The Filtering Problem
- Framework
- The Kallianpur-Striebel Formula
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3-steps numerical scheme
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Final remarks

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D. C. Salvador Ortiz-Latorre, A KLV Particle Filter
http://www2.imperial.ac.uk/~dcrisan/Crisan-Ortiz-Latorre-TBBACubFiltering.pdf
The Stochastic filtering Problem

(a, \mathcal{F}, P) probability space \( Z = (X, Y) = \{(X_t, Y_t), t \geq 0\} \)

- \( X \) the signal process - “hidden component”
- \( Y \) the observation process - “the data” - \( Y_t = f(X, \text{“noise”}) \).

The filtering problem: Find the conditional distribution of the signal \( X_t \) given \( Y_t = \sigma(Y_s, s \in [0, t]) \), i.e.,

\[ \vartheta_t(\varphi) = \mathbb{E}[\varphi(X_t)|Y_t], \quad t \geq 0, \quad \varphi \in \mathcal{B}(\mathbb{R}^d). \]

The model

\( V = (V^i_t)_{i=1}^p, t \geq 0 \), \( U = \{(U^i_t)_{i=1}^m, t \geq 0\} \) independent Brownian motions

\[
\begin{align*}
X_t &= X_0 + \int_0^t b(X_s) \, ds + \int_0^t \sigma(X_s) \, dV_s \\
Y_t &= \int_0^t \gamma(X_s) \, ds + U_t,
\end{align*}
\]
The process $Y$ becomes a Brownian motion via a change of measure (Girsanov’s theorem)

$$
\frac{d\tilde{P}}{dP} = Z_t \triangleq \exp \left( - \int_0^t \sum_{k=1}^m \gamma_k (X_s) \, dU^k_s - \frac{1}{2} \int_0^t \sum_{k=1}^m \gamma_k (X_s)^2 \, ds \right), \quad t \geq 0.
$$

Under $\tilde{P}$, $Y$ becomes a Brownian motion independent of $X$. The law of $X$ remains unchanged.

**The Kallianpur-Striebel formula**

$$
\vartheta_t (\phi) = \frac{\rho_t (\phi)}{\rho_t (1)},
$$

where

$$
\rho_t (\phi) = \tilde{E} \left[ \phi (X_t) \exp \left( \int_0^t \sum_{k=1}^m \gamma_k (X_s) \, dY^k_s - \frac{1}{2} \int_0^t \sum_{k=1}^m \gamma_k (X_s)^2 \, ds \right) \mid \mathcal{Y}_t \right] \quad \text{(1)}
$$
The measure valued stochastic process $\rho = \{\rho_t, t \geq 0\}$ satisfies the Duncan-Mortensen-Zakai equation

$$d\rho_t(x) = A^*\rho_t(x)dt + \rho_t(x) \left( \sum_{k=1}^{m} \gamma_k(x) dY^k_t \right)$$

$$d\rho_t(\varphi) = \rho_t(A\varphi)dt + \sum_{k=1}^{m} \rho_t(\gamma_k\varphi)dY^k_t,$$

where

$$A\varphi(x) = \frac{1}{2} \sum_{i,j=1}^{d} a_{ij}(x) \partial_i \partial_j \varphi(x) + \sum_{i=1}^{d} b_i(x) \partial_i \varphi(x)$$

$$a = \sigma \sigma^\top$$

Example:

$$d\rho_t(x) = \frac{1}{2} \Delta \rho_t(x) dt + \rho_t(x) \gamma(x) dY_t.$$

- **R. Frey & W. Runggaldier: Nonlinear Filtering in Models for Interest-Rate and Credit Risk**
  
  The chapter considers filtering problems that arise in Markovian factor models for the term structure of interest rates and for credit risk. The connections with the filtering problem is based on the fact that investors act on the basis of only incomplete information about the factors. The current state of the factors has to be inferred/filtered from observable financial quantities. The main goal is the pricing of derivative instruments in the interest rate and credit risk contexts.

