Computational Methods for Blood flow

Associated Team "Cardio"

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Project-team REO
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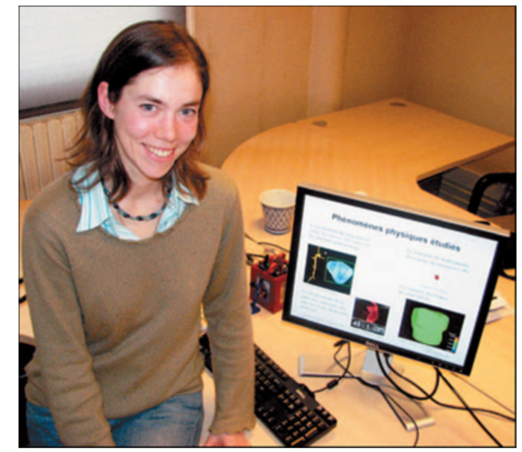
Associated Team “Cardio”

Website: https://idal-siege.inria.fr/cardio/



Coordinator

- Irène Vignon-Clementel

Members

- **Stanford** (CVBRL, Dept Bioengineering & Mech Engng)
  C. Taylor, A. Figueroa, N. Xiao, G. Troianowski,
  J Feinstein, A Marsden, S Shadden, J Kim, R. Raghu
- **INRIA**:
  - Project-team REO: J-F. Gerbeau, M. Fernández, I. Vignon-Clementel, M. Astorino, C. Bertoglio, G. Troianowski
  - Project-team MACS: D. Chapelle, P. Moireau

STANFORD UNIVERSITY
Cardiovascular Biomechanics Research Laboratory

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Similar goals and complementary approaches

- Modeling the blood flow in large arteries
- Interaction simulation / medical data

Collaboration themes

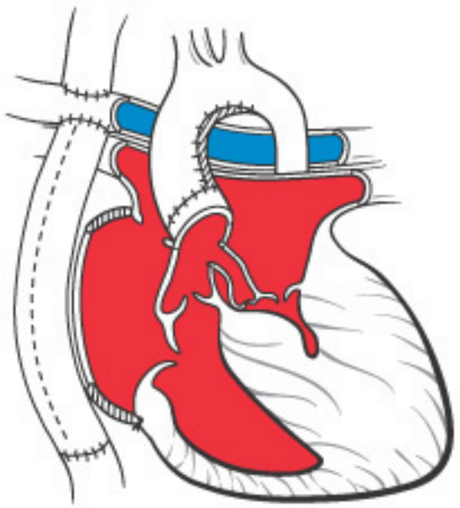
- Boundary conditions (fluid & fluid-structure)
- Advance post-processing techniques
- Image-based fluid-structure interaction

Outline

- Surgical planning
- Fluid-Structure Interaction in blood flows
- Medical Data assimilation / Inverse problems
- *Viscoelasticity (Rashmi Raghu, Stanford)*

Surgical planning

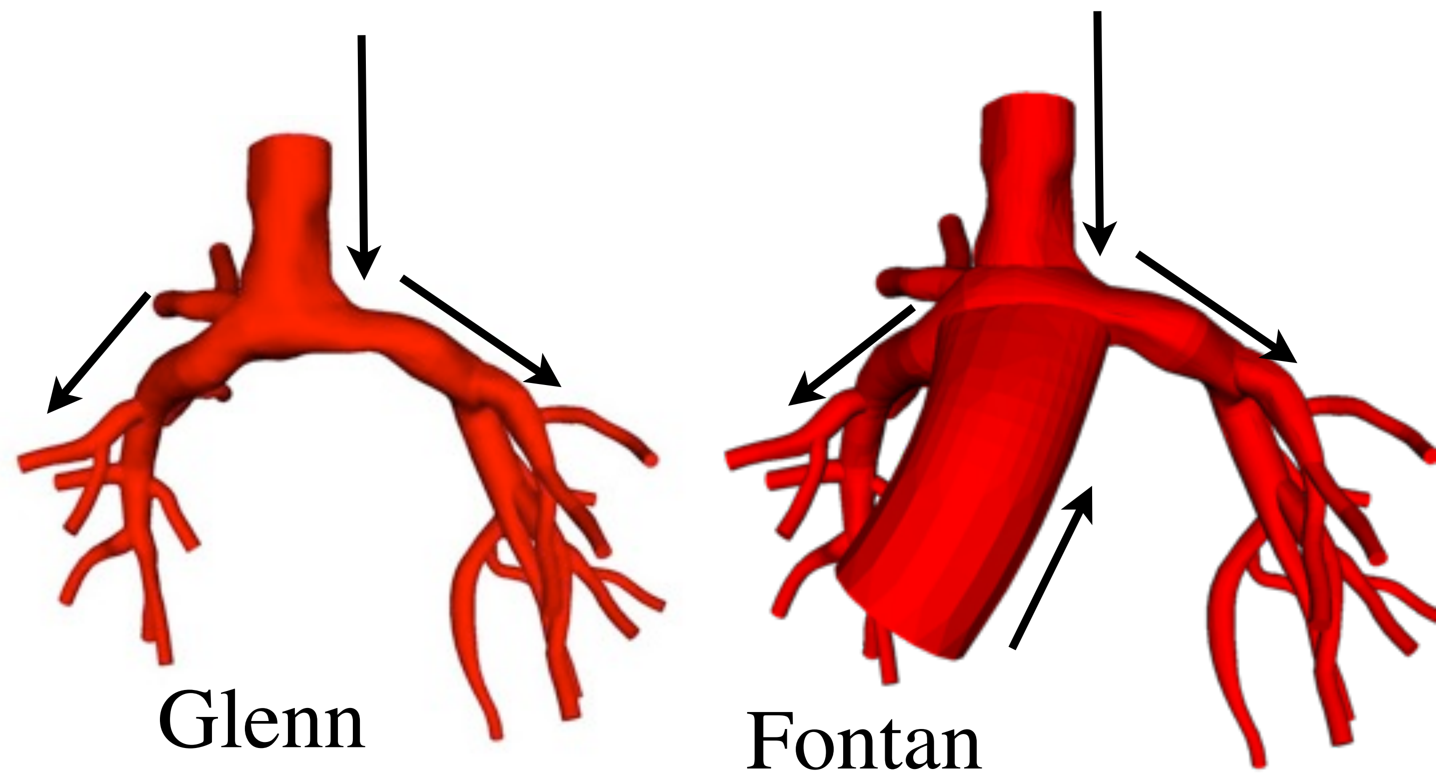
Total Extracardiac Conduit Fontan Palliation
of Hypoplastic Left Heart



