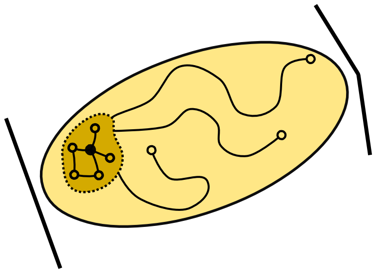
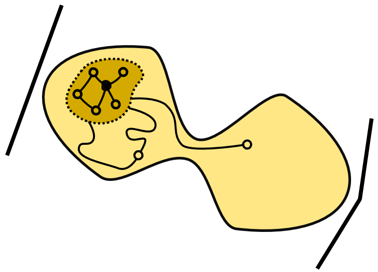
Growing Seed Sets: QW Sampling and st-Connectivity



Simon Apers
Inria, CWI

simon.apers@inria.fr



CWI-Inria Workshop – September 18, 2019 – Amsterdam

Classical vs Quantum Sampling

Classical sample:

- probability distribution $p \in \mathbb{R}^n$
- normalization $\|p\|_1 = 1$
- π = uniform distribution

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- normalization $\|p\|_1 = 1$
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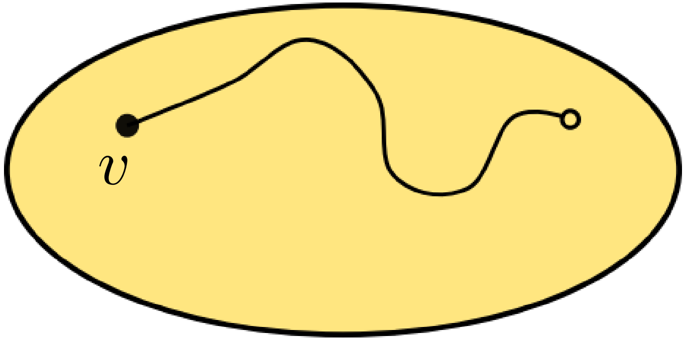
Quantum sample:

- quantum state $|q\rangle \in \mathbb{C}^n$
- normalization $\||q\rangle\|_2 = 1$
- $|\pi\rangle$ = uniform superposition

Random Walk Sampling

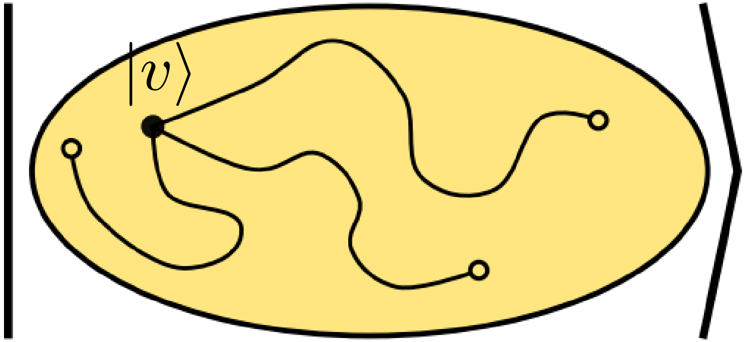
- initial node v, t-step RW $P^t v$
- stationary distribution π and $P^t v \to \pi$
- many applications in graph problems, statistical physics, $\ldots$



** unless stated otherwise, all graphs are regular, bounded degree expanders*

Quantum Walk Sampling

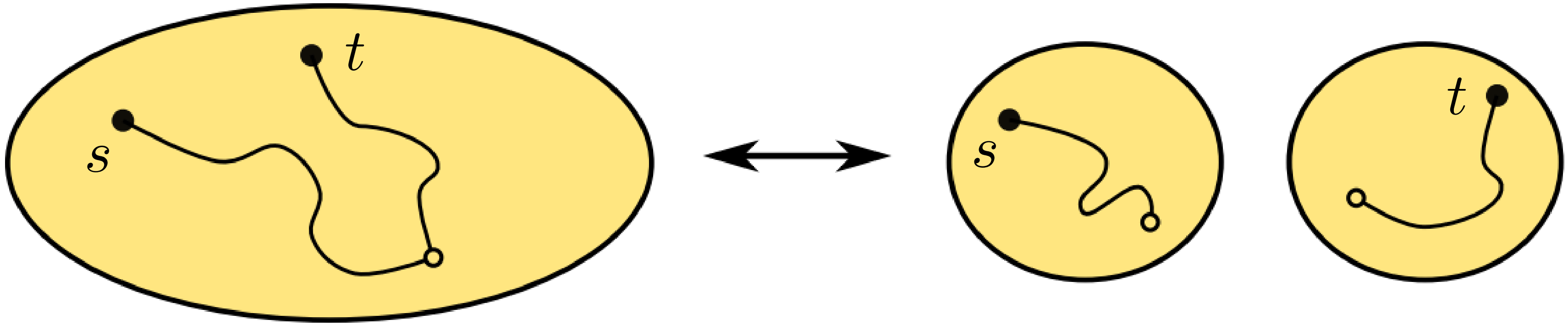
- quantum sample of RW distribution: $|P^t v\rangle = P^t|v\rangle / \|P^t|v\rangle\|$
- use QWs to create $|P^t v\rangle \to |\pi\rangle$. Complexity?
- many applications in element distinctness, formula evaluation, . . .



st-Connectivity

Classical: $O(n^{1/2})$

- use RWs from s and t
  to sample $\pi^{(s)}$ and $\pi^{(t)}$
- look for collision
  using $O(n^{1/2})$ samples