- **R. J. Elliott, H. Miao & Z. Wu: An Asset Pricing Model with Mean Reversion and Regime Switching Stochastic Volatility**
  
  The chapter introduces a generalized stochastic volatility model to help price energy-related assets by capturing two critical features: mean-reverting prices and a volatility which follows different dynamics in different states of the world. Assuming the dynamics of the states are represented by a hidden Markov chain, the authors apply filtering techniques and the EM algorithm to a time-series model for parameter estimation. Several new filters and closed form estimates for all parameters are derived in the paper. Applications of the proposed model in other fields of finance are also discussed.
H. Pham: Portfolio Optimization Under Partial Observation: Theoretical and Numerical Aspects

The chapter is a survey of the methods involved in portfolio selection with partial observation. The author describes both the theoretical and numerical aspects related to these optimization problems. The presentation is divided in two parts. The first part covers the continuous-time problem: here, the mean rates of return of the asset prices are not directly observable. Investors observe only asset prices. By the method of change of probability and innovation process in filtering theory, the partial observation portfolio selection problem is transformed into a full observation one with the additional filter state variable, for which one may apply the martingale or PDE approach. The following cases for the modeling of the unobservable mean rate of return are investigated: Bayesian, linear-Gaussian, and finite-state Markov chain. The second part covers discrete-time optimization problems: this context includes the case of unobservable volatility. The numerical approximation of the optimization problem under partial observation is studied. Several numerical experiments illustrate the results for hedging problems in the context of partially observed stochastic volatility models.


The chapter surveys the recent developments in a general filtering model with counting process observations for the micromovement of asset price and its related statistical analysis. The normalized and unnormalized filtering equations as well as the system of evolution equations for Bayes factors are reviewed. A Markov chain approximation method is used to construct recursive algorithms and their consistency is proven. The authors employ a specific micromovement model built upon the model linear stochastic differential equation to show the steps to develop a micromovement model with specific types of trading noises. The model is further utilized to show the steps to construct consistent recursive algorithms for computing the trade-by-trade Bayes estimates and the Bayes factors for model selection.


The Stochastic filtering Problem

\[ \rho_t(\varphi) = \tilde{E} \left[ \varphi(X_t) \exp \left( \int_0^t \sum_{k=1}^m \gamma_k(X_s) \, dY_s^k - \frac{1}{2} \int_0^t \sum_{k=1}^m \gamma_k(X_s)^2 \, ds \right) \bigg| Y_t \right], \]

where \( dX_t = b(X_t) \, dt + \sigma(X_t) \, dV_t \) and \( X = \Psi_t(V) \) where \( \Psi_t \) is a suitably chosen mapping.

\( \rho_t(\varphi) \) is the expected value of a functional of \( V \) which depends on \( Y \).

\[ \rho_t(\varphi) = \tilde{E}[\Lambda_t(V, Y)] = \int_{\omega \in C([0,\infty),\mathbb{R}^d)} \Lambda_t(\omega, Y) \, dP_V(\omega) \]

A three-step scheme:

- approximate \( \Lambda_{t,x} \) with an explicit/simple version \( \tilde{\Lambda}_t \)
- replace \( P_V \) with \( \tilde{P}_V = \frac{1}{n} \sum_{i=1}^n \delta_{\omega_i} \) - \( \tilde{V} \) approximates the signature of \( V \)
- control the computational effort (use the TBBA)

\[ \rho_t(\varphi) \approx \frac{1}{n} \sum_{i=1}^n \tilde{\Lambda}_{t,x}(\omega_i) \]
Picard discretization

\[
\rho_t(\varphi) = \mathbb{E} \left[ \varphi(X_t)Z_t(X, Y) | \mathcal{Y}_t \right]
\]

\[
Z_t(X, W) = \exp \left( \int_0^t \sum_{k=1}^m \gamma_k(X_s) dY_s^k - \frac{1}{2} \int_0^t \sum_{k=1}^m \gamma_k(X_s)^2 \, ds \right),
\]

Consider the equidistant partition \( \{ \frac{nt}{n} \}_{i=0}^n \) and let \( \rho^n_t \) be the approximation given by

\[
\rho^n_t(\varphi) = \mathbb{E} \left[ \varphi(X_t)Z^n_t(X, Y) | \mathcal{Y}_t \right]
\]

\[
Z^n_t(X, W) = \prod_{i=1}^n \exp \left( \sum_{k=1}^m \left( \gamma_k(X_s) (Y_{\frac{it}{n}} - Y_{\frac{(i-1)t}{n}}) - \frac{t}{2n} \gamma_k(X_s)^2 \right) \right)
\]

Finally define \( \vartheta^n_t(\varphi) \) by the formula \( \vartheta^n_t(\varphi) = \frac{\rho^n_t(\varphi)}{\rho^n_t(1)} \).
**M.** All moments of $X_0$ are finite. The functions $b, \sigma$ are Lipschitz.

