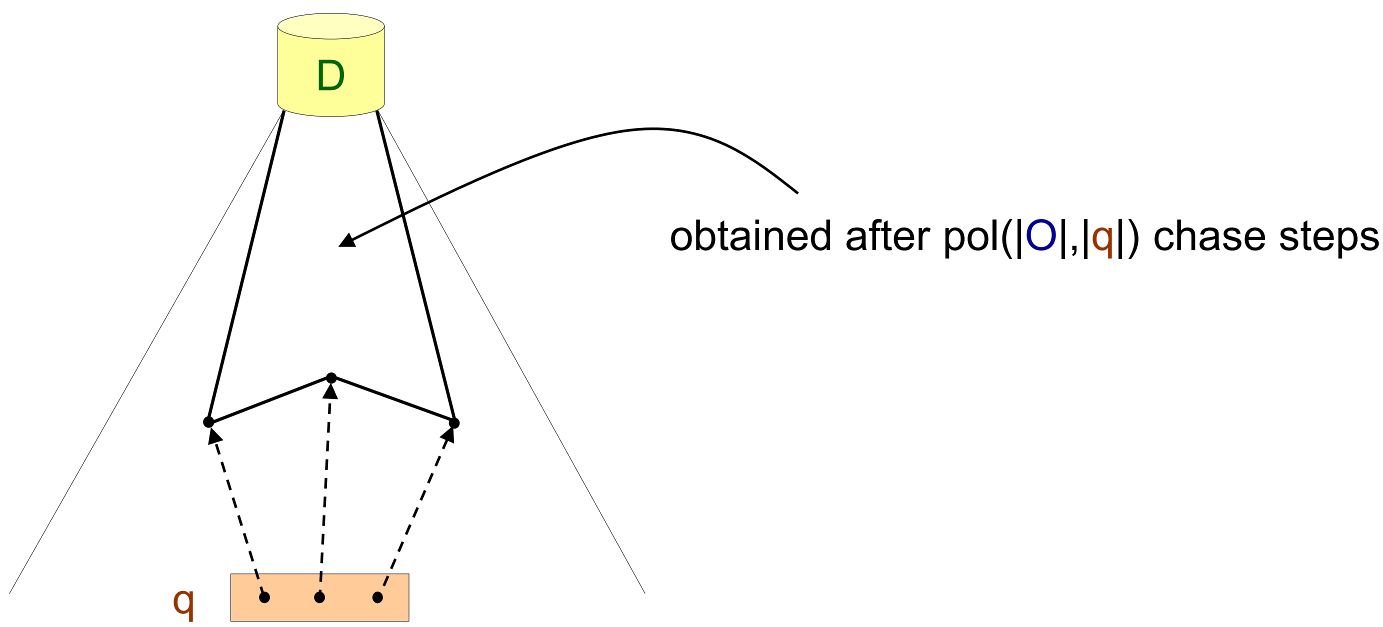
[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014] + [Gottlob, Manna & P., KR 2014]

Polynomial Witness Property (PWP)

Definition: (**L**,**CQ**) enjoys the PWP if there exists a polynomial pol(·) such that

for every Q = (**S**, O, q(**x**)) ∈ (**L**,**CQ**), **S**-database D, and **t** ∈ dom(D)$^{|\mathbf{x}|}$