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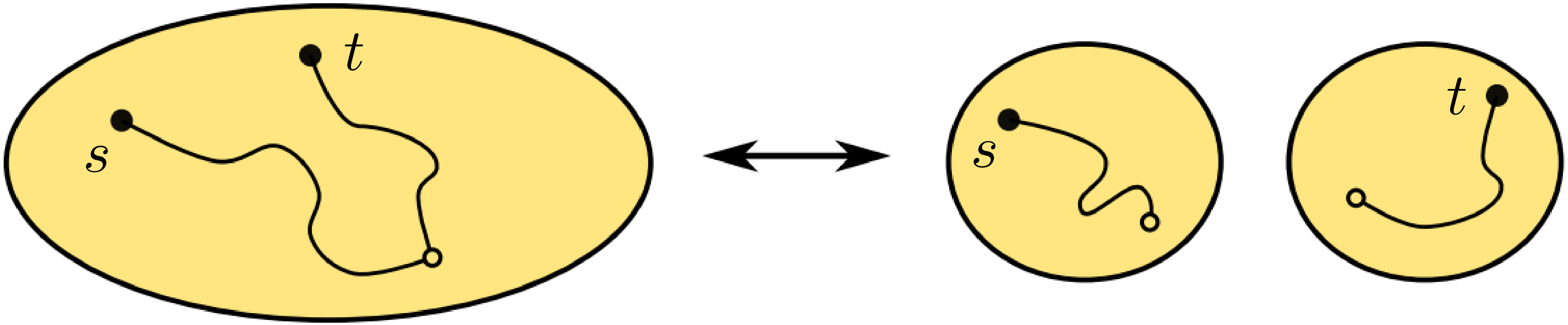

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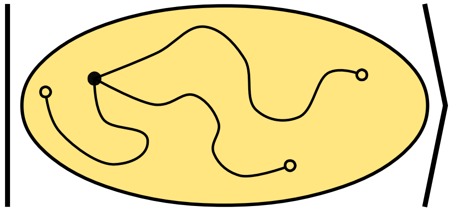
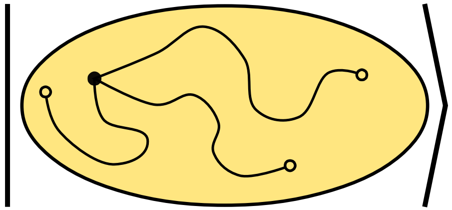


?

complexity of generating quantum sample $|\pi\rangle$

Quantum Sampling



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folklore scheme: [Watrous'98]

- t-step QW creates *subnormalized* quantum sample $P^t|v\rangle + |\Gamma\rangle$

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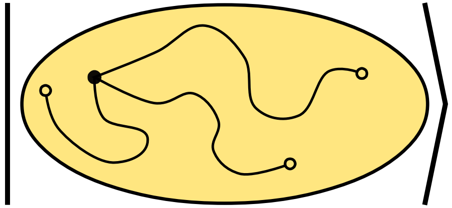

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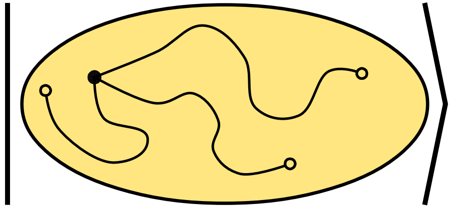

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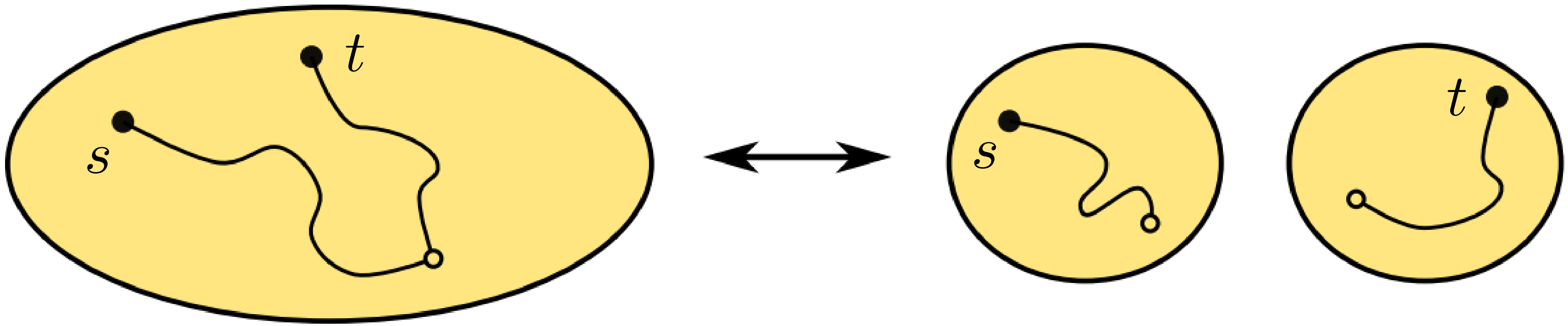

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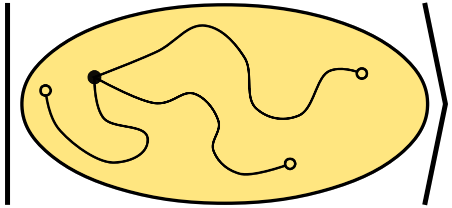

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main bottleneck of folklore approach:

number of samples

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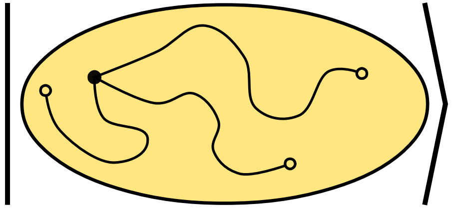