http://www.americanheart.org

Glenn-Fontan surgery

- congenital heart disease
- multi-step complex procedure

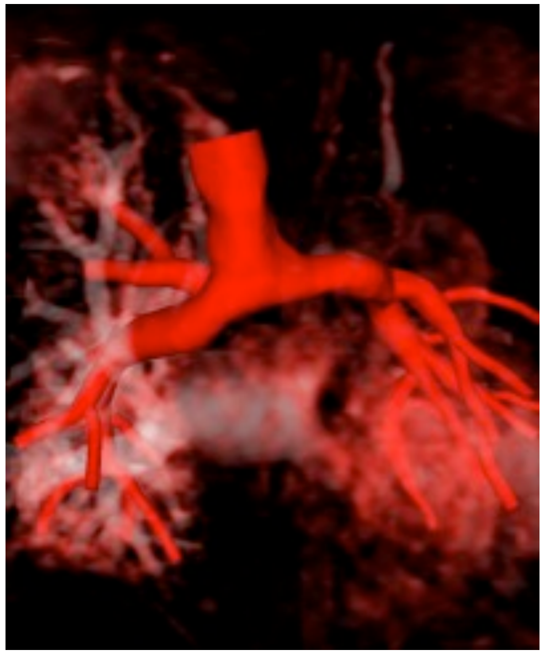


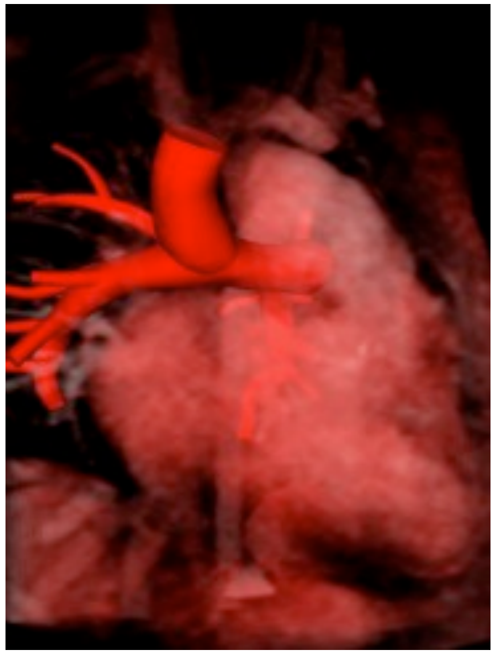
Troianowski, Taylor, Feinstein, Vignon-Clementel
Stanford/INRIA

Numerical simulations

- Patient-specific geometries
- Forecast pressure drop, flow split, wall-shear stress

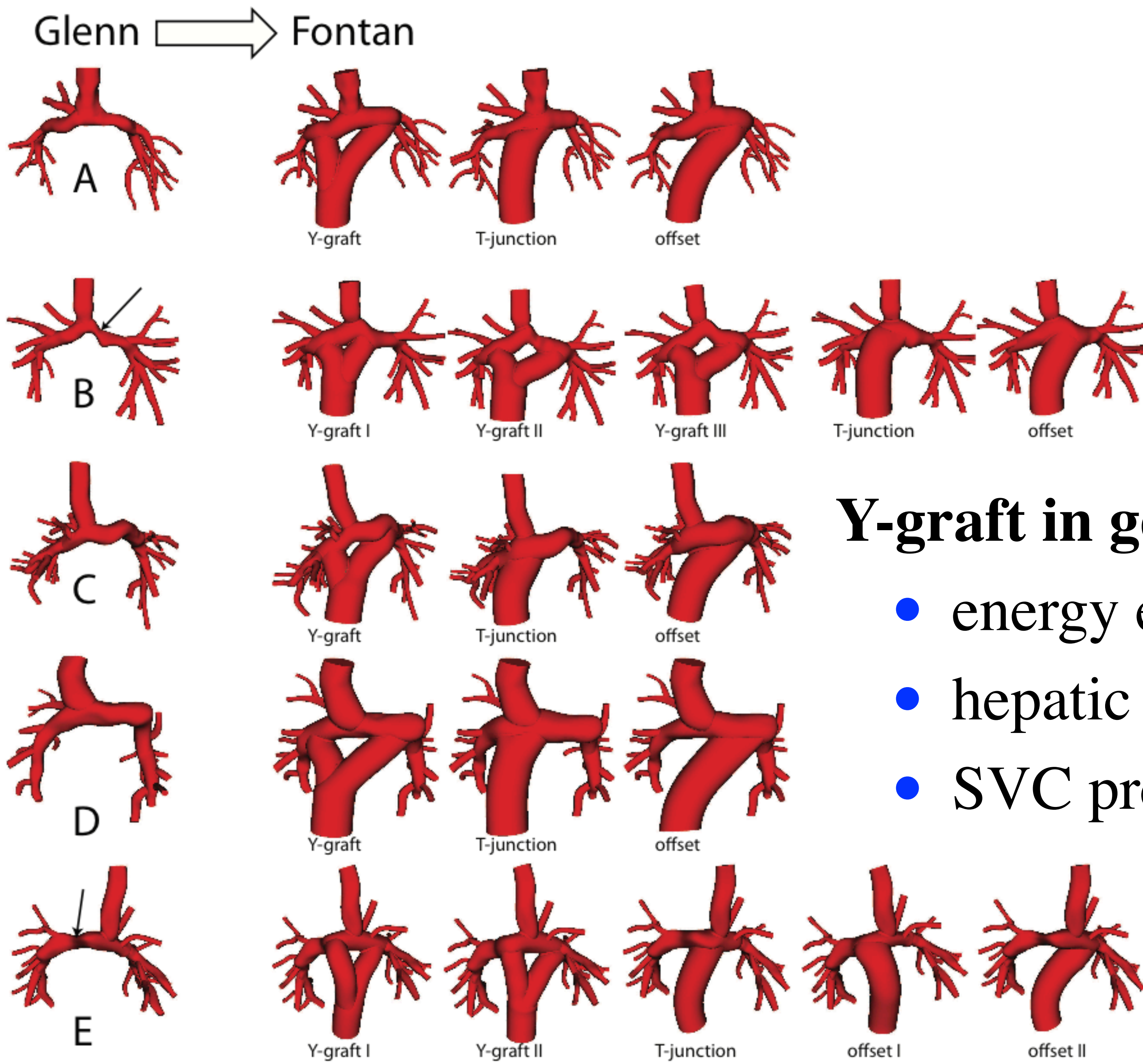
Geometry from 5 patients MRI data





Internships of A. Birolleau,
G. Troianowski

Surgical planning

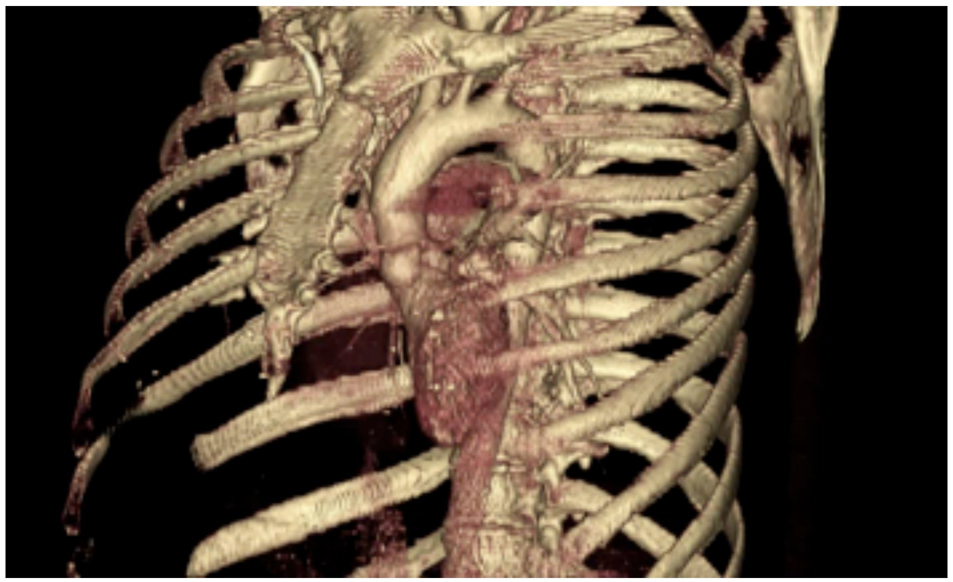


Y-graft in general improves:

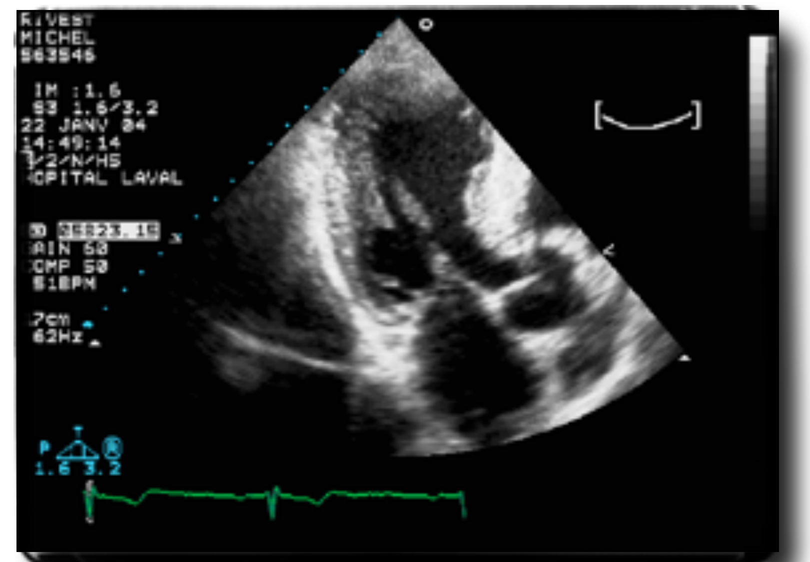
- energy efficiency
- hepatic flow distribution
- SVC pressure under rest & exercise

Troianowski, Taylor, Feinstein, Vignon-Clementel

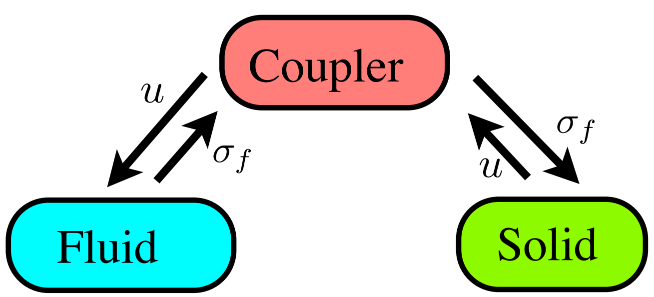
Fluid-Structure Interaction



Aorta
(CVBRL, Stanford)



Cardiac valves
(Hôpital Laval)



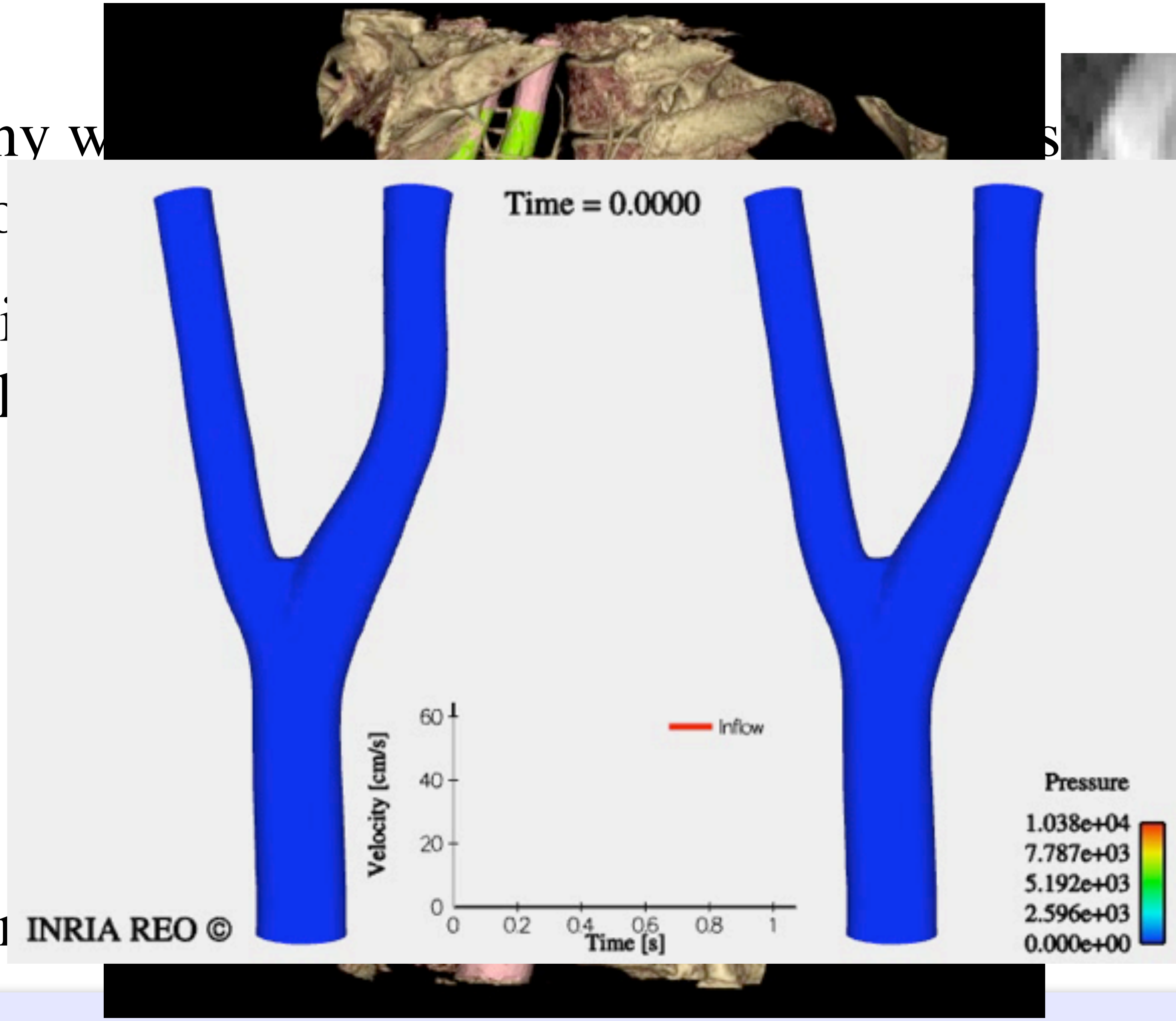
$$\rho^{\mathrm{f}} \left(\frac{\partial \boldsymbol{u}}{\partial t}_{|\widehat{\boldsymbol{x}}} + (\boldsymbol{u} - \boldsymbol{w}) \cdot \boldsymbol{\nabla} \boldsymbol{u} \right) - 2\mu \mathbf{div} \boldsymbol{\epsilon}(\boldsymbol{u}) + \boldsymbol{\nabla} p = \mathbf{0}, \quad \text{in} \quad \Omega^{\mathrm{f}}(t)$$

