Reliable, efficient, and robust a posteriori estimates for nonlinear elliptic problems. An orthogonal decomposition result based on iterative linearization

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Abstract

We consider strongly monotone and Lipschitz-continuous nonlinear elliptic problems. We apply a finite element discretization in conjunction with an iterative linearization such as the fixed-point scheme or the Newton scheme. In this setting, we derive a posteriori error estimates that are robust with respect to the ratio of the continuity over monotonicity constants in the dual energy norm invoked by the linearization iterations. This is linked to an orthogonal decomposition of the total error into a linearization error component and a discretization iterations for efficient error balancing. The applications cover diverse physical phenomena such as mean curvature flow, flow through porous media, and biological processes. Numerical experiments for the time-discrete Richards equation illustrate the theoretical results.

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