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improve projection on $|\pi\rangle$
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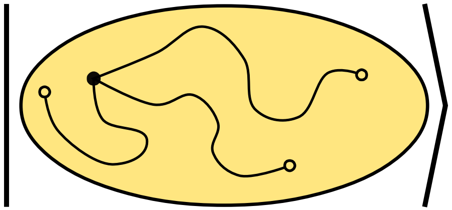

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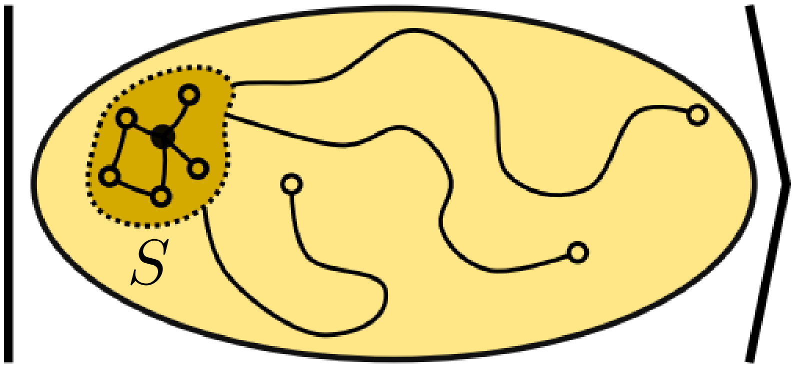

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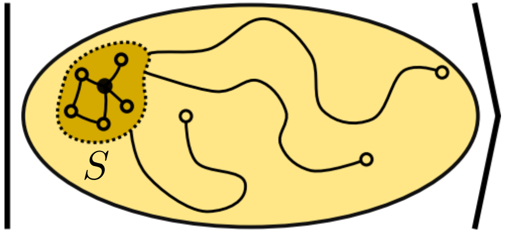
$$S \subset \mathcal{V},\ |S| = n^{1/3}:$$

$$\|P^t|\pi_S\rangle\| \geq |\langle\pi|\pi_S\rangle| \geq n^{-1/3}$$

Quantum Sampling



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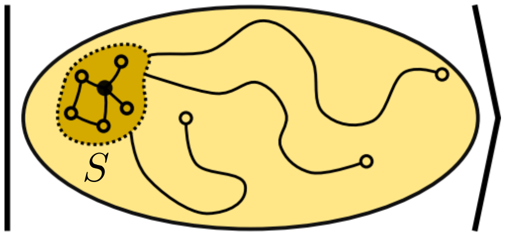

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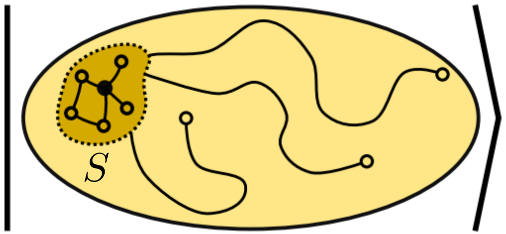

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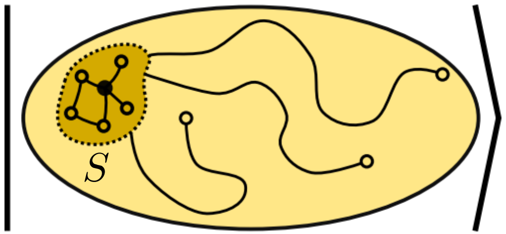

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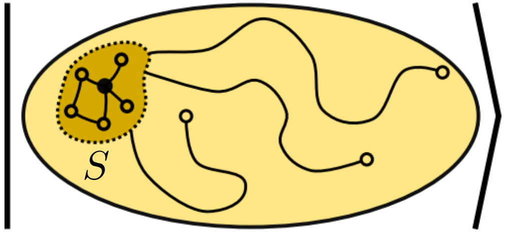

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quantum sample $|\pi\rangle$ in $O(n^{1/3})$ QW steps

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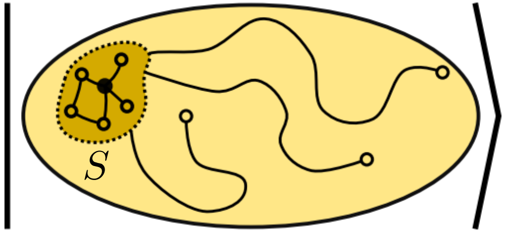

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- use "a … $s\rangle\|$

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quantum sample $|\pi\rangle$ in $O(n^{1/3})$ QW steps