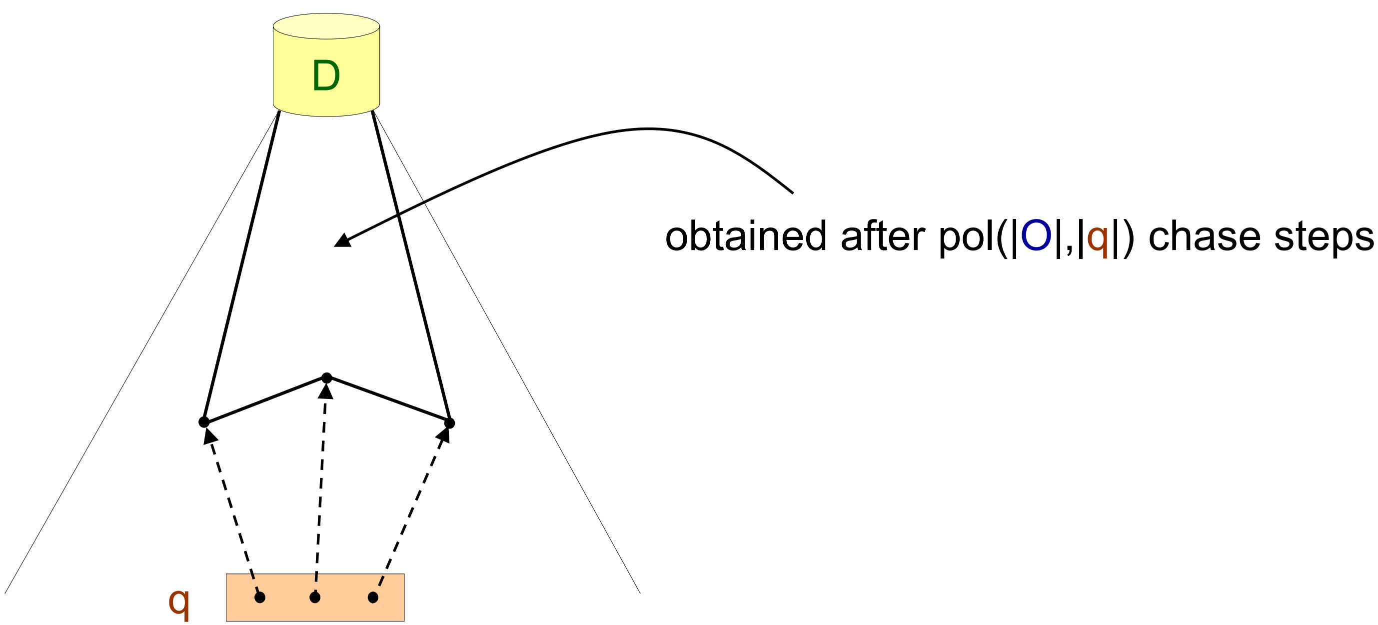
t ∈ Q(D) ⇒ q(**t**) can be entailed after pol(|O|,|q|) chase steps



[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014]

Polynomial Witness Property (PWP)

Proposition: PWP $\Rightarrow$ PE/NDL-rewritings constructible in polynomial time, assuming databases with at least two constants



[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014]

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	**Linear**	**Acyclic**	**Sticky**	**Guarded**	**Fr-Guarded**
∞	∞	✓	[×]	[[×]]	×	×
∞	≤ k	✓	[×]	✓	[[×]]	×
≤ k	∞	✓	✓	[[×]]	×	×
≤ k	≤ k	✓	✓	✓	✓	?

via the Polynomial Witness Property

[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014] + [Gottlob, Manna & P., KR 2014]

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	Linear	Acyclic	Sticky	Guarded	Fr-Guarded
∞	∞	✓	[×]	[[×]]	×	×
∞	≤ k	✓	[×]	✓	[[×]]	×
≤ k	∞	✓	✓	[[×]]	×	×
≤ k	≤ k	✓	✓	✓	✓	?

via the Polynomial Witness Property?

[Gottlob, Kikot, Kontchakov, Podolskii, Schwentick & Zakharyaschev, Artif. Intell. 2014] + [Gottlob, Manna & P., KR 2014]