**FLp.** $\mathbb{E} [Z_t(X, W)^p] < \infty$ and $\sup_n \mathbb{E} [Z^n_t(X, W)^p] < \infty$ for some $p > 2$.

Condition **FLp** holds true if $\gamma$ is bounded. If $\gamma$ is unbounded, but it has linear growth, then the condition is satisfied if $X$ has exponential moments uniformly bounded on $[0, t]$.

**Theorem**

Assume that conditions **M** and **FLp** hold true and $\gamma_k$, $k = 1, \ldots, m$ are Lipschitz. Then, if $\varphi$ has polynomial growth, there exists a constant $c = c(\varphi, t)$ independent of $n$ such that

$$
\mathbb{E} [|\rho^n_t \varphi - \rho_t \varphi|^2] \leq \frac{c}{n}.
$$

Moreover, if $\sup_n \mathbb{E} [(\theta^n_t (\varphi))^2] < \infty$, then

$$
\mathbb{E} [|\vartheta^n_t \varphi - \vartheta_t \varphi|] \leq \frac{c}{\sqrt{n}},
$$

where, again, $c = c(\varphi, t)$ is a constant independent of $n$. 
Let \( (P_s)_{s \geq 0} \) be the semigroup associated to the Markov process \( X \). We will assume that, for any Lipschitz continuous function \( \psi : \mathbb{R}^d \to \mathbb{R} \), \( P_s \psi \) is twice differentiable for any \( s \in [0, t] \). Moreover, if

\[
P_{a,b} \psi \triangleq P_a \psi - P_b \psi, \quad a, b \in [0, t],
\]

we will assume that there exists a constant \( c_7 = c_7(t) \) independent of \( a \) and \( b \) such that

\[
\sup_{x \in \mathbb{R}^d} |P_{a,b} \psi(x)| \leq c k_\psi \left( \sqrt{a} - \sqrt{b} \right) \tag{4}
\]

\[
\sup_{x \in \mathbb{R}^d} |\partial_i P_{a,b} \psi| \leq \frac{c}{b} k_\psi (a - b), \quad i = 1, \ldots, d, \tag{5}
\]

where \( k_\psi \) is the Lipschitz constant of \( \psi \).

Inequalities (4) and (5) are satisfied if, for example, \( f, \sigma = (\sigma^i)_{i=1}^d \in C^\infty_b(\mathbb{R}^d) \) and the vector fields \( (\sigma^i)_{i=1}^d \) satisfy the Hörmander condition.
Theorem

Assume that conditions $M$, $FL_p$ and $AP$ hold true. Assume also that the functions $\varphi$ and $\gamma_i$, $i = 1, \ldots, m$ are Lipschitz. Then there exists $N > 0$ such that for all $n > N$ and $\varepsilon \in (0, 1)$, there exists a constant $c = c(\varphi, t, N, \varepsilon)$ independent of the partition such that

$$
E \left[ |\rho^n_t \varphi - \rho_t \varphi|^2 \right] \leq \frac{c}{n^{2-\varepsilon}}.
$$

Moreover, if $\sup_n E[(\vartheta^n_t (\varphi))^2] < \infty$, then for all $n > N$ and $\varepsilon \in (0, 1)$, there exists a constant $c = c(\varphi, t, N, \varepsilon)$ independent of the partition such that

$$
E \left[ |\vartheta^n_t \varphi - \vartheta_t \varphi| \right] \leq \frac{c}{n^{1-\varepsilon}}.
$$
Theorem

Assume that conditions $M, FLp$ are satisfied and that the functions 
$\gamma_i \in C_b^2(\mathbb{R}^d)$ for $i = 1, \ldots, m$. Then, if $\varphi$ has polynomial growth, there exists a 
constant $c = c(\varphi, t)$ independent of $n$ such that

$$\mathbb{E} \left[ |\rho^n_t \varphi - \rho_t \varphi|^2 \right] \leq \frac{c}{n^2}. \quad (6)$$