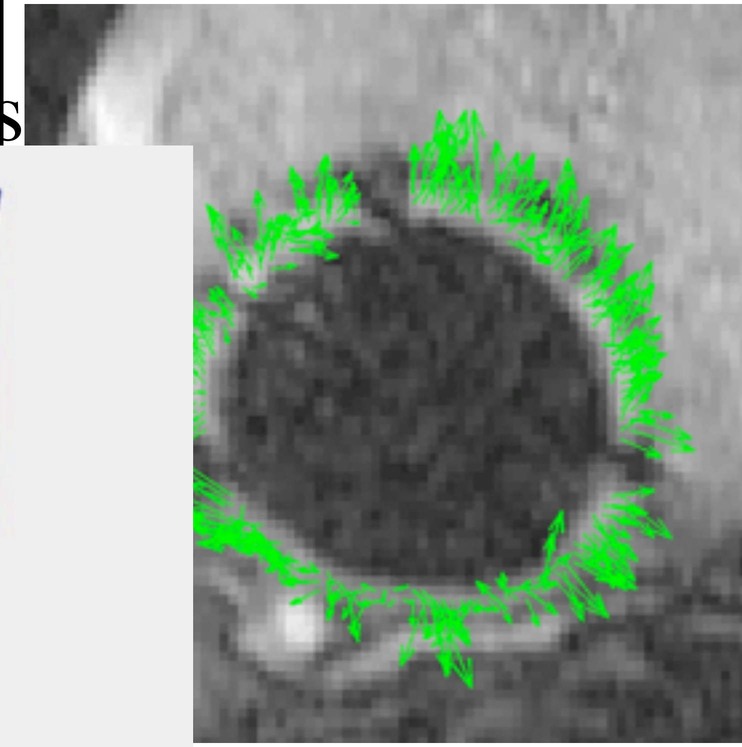
$$\operatorname{div} \boldsymbol{u} = 0, \quad \text{in} \quad \Omega^{\mathrm{f}}(t)$$

$$\rho^{\mathrm{s}} \frac{\partial^2 \boldsymbol{d}}{\partial t^2} - \mathbf{div}\big(\boldsymbol{F}(\boldsymbol{d})\boldsymbol{S}(\boldsymbol{d})\big) = \mathbf{0}, \quad \text{in} \quad \widehat{\Omega}^{\mathrm{s}}$$

Blood flow in aorta

- Many w s
but no
- Typi
vessel





Abdominal aorta
. Taylor et al. , Stanford

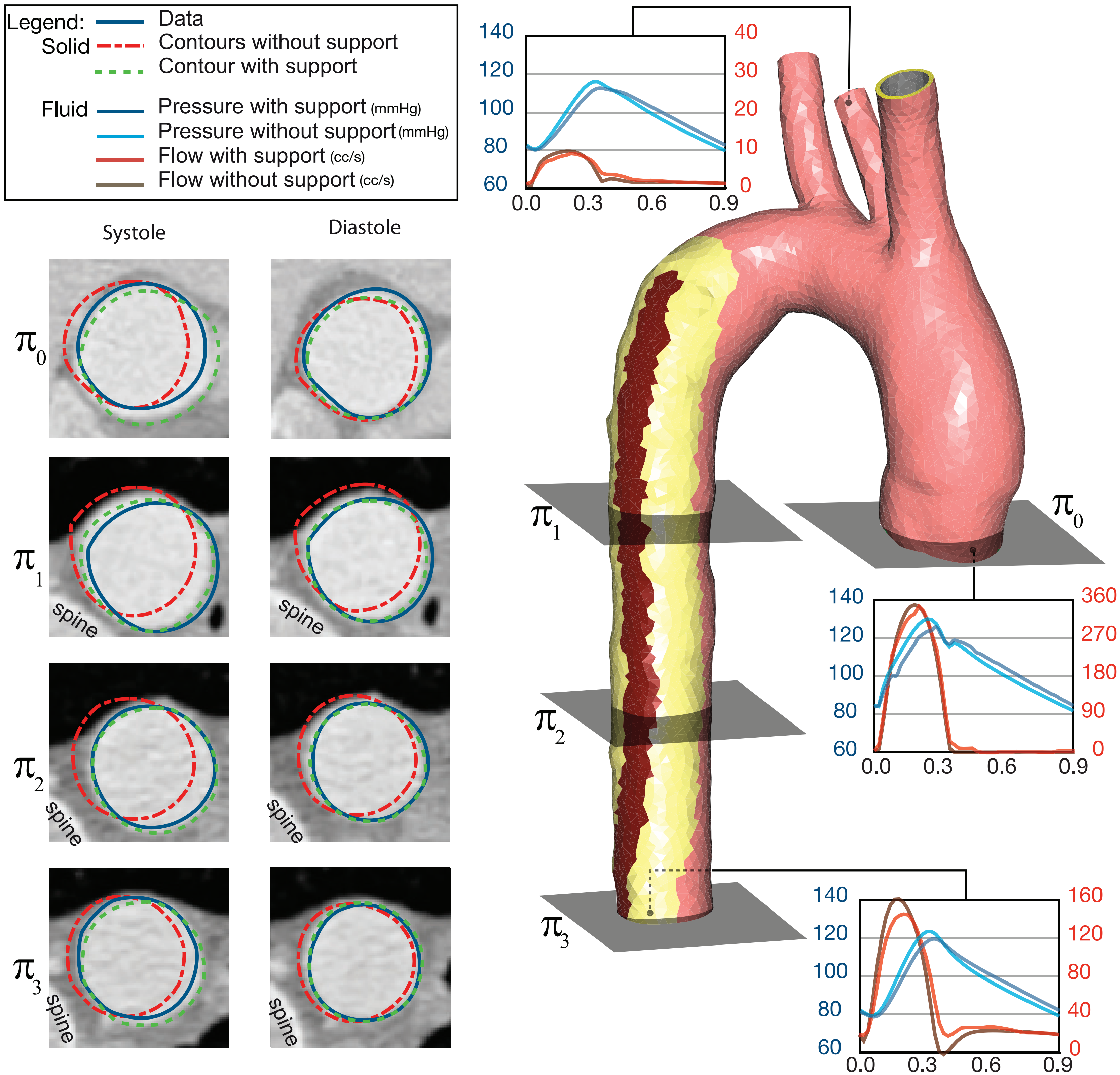
- A si l tissues :

$$\boldsymbol{\sigma}_s \boldsymbol{n} = -k_s \boldsymbol{d} - c_s \frac{\partial \boldsymbol{d}}{\partial t}$$

Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)

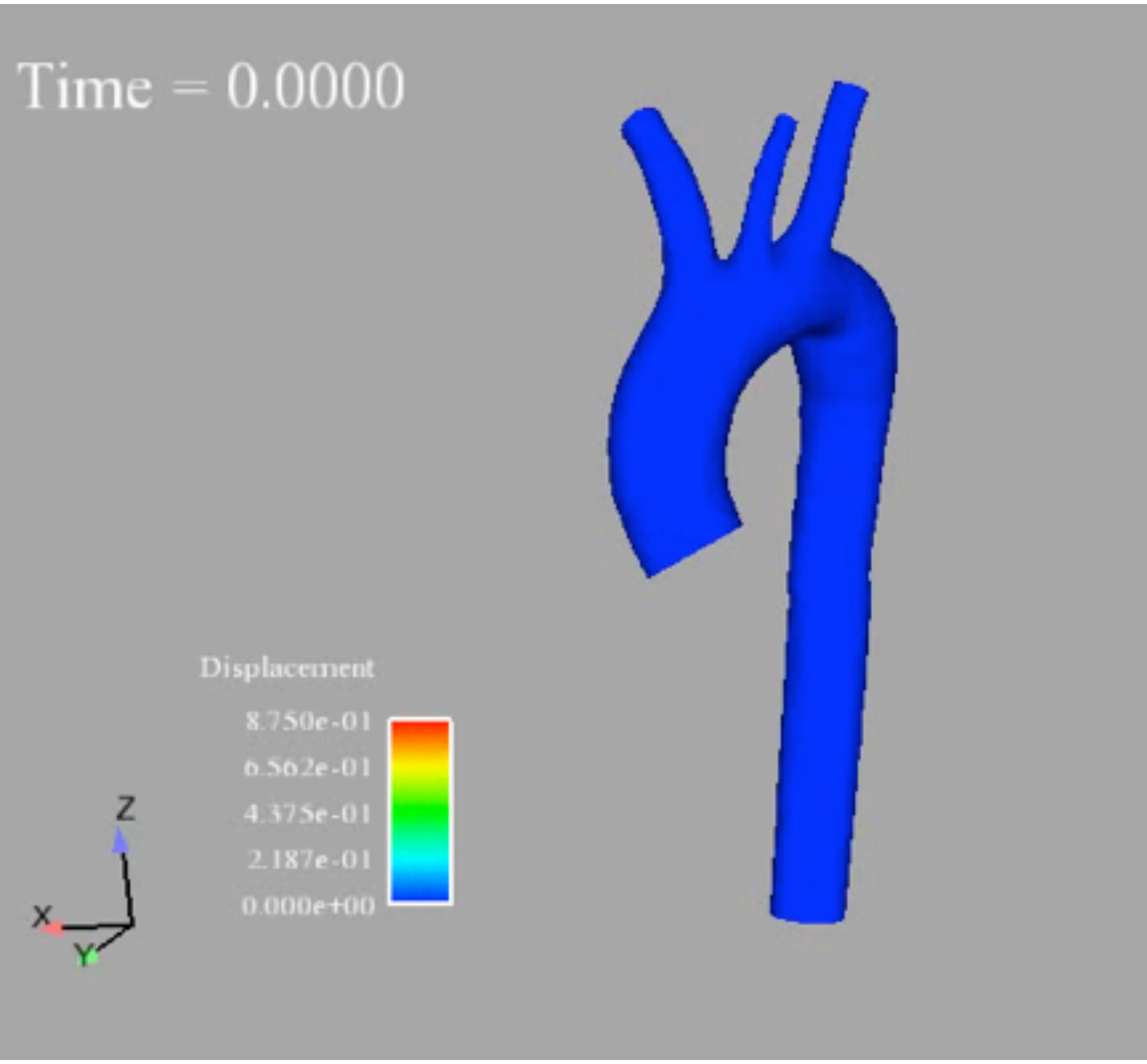
Blood flow in aorta

Comparison simulation / images

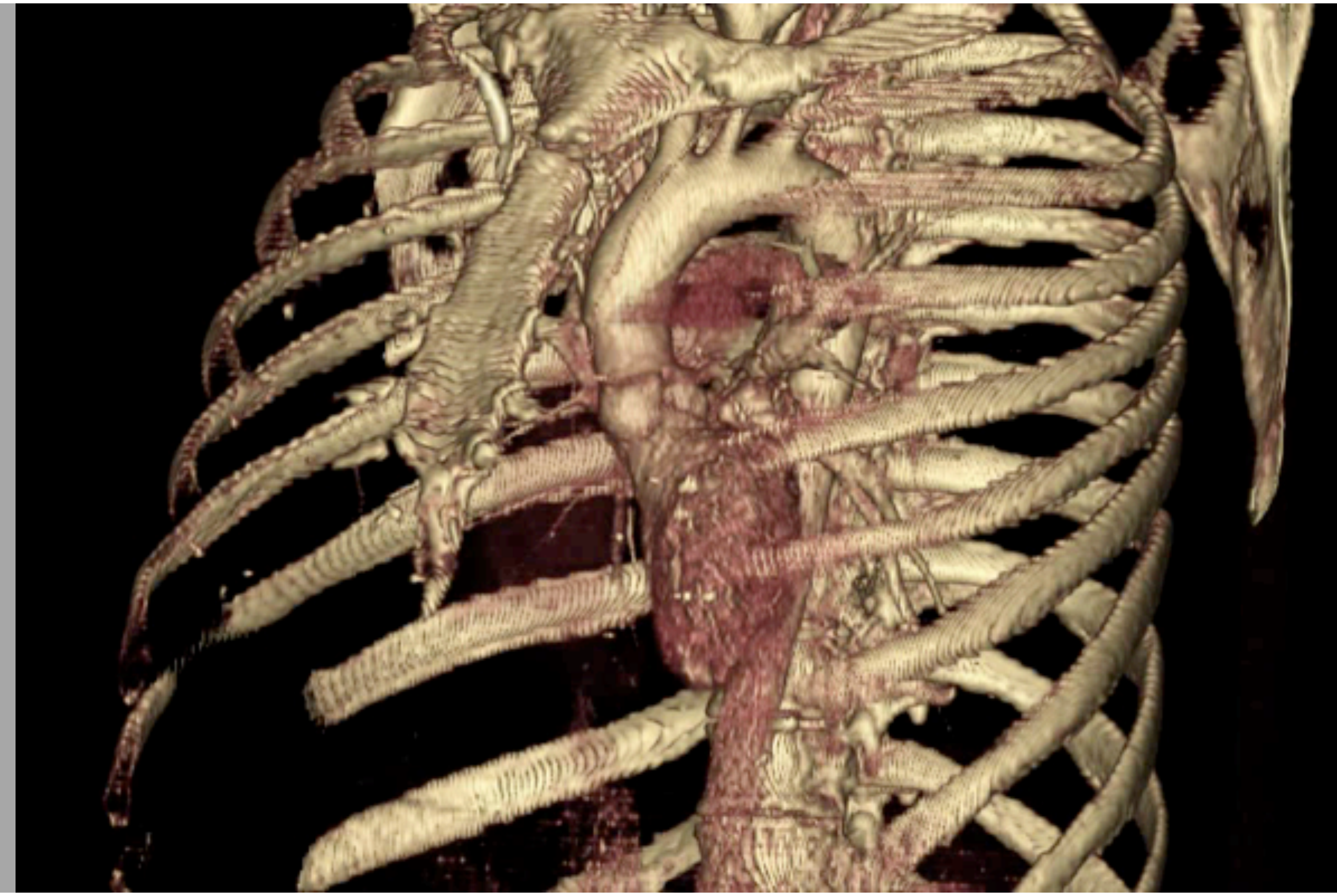


Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)

Medical Data Assimilation



INRIA



CVBRL, Stanford

Data assimilation

- Reduce model uncertainties using observations
- Access to "hidden" quantities
- Smooth the data

Data assimilation in FSI

- FSI dynamical system: $\left\{ \begin{array}{rcl} B\dot{X} & = & A(X,\theta) + R \\ X(0) & = & X_0 \end{array} \right.$

- Time discretization: $X^{n+1} = F^{n+1}(X^n, \theta)$

- **State variable**: X=$[\boldsymbol{u}, p, \boldsymbol{d}^f, \boldsymbol{d}, \boldsymbol{v}]$

- **Parameters**: θ = [Young modulus, viscosity, boundary conditions, …]

- **Uncertainties** on the initial condition X_0 and the parameters θ

- Partial **observations** of X: $Z = H(X)$

Data assimilation

- **State estimation** through "simple" feedback terms.