space-time tradeoff: exchange $(1, n^{1/2})$ for $(n^{1/3}, n^{1/3})$

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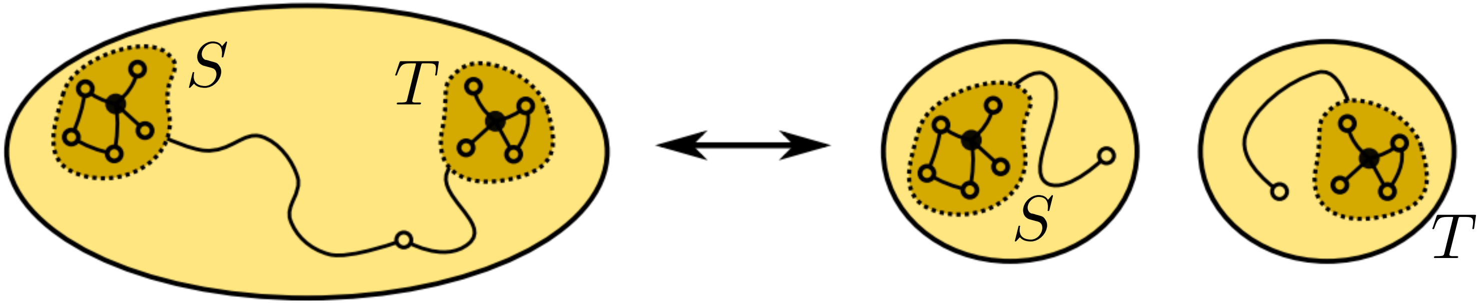
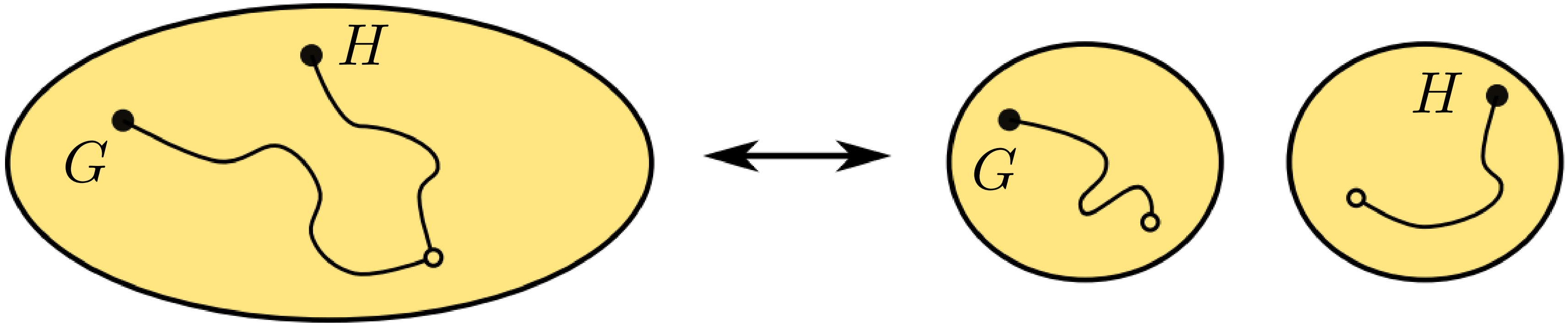


Graph Isomorphism

instance of st-connectivity:
pairwise permutations describe RW over isomorphisms

$$G \to G' \to G'' \to \dots$$

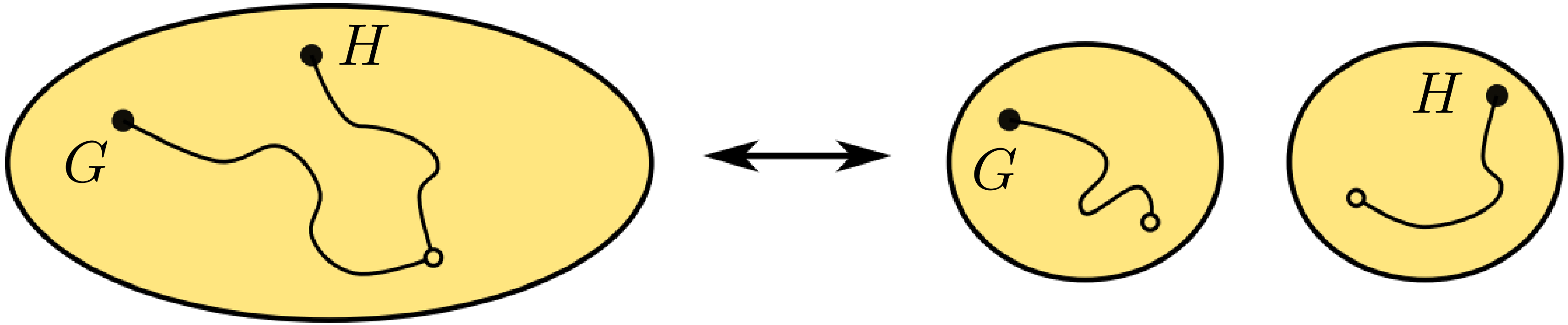
? is there permutation that turns G into H?



Graph Isomorphism

quantum approach:

1. create superpositions $|G\rangle, |H\rangle$ over isomorphisms of G, H
2. compare states using swap test



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- [AMRR'11]: $\Omega(2^{n/2})$ lower bound for generalized approach

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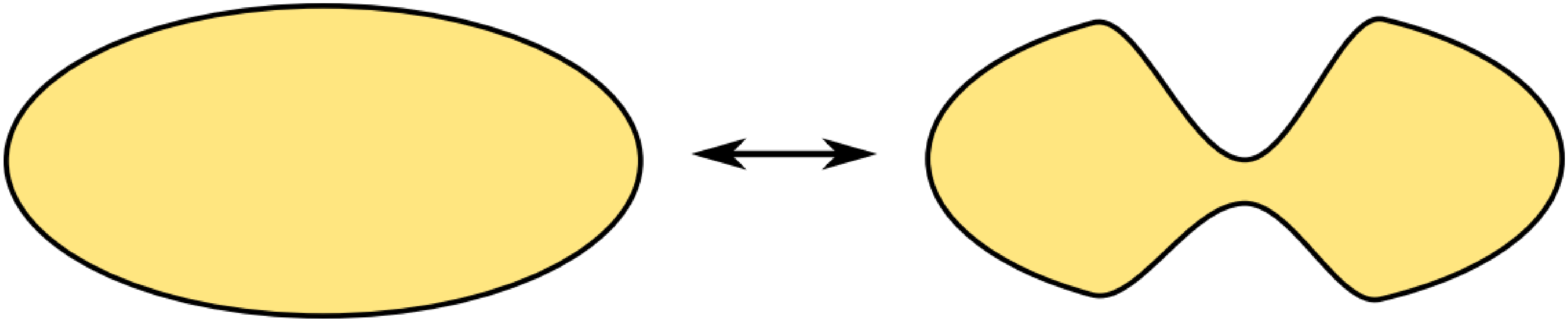
- this work: $O(2^{n/3})$ using QW sampling