Moreover, if $\sup_n \mathbb{E}[(\vartheta^n_t (\varphi))^2] < \infty$,

$$\mathbb{E} \left[ |\vartheta^n_t \varphi - \vartheta_t \varphi| \right] \leq \frac{c}{n},$$

where, again, $c = c(\varphi, t)$ is a constant independent of $n$. 
Chen’s iterated integrals expansion - signature of a path

Let $T(\mathbb{R}^d) = \bigoplus_{i=0}^{\infty} (\mathbb{R}^d)^{\otimes i}$ and $T^{(m)}(\mathbb{R}^d) = \bigoplus_{i=0}^{m} (\mathbb{R}^d)^{\otimes i}$ be the tensor algebra of all non-commutative polynomials over $\mathbb{R}^d$ and, respectively the tensor algebra of all non-commutative polynomials of degree less than $m + 1$. For a path $\omega \in C_{bv}([0, \infty), \mathbb{R}^d)$ define its signature $S_t(\omega)$ and, respectively, its truncated signature $S^m_t(\omega)$ to be the corresponding Chen’s iterated integrals expansion:

$$S : C_{bv}([0, \infty), \mathbb{R}^d) \to T(\mathbb{R}^d) \quad S_t(\omega) = \sum_{k=0}^{\infty} \int_{0 < t_1 \ldots t_k < t} d\omega_{t_1} \otimes \ldots \otimes d\omega_{t_k}$$

$$S^m : C_{bv}([0, \infty), \mathbb{R}^d) \to T^{(m)}(\mathbb{R}^d) \quad S^m_t(\omega) = \sum_{k=0}^{m} \int_{0 < t_1 \ldots t_k < t} d\omega_{t_1} \otimes \ldots \otimes d\omega_{t_k}.$$

The (random) signature and, respectively, the truncated signature of the Brownian motion are

$$S_t(W) = \sum_{k=0}^{\infty} \int_{0 < t_1 \ldots t_k < t} dW_{t_1} \otimes \ldots \otimes dW_{t_k}, \quad S^m_t(W) = \sum_{k=0}^{m} \int_{0 < t_1 \ldots t_k < t} dW_{t_1} \otimes \ldots \otimes dW_{t_k}.$$
- $E[S_t(W)]$ uniquely identifies the Wiener measure $P_W$.
- If $\tilde{W}$ is another process such that $E[S_t^m(W)] = E[S_t^m(\tilde{W})]$, then

$$E[\Lambda_{t,x}(W)] \simeq E[\Lambda_{t,x}(\tilde{W})]$$  

high order approximation.

See Crisan and Ghazali [2007] for conditions.

- Several choices for $\tilde{W}$: Kusuoka [2001,2004], Kusuoka and Ninomiya [2004], Lyons and Victoir [2004], Ninomiya and Victoir [2004], etc.
Theorem (Lyons & Victoir (2004))

For any \( t > 0 \), there exists paths \( \omega_1, \ldots, \omega_N \in C^0_{0,bv}([0, t]; \mathbb{R}^d) \) and \( \lambda_1, \lambda_2, \ldots, \lambda_N \) (\( \sum_{i=1}^{N} \lambda_i = 1 \)), such that if \( P(\tilde{W} = \omega_i) = \lambda_i \) then

\[
E \left[ S_t^m(W) \right] = E \left[ S_t^m(\tilde{W}) \right].
\]

If the above is true, we call \( L_{\tilde{W}} = \sum_{i=1}^{N} \lambda_i \delta_{\omega_i} \) the cubature measure and denote it by \( \mathcal{Q}_t^m \).