FO-Rewritability: Combined Approach

schema assumptions

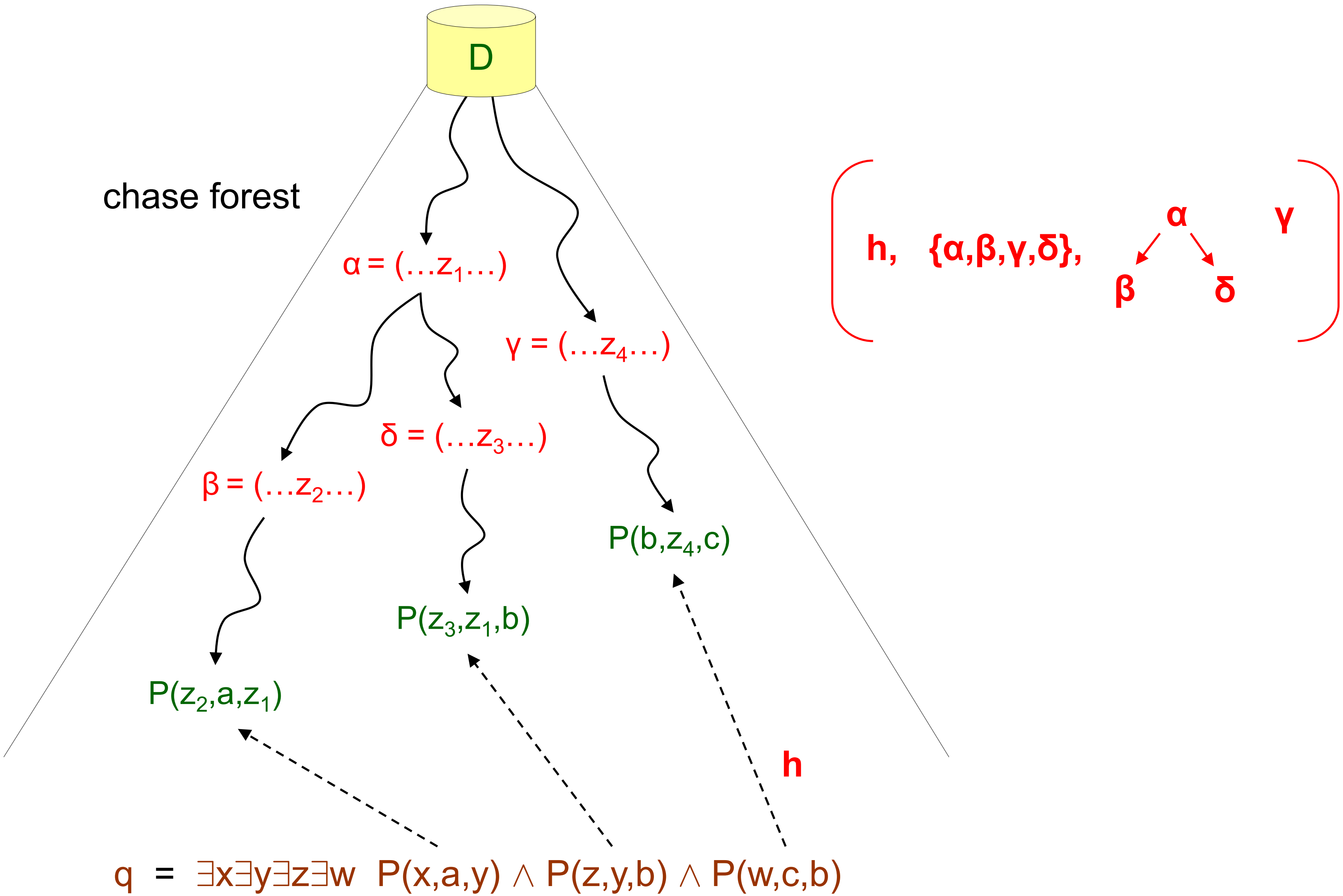
Size	Arity	Linear	Acyclic	Sticky	Guarded	Fr-Guarded
∞	∞	✓	[×]	[[×]]	×	×
∞	≤ k	✓	[×]	✓	[[×]]	×
≤ k	∞	✓	✓	[[×]]	×	×
≤ k	≤ k	✓	✓	✓	✓	?