Expansion Testing



$$\Phi = \min_{\mathcal{S} \subset \mathcal{V} : |\mathcal{S}| \leq n/2} |\partial \mathcal{S}| / |\mathcal{S}|$$

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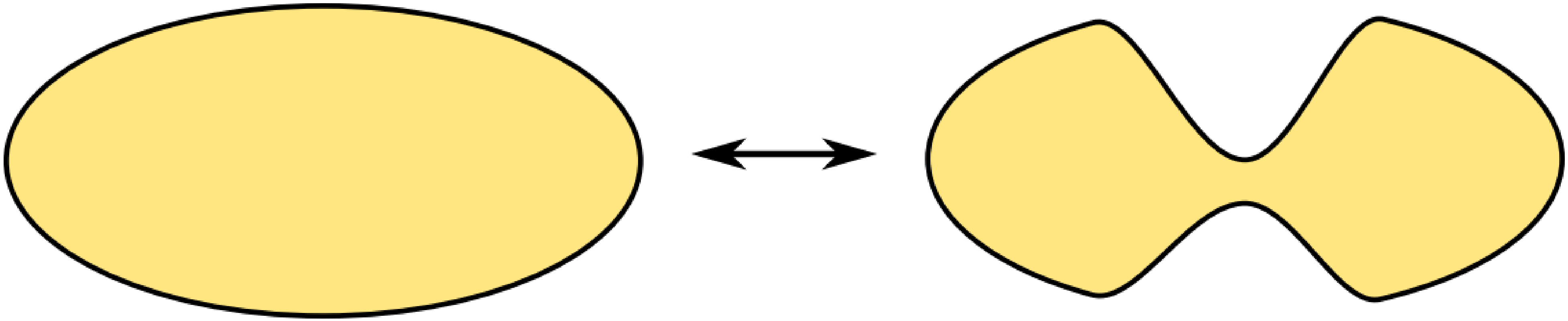

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[Goldreich-Ron'97]:

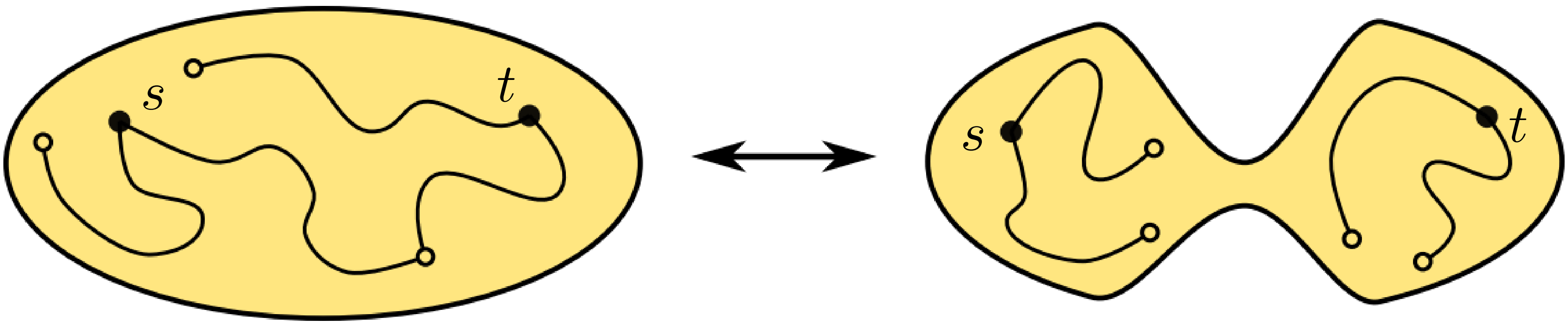
does G have expansion $\geq \Upsilon$, or is G far from any such graph?

Expansion Testing

GR expansion tester:

- pick uniformly random nodes s, t
- perform RWs of length Υ^{-2}
- count collisions between $O(n^{1/2})$ samples

if none, `reject`; otherwise, `accept`



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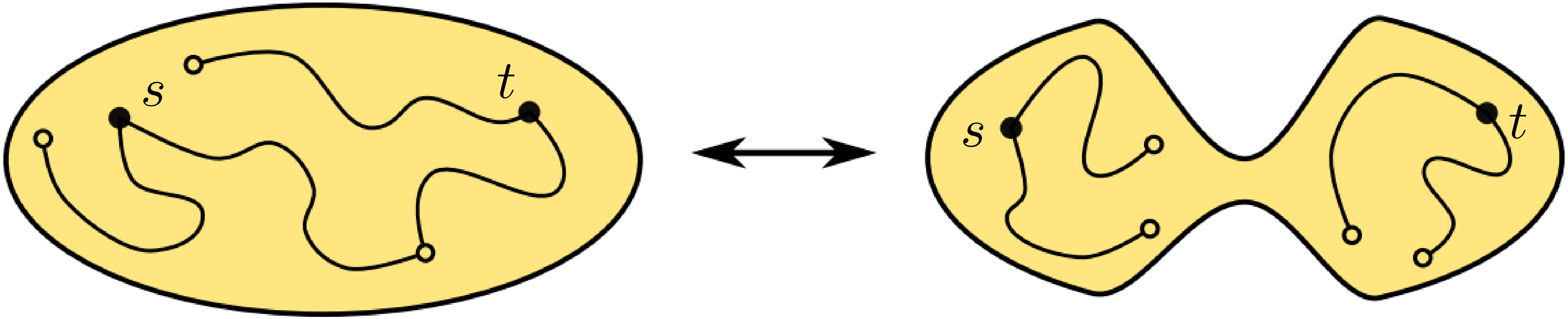