If we want to approximate \( \mathbb{E}[\alpha(X_t)] \), where \( X \) is the solution of the following SDE

\[
dX_t = V_0(X_t) dt + \sum_{i=1}^{d} V_i(X_t) \circ dW_t^i
\]

Then \( X \) can be expressed as \( X = \Psi_t(W) \) giving a representation of the form \( \mathbb{E}[\Lambda_t(W)] \). Choose \( X^j \) to be the solution of the following ODE

\[
dX^j_t = V_0(X^j_t) dt + \sum_{i=1}^{d} V_i(X^j_t) d\omega^i_t.
\]
In this case:

\[ \mathbb{E}[\Lambda_t(\tilde{W})] = \mathbb{E}_{\mathbb{Q}^m}[\alpha(X_t)] = \sum_{i=1}^{N} \lambda_i g(X_t^i) \]

and

\[ |\mathbb{E}[\Lambda_t(W)] - \mathbb{E}[\Lambda_t(\tilde{W})]| \leq C\delta^{m-1}. \]

**Cubature of order 3:** For d=1, we can use 2 paths with equal weights \( \lambda_j = \frac{1}{2} \) defined as

\[ \omega_t^j = tz^j, \]

where \( z^j \in \{-1, 1\} \). For \( d \geq 2 \), we can use \( \left\lfloor \frac{d(d+2)}{6} \right\rfloor + 1 \) linear paths.

**Cubature of order 5:** For d=1, we can use 3 paths, \( \omega, -\omega \) and 0 with \( \omega \) is defined as

\[ \omega(t) = \begin{cases} 
\frac{\sqrt{3}}{2} \left(4 - \sqrt{22}\right) t & t \in [0, \frac{1}{3}] \\
\frac{\sqrt{3}}{6} \left(4 - \sqrt{22}\right) + \sqrt{3} \left(-1 + \sqrt{22}\right) \left(t - \frac{1}{3}\right) & t \in \left[\frac{1}{3}, \frac{2}{3}\right] \\
\frac{\sqrt{3}}{6} \left(2 + \sqrt{22}\right) + \frac{\sqrt{3}}{2} \left(4 - \sqrt{22}\right) \left(t - \frac{2}{3}\right) & t \in \left[\frac{2}{3}, 1\right]
\end{cases} \]
Introduce the operators

\[
\bar{R}_s^n \varphi(x) = \mathbb{E}_{Q_m}[\varphi(X_s^x)] \\
\bar{R}_s^i \varphi(x) = \mathbb{E}_{Q_m}[\varphi(X_s^x) \exp(g_i(X_s^x))]
\]

Recall that \( \rho_t^n(\varphi) = \mathbb{E} \left[ R_t^0 R_t^1 \ldots R_t^n \varphi(X_0) \mid \mathcal{W}_t \right] \) with corresponding approximation

\[
\bar{\rho}_t^n(\varphi) = \mathbb{E}_{Q_m} \left[ \bar{R}_t^0 \bar{R}_t^1 \ldots \bar{R}_t^n \varphi(X_0) \mid \mathcal{W}_t \right].
\]

**Theorem**

*For all \( \varphi \in C_b^{m+2}(\mathbb{R}^d) \) and \( p \geq 1 \),

\[
\|\rho_t^n(\varphi) - \bar{\rho}_t^n(\varphi)\|_p \leq \frac{c}{n^{(m-1)/2}} \sum_{i=1}^{m+2} \|\nabla^i \varphi\|_\infty.
\]

where \( c = c(t, m, p) \) is independent of \( n \).
• An additional procedure is used to control the computational effort.
• The measure $Q^m$ is replaced by a measure $\tilde{Q}^{m,N}$ with support of size $N$ by using a *tree based branching algorithm* (TBBA).
• The TBBA was introduced as correction procedure in the context of the filtering problem. The method has a wider applicability: it is applicable to any method that uses branching trees.
• By merging the TBBA with the cubature method we keep the number of particles on the support of the intermediate measures constant.
The TBBA is a minimal variance Monte Carlo method:

- Assume that we constructed a measure on paths

\[ Q^i = \frac{\sum_{j=1}^{M} \lambda_j^i w_j^i \delta_{\gamma_j}}{\sum_{j=1}^{M} \lambda_j^i w_j^i}, \]

where

\[ w_j^i = \exp \left( \sum_{k=1}^{m} \left( \gamma_k \left( \beta_{j_{n_t}}^j \right) (Y_{(i+1)t} - Y_{it}) - \frac{t}{2n} \gamma_k \left( \beta_{j_{n_t}}^j \right)^2 \right) \right). \]

with \( M \) paths and we want to reduce the number to at most \( N \) paths.

