Numerical approximation of the spectrum of self-adjoint operators and operator preconditioning

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Abstract

We consider operator preconditioning $\mathcal{B}^{-1}\mathcal{A}$, which is employed in the numerical solution of boundary value problems. Here, the self-adjoint operators $\mathcal{A}, \mathcal{B} : H_0^1(\Omega) \to H^{-1}(\Omega)$ are the standard integral/functional representations of the partial differential operators $-\nabla \cdot (k(x)\nabla u)$ and $-\nabla \cdot (g(x)\nabla u)$, respectively, and the scalar coefficient functions $k(x)$ and $g(x)$ are assumed to be (piecewise) continuous throughout the closure of the solution domain. The function $g(x)$ is also assumed to be uniformly positive. When the discretized problem, with the preconditioned operator $\mathcal{B}_n^{-1}\mathcal{A}_n$, is solved with Krylov subspace methods, the convergence behavior depends on the distribution of the eigenvalues. Therefore it is crucial to understand how the eigenvalues of $\mathcal{B}_n^{-1}\mathcal{A}_n$ are related to the spectrum of $\mathcal{B}^{-1}\mathcal{A}$. Following the path started in the two recent papers published in SIAM J. Numer. Anal. [57 (2019), pp. 1369-1394 and 58 (2020), pp. 2193-2211], this talk also addresses the open question concerning the distribution of the eigenvalues of $\mathcal{B}_n^{-1}\mathcal{A}_n$ formulated at the end of the second paper.

The presented spectral approximation problem includes the continuous part of the spectrum and it differs from the eigenvalue problem studied in the classical PDE literature which addresses compact (solution) operators.

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