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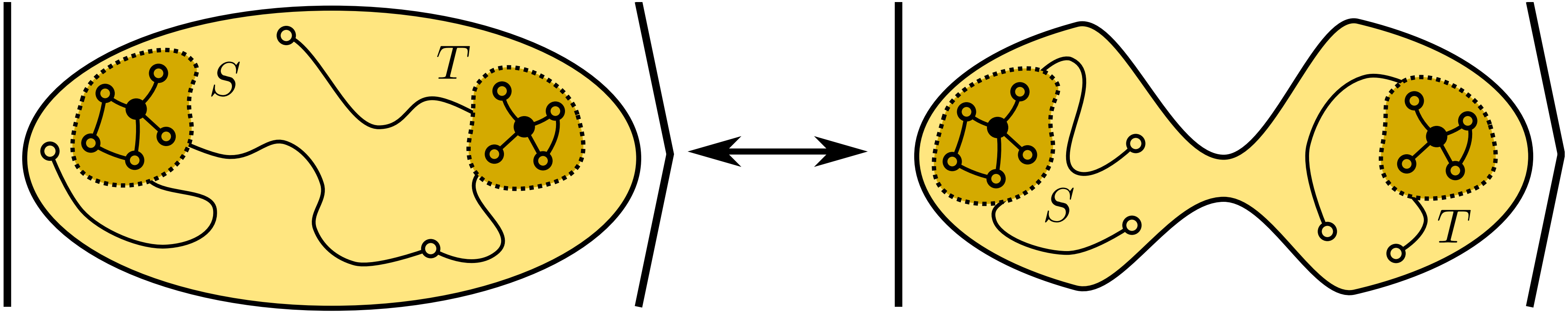

How to Grow a Seed Set?

◇ use tool from local graph clustering!

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= Markov chain on node subsets: from $\mathcal{S} \subseteq \mathcal{V}$

- pick $Z \in [0, 1]$ u.a.r.
- $\forall u \in \mathcal{V} : u \in \mathcal{S}'$ iff $|E(u, \mathcal{S})|/d(u) \geq Z$

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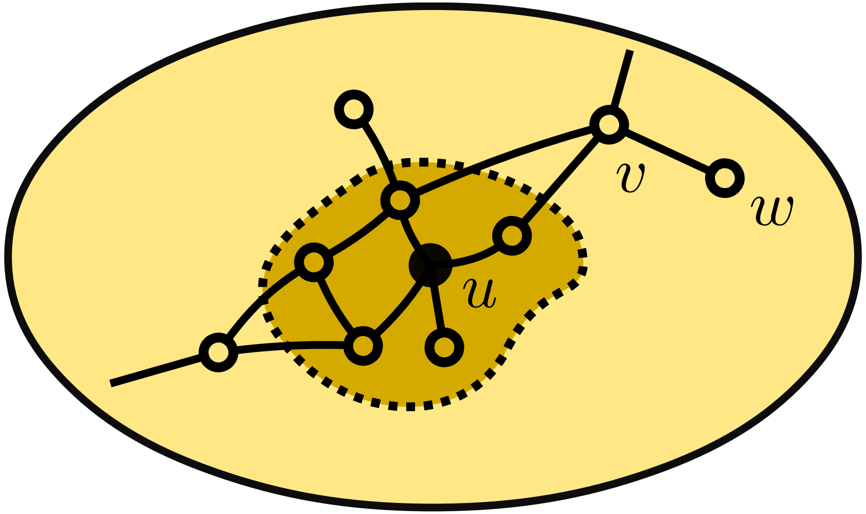

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$|E(u,\mathcal{S})|/d(u) = 1, \ |E(v,\mathcal{S})|/d(v) = 1/2, \ |E(w,\mathcal{S})|/d(w) = 0$