- We replace \( Q^i \) with a random measure \( \tilde{Q}^i \) such that \( \text{supp}(\tilde{Q}^i) \subseteq \text{supp}(Q^i) \) and that the size of its support is at most \( N \). For an arbitrary path \( \gamma \in \text{supp}(Q^i) \), we will have

\[ \hat{Q}^i(\gamma) = \begin{cases} \left\lfloor \frac{NQ^i(\gamma)}{N} \right\rfloor & \text{with probability } 1 - \{NQ^i(\gamma)\} \\ \left\lfloor \frac{NQ^i(\gamma)}{N} \right\rfloor + 1 & \text{with probability } \{NQ^i(\gamma)\} \end{cases} \]
The Stochastic filtering Problem
Computational effort reduction

- In addition, \( \tilde{Q}^i \) is constructed so that it is a (random) probability measure, i.e.,
  \[
  \sum_{\gamma \in \text{supp}(Q)} \hat{Q}^i(\gamma) = 1. \tag{8}
  \]
- The mass allocated to each path \( \gamma \in \text{supp}(Q) \) is either 0 or an integer multiple of \( 1/N \) \( \Rightarrow \) the support of any realization of \( \tilde{Q}^i \) has size at most \( N \).
- The maximum number of paths is achieved when \( \tilde{Q}^i \) puts mass \( 1/N \) on \( N \) of the \( M \) paths. This is the basis of the control of the computational effort.
- Let \( \tilde{\rho}^n_t \) be a suitable unnormalized version of the probability measure obtain by applying \( n \) times this procedure.

**Theorem (D.C. S. Ortiz-Latorre 2011)**

\[
\mathbb{E}[|\tilde{\rho}^n_t(\varphi) - \bar{\rho}^n_t(\varphi)|^2] \leq \frac{C}{N}.
\]

**Corollary**

\[
\mathbb{E}[|\tilde{\rho}^n_t(\varphi) - \rho_t(\varphi)|^2] \leq C \left( \frac{1}{n^\alpha} + \frac{1}{n^{m-1}} + \frac{1}{N} \right).
\]
Consider the 1-dimensional Benes filter:

\[
dX_t = \mu \sigma \tanh \left( \frac{\mu X_t}{\sigma} \right) dt + \sigma dV_t
\]
\[
dY_t = (h_1 X_t + h_2) dt + dU_t,
\]

Then

\[
\rho_t \simeq w^+ \mathcal{N}(A^+_t/(2B_t), 1/(2B_t)) + w^- \mathcal{N}(A^-_t/(2B_t), 1/(2B_t)),
\]

where

\[
w^\pm_t \triangleq \exp \left( \frac{(A^\pm_t)^2}{4B_t} \right) / \left( \exp \left( \frac{(A^+_t)^2}{4B_t} \right) + \exp \left( \frac{(A^-_t)^2}{4B_t} \right) \right)
\]
\[
A^\pm_t \triangleq \pm \frac{\mu}{\sigma} + h_1 \psi_t + \frac{h_2 + h_1 x_0}{\sigma \sinh(h_1 \sigma t)} - \frac{h_2}{\sigma} \coth(h_1 \sigma t),
\]
\[
B_t \triangleq \frac{h_1}{2\sigma} \coth(h_1 \sigma t),
\]
\[
\psi_t \triangleq \int_0^t \frac{\sinh(h_1 \sigma s)}{\sinh(h_1 \sigma t)} dW_s,
\]
Comparison with the classical particle filter implemented using the Euler scheme plus the TBBA to perform the resampling at each time step.

Number of launches $M = 10$

Number of particles used in the KLV algorithm $N = 100$

Number of particles used for the classical particle $N = 10000$

Parameter values

$$\mu = 0.05, \quad h_1 = 0.8, \quad h_2 = 0.0, \quad \sigma = 1.0, \quad x_0 = 0.0 \quad T = 20.0.$$
The solution of the filtering problem is approximated through the Kallianpur-Striebel representations.

The common method contains three steps: functional discretization, Wiener measure approximation (cubature method) and computational effort reduction.

The cubature method is essentially deterministic. The diffusion approximation uses a set of ordinary differential equations to approximate the distribution of the solution of the SDE.

The (exponentially) increase in the computational effort is controlled by the TBBA (a random method).