via proof generators

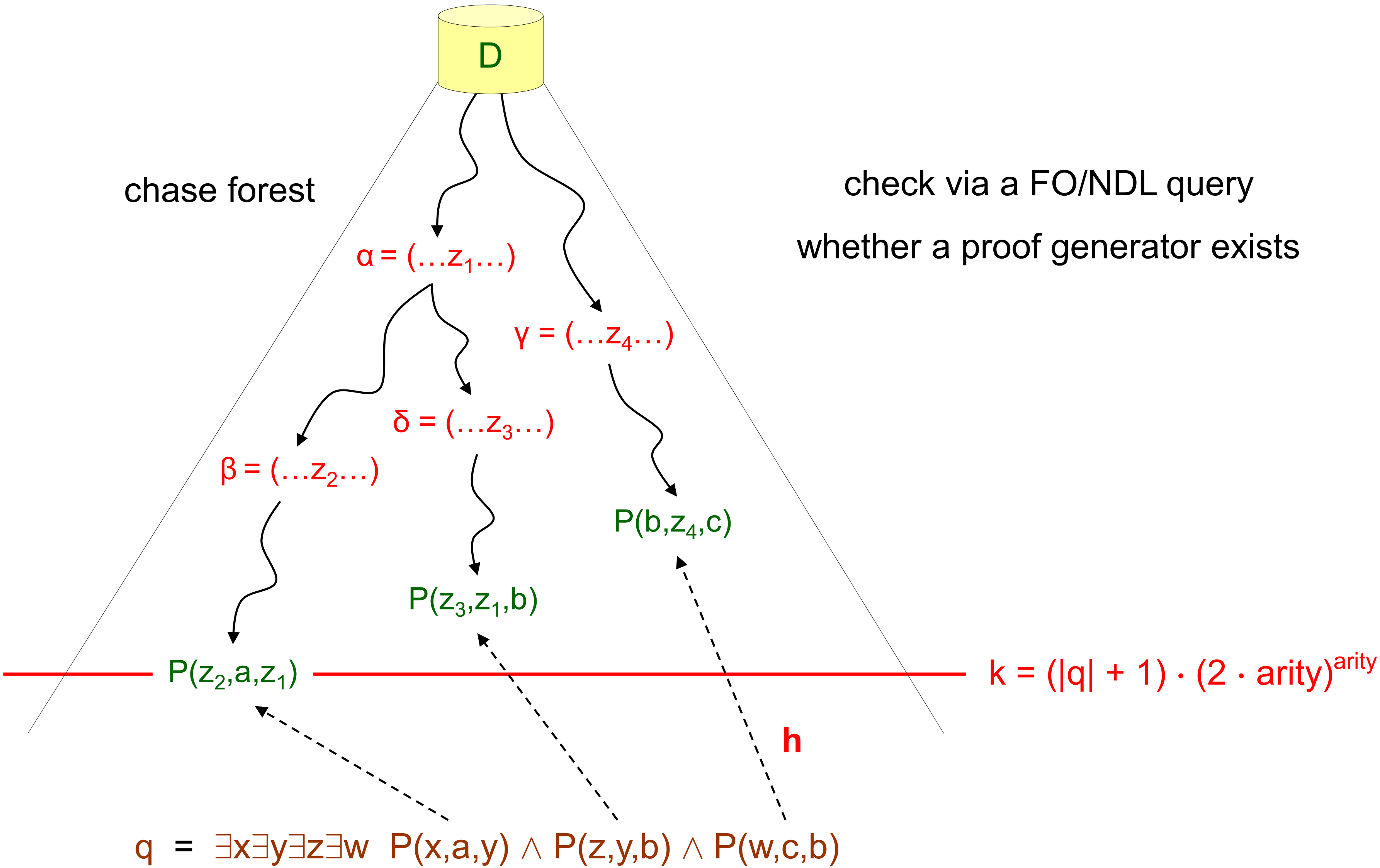
a compact representation of an exponentially-sized witness

[Gottlob, Manna & P., IJCAI 2015]

Proof Generator



Proof Generator



FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	Linear	Acyclic	Sticky	Guarded	Fr-Guarded
∞	∞	✓	[×]	[[×]]	×	×
∞	$\leq k$	✓	[×]	✓	[[×]]	×
$\leq k$	∞	✓	✓	[[×]]	×	×
$\leq k$	$\leq k$	✓	✓	✓	✓	?

a unique positive case without polynomially-sized witnesses

[Gottlob, Manna & P., IJCAI 2015]

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	Linear	Acyclic	Sticky	Guarded	Fr-Guarded
∞	∞	✓	[×]	[[×]]	×	×
∞	≤ k	✓	[×]	✓	[[×]]	×
≤ k	∞	✓	✓	[[×]]	×	×
≤ k	≤ k	✓	✓	✓	✓	?

via linearization

encode the type of the guard-atom in a single predicate

[Gottlob, Manna & P., KR 2014]

FO-Rewritability: Combined Approach

schema assumptions

Size	Arity	**Linear**	**Acyclic**	**Sticky**	**Guarded**	**Fr-Guarded**
∞	∞	✓	[×]	[[×]]	×	×
∞	$\leq k$	✓	[×]	✓	[[×]]	×
$\leq k$	∞	✓	✓	[[×]]	×	×
$\leq k$	$\leq k$	✓	✓	✓	✓	?

fixing the schema is not enough

we should fix the ontology, and then adapt the linearization technique

[Thomazo, Personal Communication 2017]

Some Final Remarks

- **FO-Rewritable languages**
  - Practical resolution-based algorithms exist (XRewrite, Piece-based rewriting)
  - Prototype systems exist (Nyaya, Graal)
- **Far from practical algorithms for checking FO rewritability**
  - Notable exception the algorithm for (**EL**,**CQ**)
  - Prototype system Grind
- **Polynomial combined FO rewriting algorithms are of theoretical nature**
  - Can we construct compact UCQs?

Thank you!