

A-posteriori-steered and adaptive p -robust multigrid solvers

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Abstract

We study a symmetric second-order linear elliptic PDE discretized by piecewise polynomials of arbitrary degree $p \geq 1$. To treat the arising linear system, we propose a geometric multigrid method with zero pre- and one post-smoothing by an overlapping Schwarz (block Jacobi) method [Miraçi, Papež, Vohralík. SIAM J. Sci. Comput. 2021]. Introducing optimal step sizes which minimize the algebraic error in the level-wise error correction step of multigrid, one obtains an explicit Pythagorean formula for the algebraic error. Importantly, this inherently induces a fully computable a posteriori estimator for the energy norm of the algebraic error. We show the two following results and their equivalence: 1) the solver contracts the algebraic error independently of the polynomial degree p ; 2) the estimator represents a two-sided p -robust bound on the algebraic error. The p -robustness results are obtained by carefully applying the results of [Schöberl, Melenk, Pechstein, Zaglmayr. IMA J. Numer. Anal. 2008] for one mesh, combined with a multilevel stable decomposition for piecewise affine polynomials of [Chen, Nochetto, Xu. Numerical. Math. 2012]. Two adaptive variants of this approach are also presented: a multigrid solver with an adaptive number of smoothing steps per level and a multigrid solver with adaptive local smoothing per patches. Moreover, recent developments in [Miraçi, Praetorius, Streitberger. In preparation] allow to prove that a local variant of the solver is robust also with respect to the number of mesh levels used for rate-optimal adaptive finite element method. Finally, we present a variety of numerical tests to confirm the theoretical results and to illustrate the advantages of our approach.

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