For example, for velocity measurements:

$$M\ddot{X} + KX = f \; - \gamma H^T (H\dot{X} - Y_{obs})$$

Moireau, Chapelle, Le Tallec, 2008

- **Parameter estimation** : reduced Unscented Kalman filter (SEIK)

Dinh Tuan Pham, 2001
Moireau, Chapelle, 2009

- With respect to a variational approach: no tangent, no adjoint
- Counterpart : as many resolutions as parameters ("particles")...
- ... but easy to run them in parallel

Data assimilation

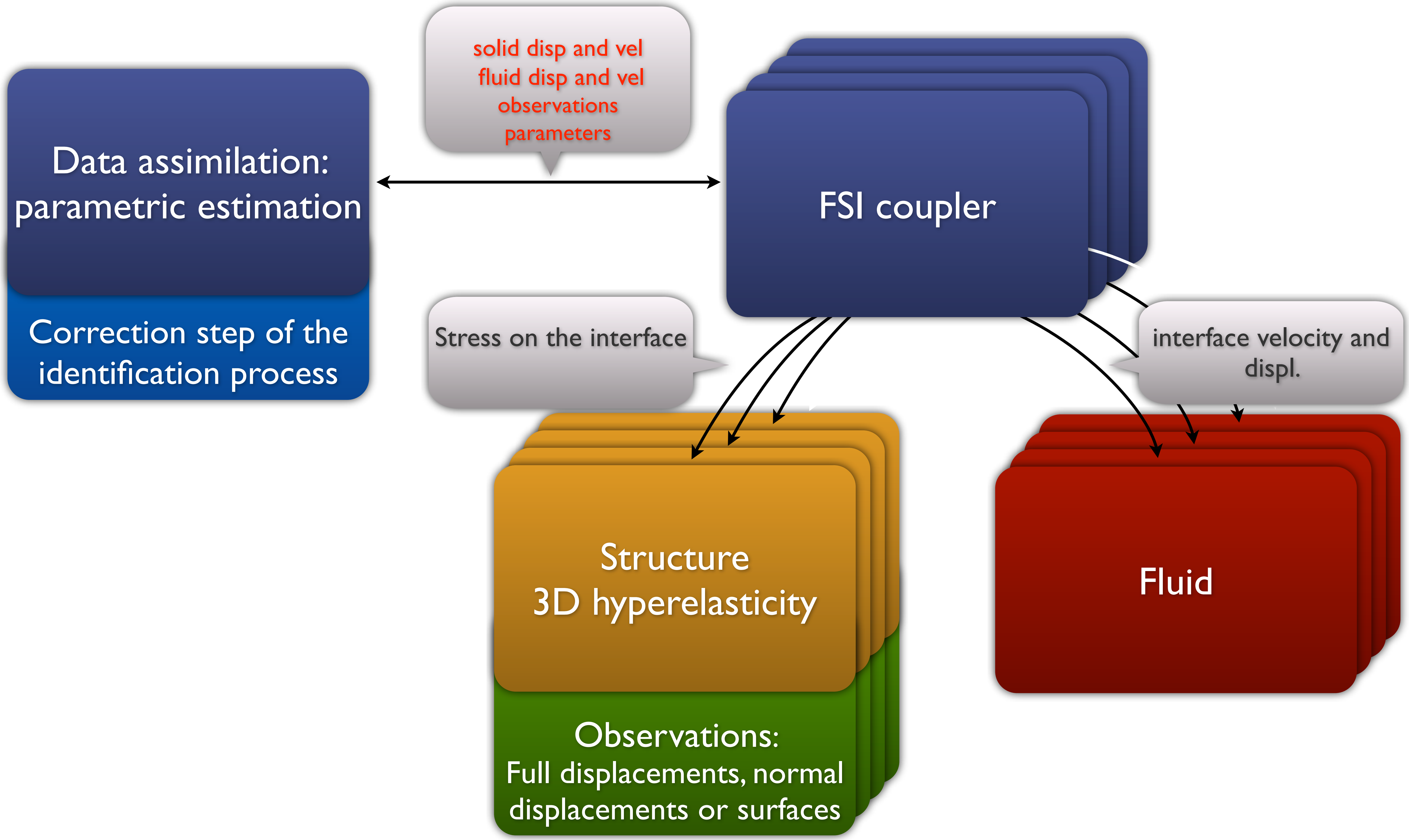
Luenberger filter
for the state

Summary

- Prediction: $$\left\{ \begin{array}{rcl} \hat{X}_-^{n+1} & = & \hat{F}^{n+1}(\hat{X}_+^n, \theta_+^n, Z^{n+1}, K) \\ \hat{\theta}_-^{n+1} & = & \hat{\theta}_+^n \end{array} \right.$$
- Correction: $$\left\{ \begin{array}{rcl} \hat{X}_+^{n+1} & = & \hat{X}_-^{n+1} + \hat{K}_X^{n+1}(Z^{n+1} - H(\hat{X}_-^{n+1})) \\ \hat{\theta}_+^{n+1} & = & \hat{\theta}_-^{n+1} + \hat{K}_\theta^{n+1}(Z^{n+1} - H(\hat{X}_-^{n+1})) \end{array} \right.$$

Kalman-like filter for
for the parameters

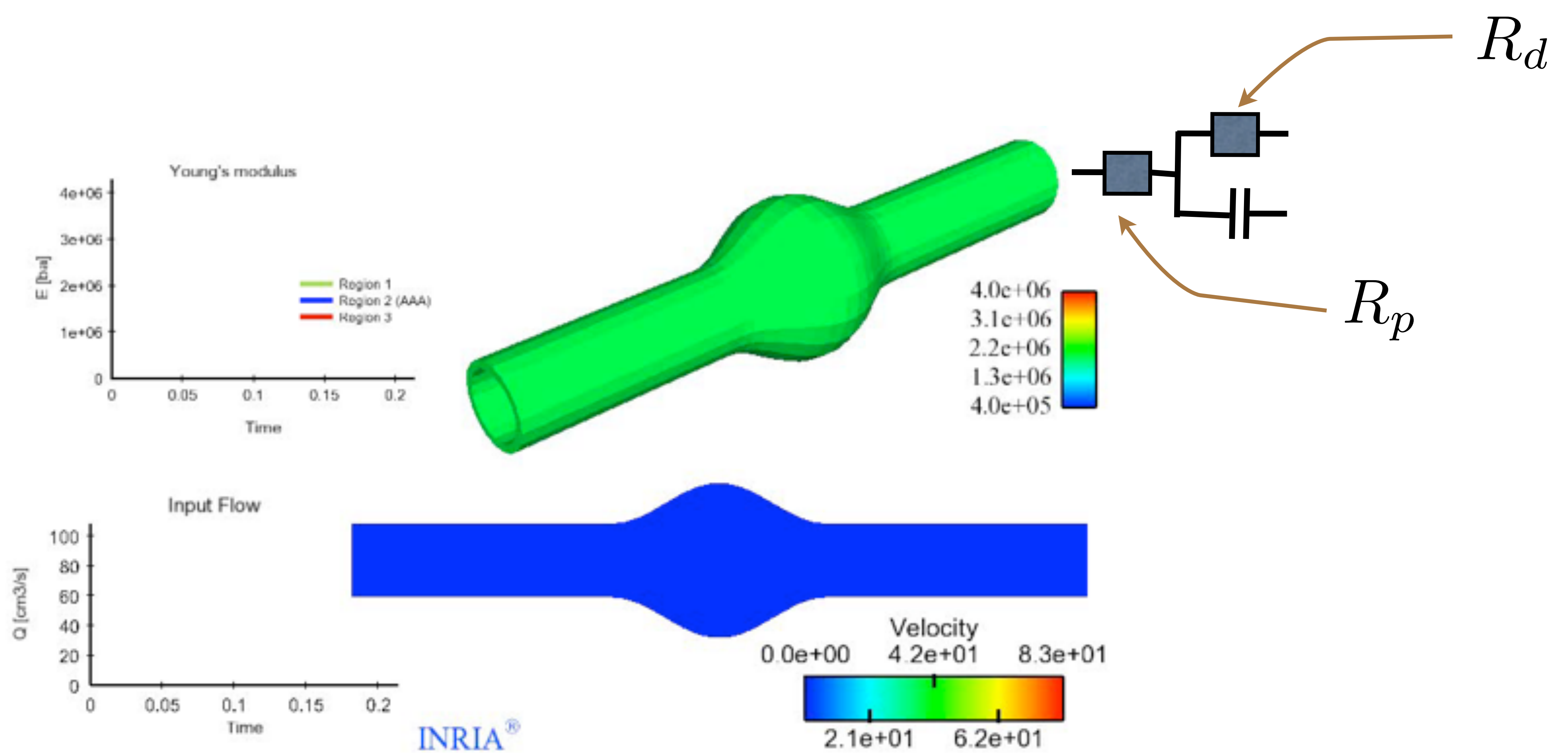
Implementation



Stiffness estimation

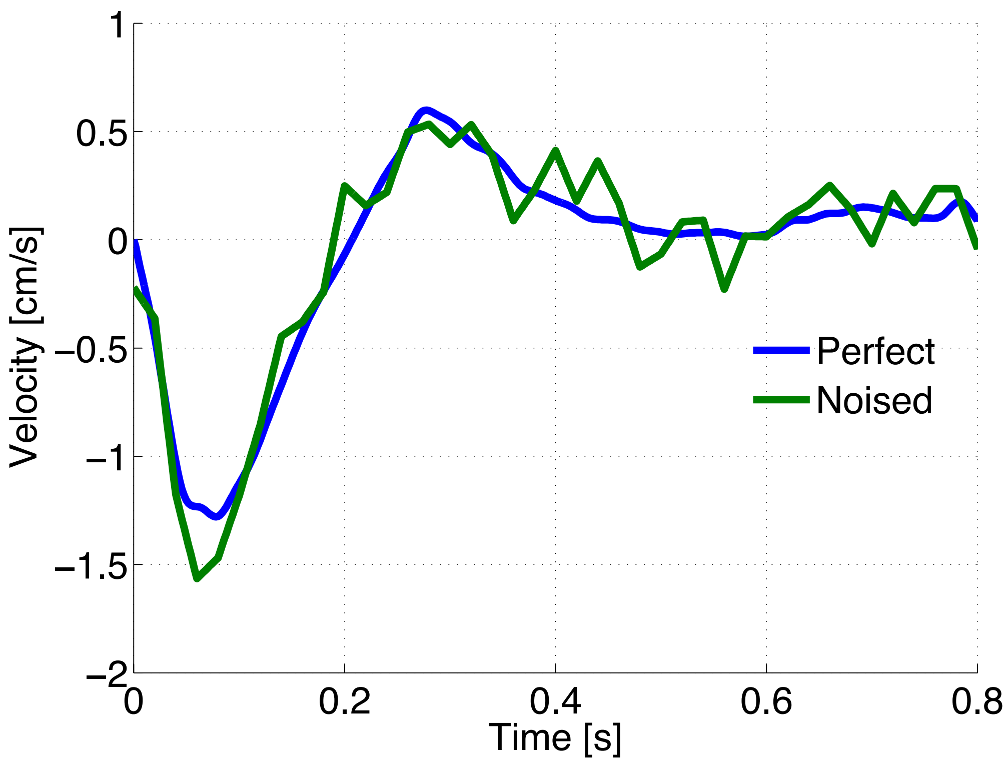
Parameter estimation

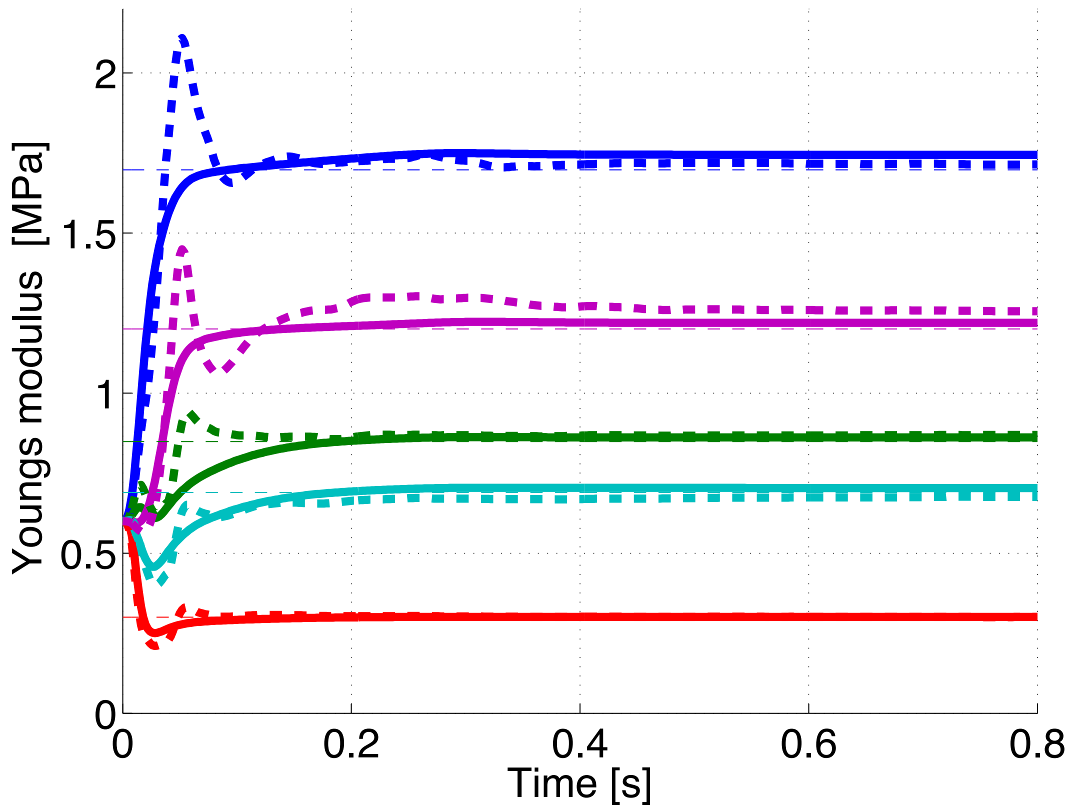
- Parameter estimation: Young modulus E in 3 regions
- Synthetic data with $E_1 = 0.5, E_2 = 2, E_3 = 4MPa$
- Initial guess: $E = 2MPa$ in the three regions
- Observations: wall velocity



Simulation : C.Bertoglio (INRIA)

- Similar experiment with 5 regions
- With noise (10%) and resampling:

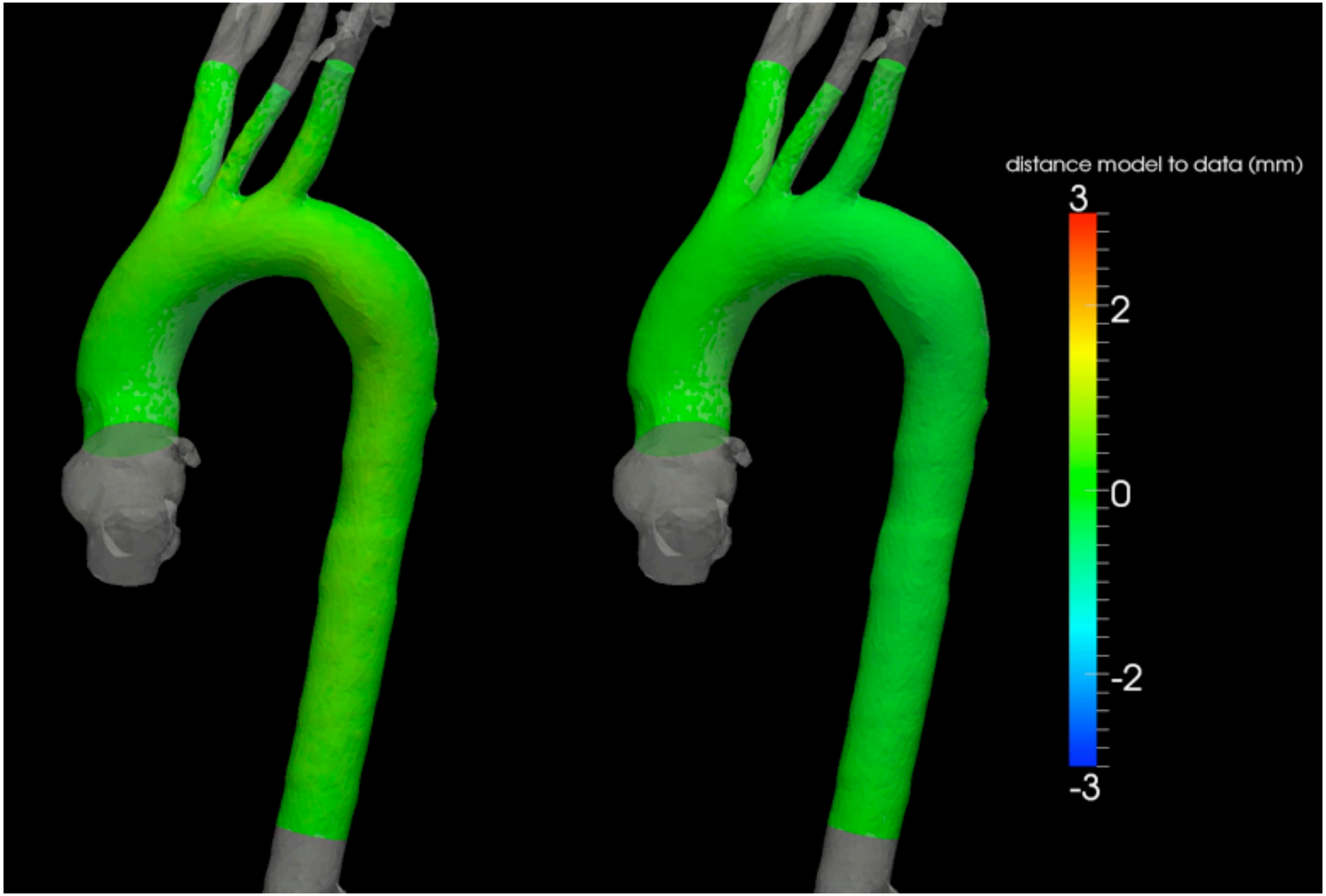




Simulation : C.Bertoglio (INRIA)

Example 2

State estimation



Data assimilation (state only)

Direct similation

Simulation: N. Xiao (Stanford)

Future

- Charley Taylor & col. are about to leave Stanford
- ...but not the end of the collaboration !
  - ★ With A. Figueroa & N. Xiao :
    - inverse problems in full-body-scale 3D vasculature
  - ★ With A. Marsden (UC San Diego) :
    - Pulmonary arteries in the *Transatlantic Network* (Leducq foundation)
    - Respiration modeling