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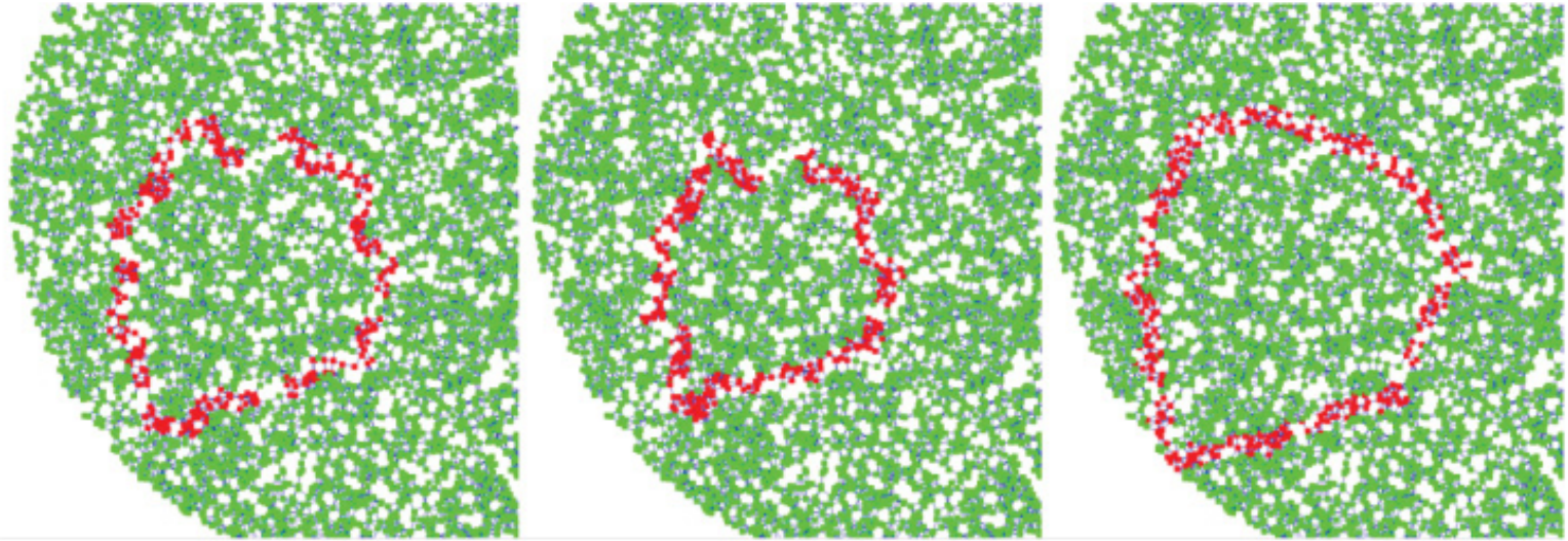

232 steps | 235 steps | 273 steps

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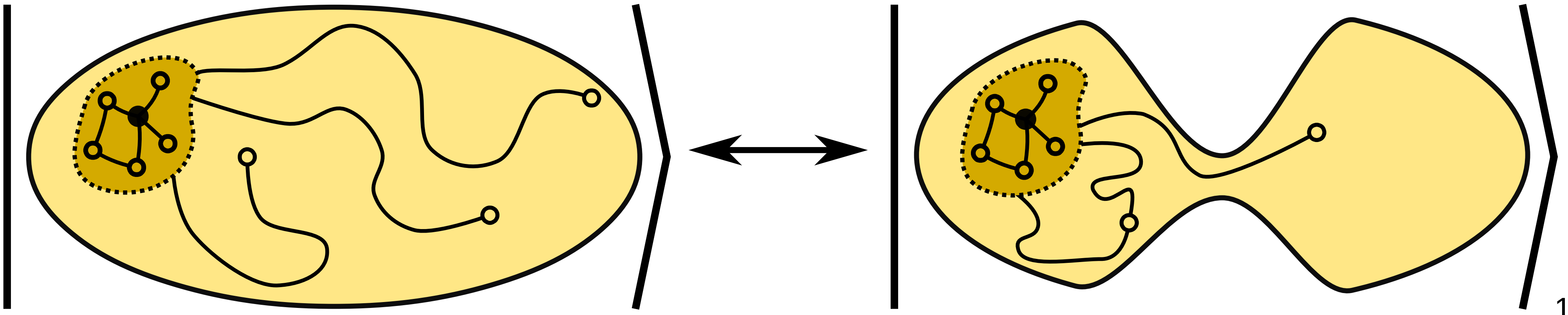
prop.: ESP returns a set of size $n^{1/3}$ within cluster in $O(n^{1/3}\Upsilon^{-1})$ steps



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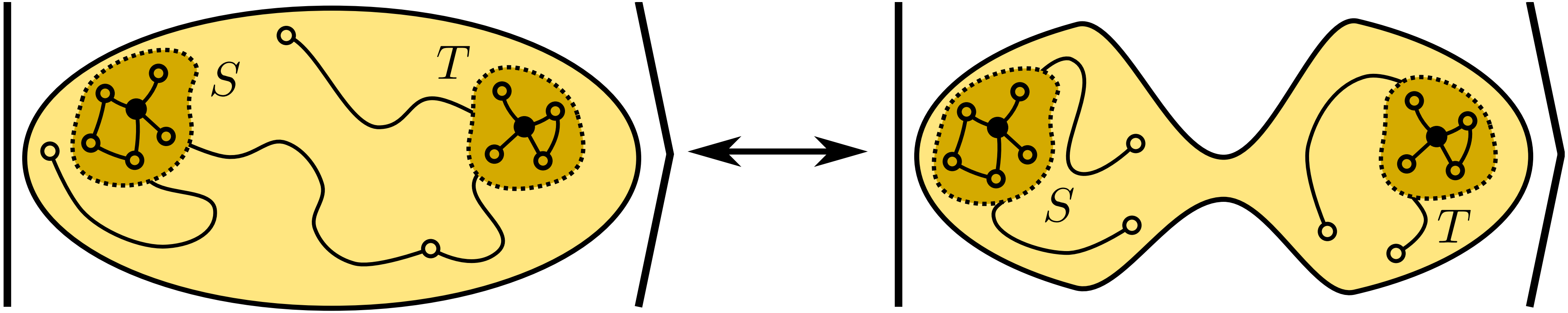

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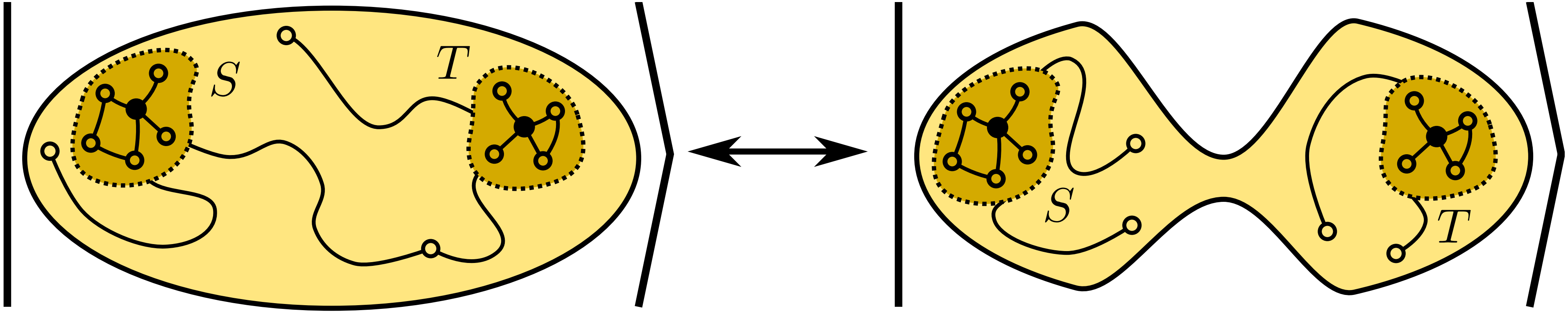

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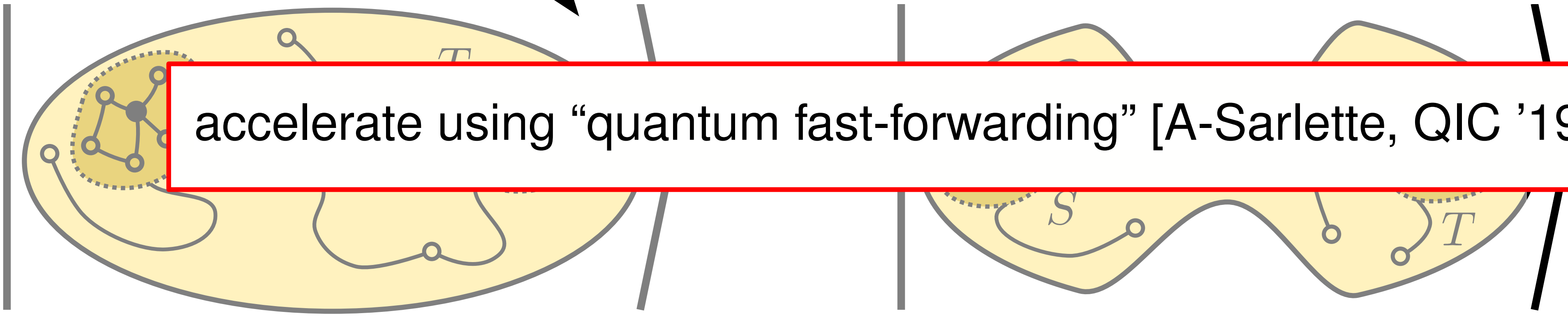

accelerate using "quantum fast-forwarding" [A-Sarlette, QIC '19]

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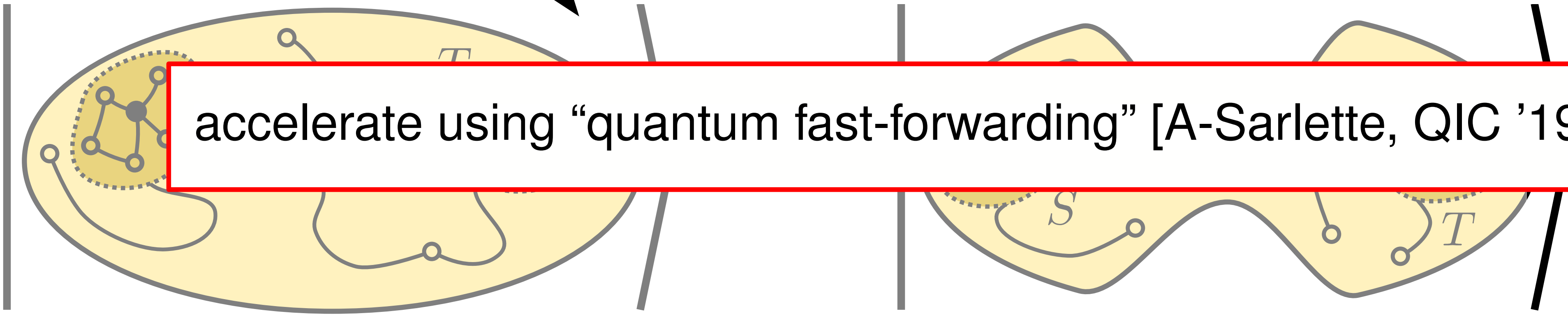

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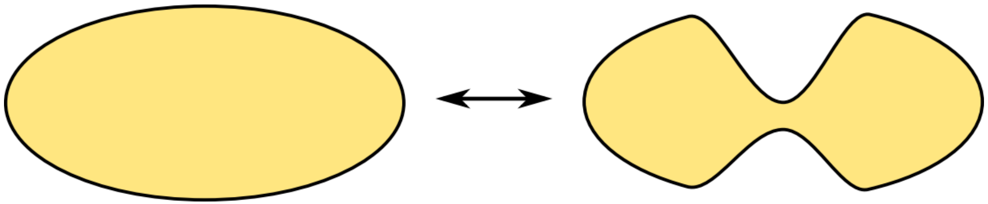

[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets