Causal Discovery in Observational Time Series

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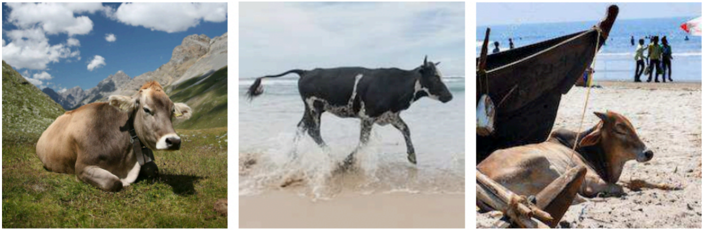

(A) **Cow: 0.99**, Pasture: 0.99, Grass: 0.99, No Person: 0.98, Mammal: 0.98

(B) No Person: 0.99, Water: 0.98, Beach: 0.97, Outdoors: 0.97, Seashore: 0.97

(C) No Person: 0.97, **Mammal: 0.96**, Water: 0.94, Beach: 0.94, Two: 0.94

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    - What if my train had not been late?
    - How effective is a treatment in preventing a disease?
    - Can hiring records prove an employer guilty of gender discrimination?

Why causality for time series?

- Causality is crucial for explanatory purpose, since an effect can be explained by its causes, regardless of the correlations it may have with other variables
- Time series are everywhere but you know that ;)

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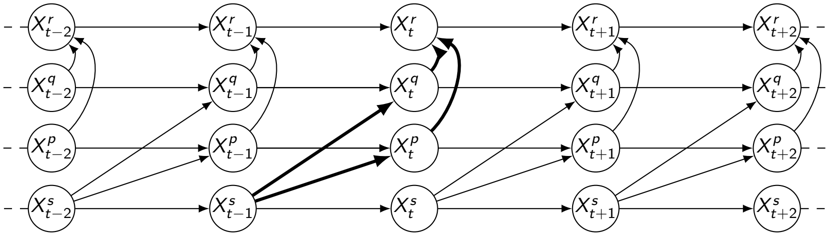

Figure: *Running example*: a diamond structure with self causes.

Outline

Causal graphs for time series

A d-variate time series X of continuous values
For a fixed t, each X_t is a vector $(X_t^1, \ldots, X_t^d)$,
in which X_t^p is the measurement of the pth time series at time t.

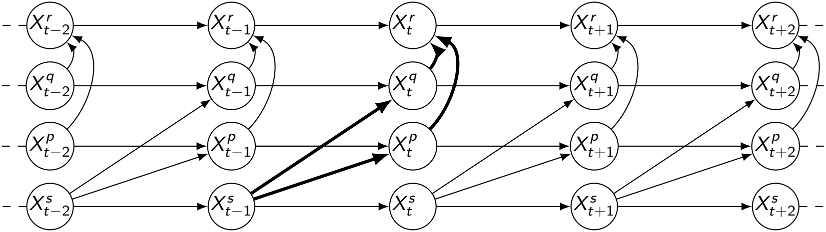

Figure: Full time causal graph.

Causal graphs for time series

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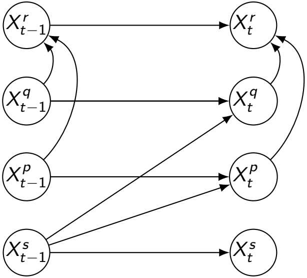
A causal graph for a multivariate time series X is said to be *consistent throughout time* if all the causal relationships remain constant in direction throughout time.



Window Causal Graph

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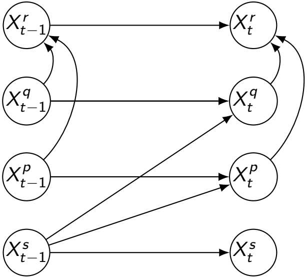

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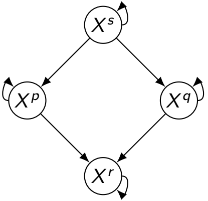
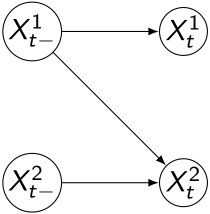


Summary Causal Graph

Classical Assumptions

- A set of variables is said to be *causally sufficient* if all common causes of all variables are observed.
- A causal relation between two variables is said to satisfy the *temporal priority* if it is oriented in such a way that the cause occurred before its effect.
- *Causal Markov Condition*: (conditional) independence in the graph leads to (conditional) independence in the data.



$$X^1_t \perp\!\!\!\perp X^2_t \mid X^1_{t-}$$

Classical Assumptions

- A set of variables is said to be *causally sufficient* if all common causes of all variables are observed.
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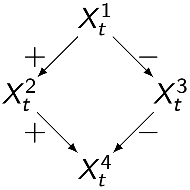
- *Causal Markov Condition*: (conditional) independence in the graph leads to (conditional) independence in the data.
- *Minimality condition*: the graph does not contain dependencies not present in the observational data.
- *Faithfulness*: only the conditional independence relations true in the data are entailed by the Causal Markov condition applied to the graph.



Outline

Granger Causality

X^p Granger-causes X^q if past values of X^p provide unique statistically significant information about future values of X^q.

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Pairwise Granger causality

$$X_t^q = a_{q,0} + \sum_{i=1}^{\tau} a_{q,i} X_{t-i}^q + \xi_t^q, \qquad \text{(Mres)}$$

$$X_t^q = a_{q,0} + \sum_{i=1}^{\tau} a_{q,i} X_{t-i}^q + \sum_{i=1}^{\tau} a_{p,i} X_{t-i}^p + \xi_t^q, \qquad \text{(Mfull)}$$

Statistical test (e.g. F-test) can be used to determine whether (Mfull) is significantly better than (Mres),

H_0: X^p does not Granger-cause X^q.

Granger Causality

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Pairwise Granger causality

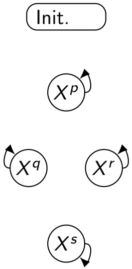

Figure: Running example: structure inferred by the pairwise Granger method (an arbitrary order has been chosen for the example).

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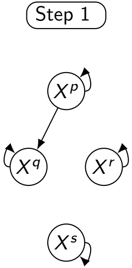

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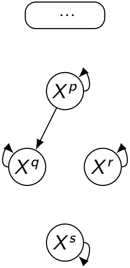

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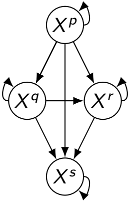
Pairwise Granger causality

Step 6



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Granger Causality

Multivariate Granger causality

$$X_t^q = a_{q,0} + \sum_{\substack{r=1\\r\neq p}}^{d} \sum_{i=1}^{\tau} a_{r,i} X_{t-i}^p + \xi_t^q, \qquad \text{(mvMres)}$$

$$X_t^q = a_{q,0} + \sum_{r=1}^{d} \sum_{i=1}^{\tau} a_{r,i} X_{t-i}^r + \xi_t^q, \qquad \text{(mvMfull)}$$

Extensions

- Non-linear associations
- Nonstationnarity

Constraint-based approaches

Constraint-based approaches exploit conditional independencies to build a skeleton between variables. This skeleton is then oriented according to a set of rules that define constraints on admissible orientations.

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Assumptions

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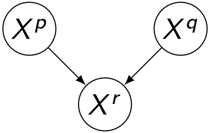
v-structures (colliders): only structures which can be oriented without ambiguity.



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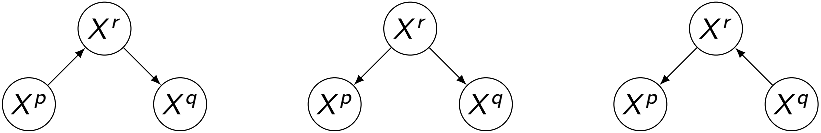


Figure: Three equivalent structures: $X^p \perp\!\!\!\perp X^q | X^r$

Markov equivalence class: set of DAGs that encode the same set of conditional independencies.

Constraint-based approaches

PCMCI

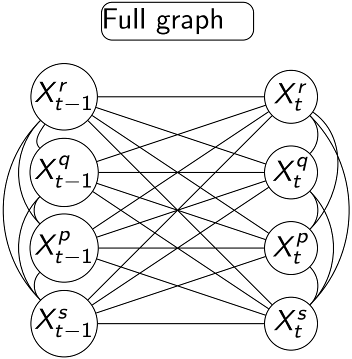
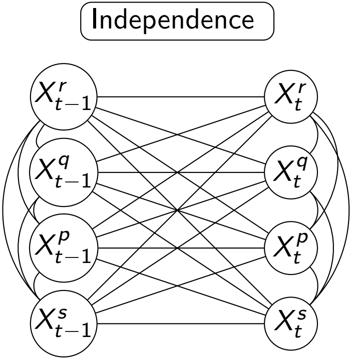


Figure: Running example: structure inferred by PCMCI with instantaneous relations.

Constraint-based approaches
PCMCI



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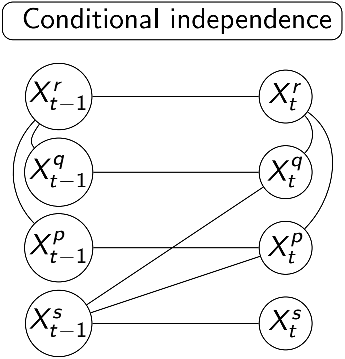

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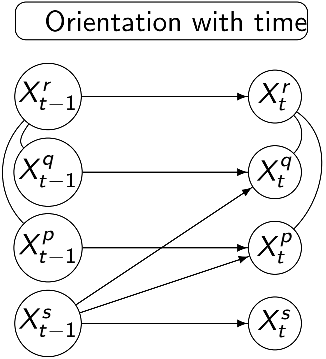

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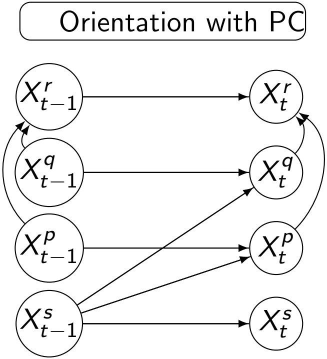

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Noise-based approaches

Causal system described by a set of equations, where each equation explains one variable of the system in terms of its direct causes and some additional noise.

Assumptions

- Causal Markov Condition
- Minimality

Can deal with 2 variables

Additive noise model with nonlinear functions

$$X^p = \xi^p,$$
$$X^q = f_q(X^p) + \xi^q \qquad \text{with } X^p \perp\!\!\!\perp \xi_q.$$

Noise-based approaches

Additive Noise Models

Additive noise model with nonlinear functions

$$X^p = \xi^p,$$
$$X^q = f_q(X^p) + \xi^q \qquad \text{with } X^p \perp\!\!\!\perp \xi_q.$$

Theorem (Identifiability of ANMs)

Assume that the conditional distribution of $X^q \mid X^p$ admits a smooth ANM, and that there exists $x_q \in \mathbb{R}$ such that, for almost all $x_p \in \mathbb{R}$,

$$(\log p_{\xi^q})''(x_q - f_q(x_p))f_q'(x_p) \neq 0.$$

Then, the set of log densities $\log p_X$ for which the obtained joint distribution P_{X^p,X^q} admits a smooth ANM from X^q to X^p is contained in a 3-dimensional affine space.

Additive noise model with nonlinear functions

$$X^p = \xi^p,$$
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Principle (Multivariate additive noise principle)

Suppose we are given a joint distribution $P(X^1, \cdots, X^d)$. *If it satisfies an identifiable Additive Noise Model such that* $\{(X^p_{t-j})_{1\leq p\neq q\leq d, 0\leq j\leq \tau}, (X^q_{t-j})_{1\leq j\leq \tau}\} \rightarrow X^q$, *then it is likely that* $\{(X^p_{t-j})_{1\leq p\neq q\leq d, 0\leq j\leq \tau}, (X^q_{t-j})_{1\leq j\leq \tau}\}$ *precedes* X^q *in the causal order.*

Noise-based approaches

VarLINGAM

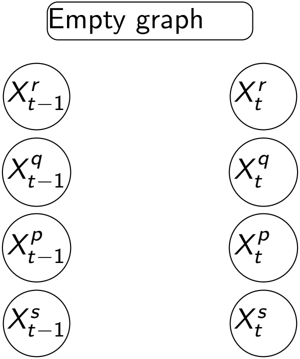

Figure: Running example: structured inferred by VarLiNGAM.

Noise-based approaches

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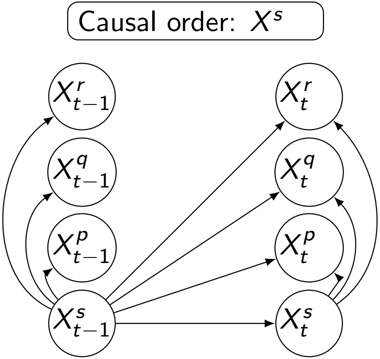

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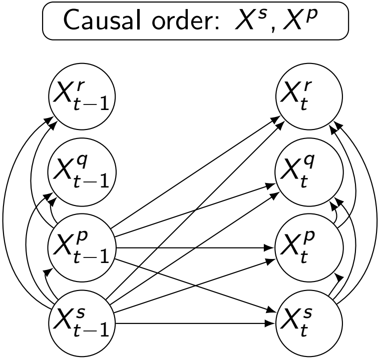

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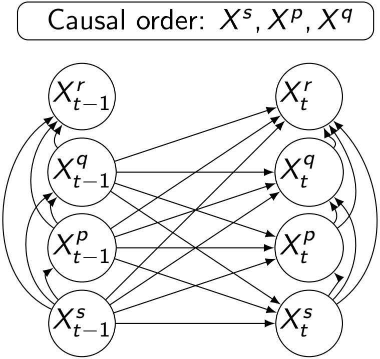
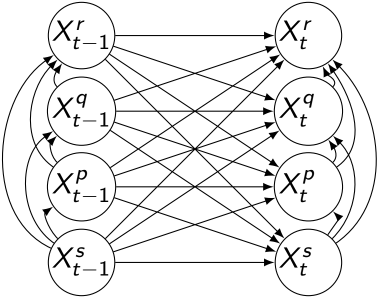


Figure: Running example: structured inferred by VarLiNGAM.

Noise-based approaches
VarLINGAM

Causal order: X^s, X^p, X^q, X^r



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Noise-based approaches

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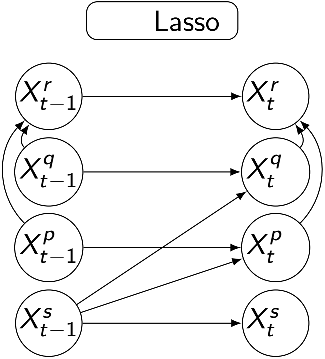

Figure: Running example: structured inferred by VarLiNGAM.

Outline

NBCB[1]

A mix between noise-based and constraint-based approaches

Assumptions

- Causal Markov Condition
- Adjacency faithfulness: if X^p and X^q are adjacent, then they are not conditionally independent given any subset of vertices except Xp, Xq.
- Minimality

[1]ECMLPKDD 2021

NBCB[1]

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Representation of the time series

Optimal lag γ_{pq} and $(\lambda_{pq}, \lambda_{qp})$ the *optimal* windows:

$$\gamma_{pq}, \lambda_{pq}, \lambda_{qp} = \underset{\gamma \geq 0, \lambda_1, \lambda_2}{\text{argmax}} \ h(X^q_{t:t+\lambda_2} \mid X^q_{t-1}, X^p_{t-\gamma-1}) - h(X^q_{t:t+\lambda_2} \mid X^p_{t-\gamma-1:t-\gamma+\lambda_1}, X^q_{t-1}).$$

where h denotes the entropy.

[1] ECMLPKDD 2021

NBCB

A mix between noise-based and constraint-based approaches

Step 1: causal ordering (additive noise model)
Last place: time series which yields the residuals that are more independent to the other time series.
Step 2: pruning to remove spurious relations

$$\begin{aligned}
&\mathrm{TCE}(X^p \rightarrow X^q \mid X^{\mathbf{R}}) = \\
&\min_{\Gamma_{r_i} \geq 0,\, 1 \leq i \leq K} \; h(X^q_{t:t+\lambda_{qp}} \mid (X^{r_i}_{t-\Gamma_{pq|r_i}})_{1 \leq i \leq K}, X^q_{t-1}, X^p_{t-\gamma_{pq}-1}) \\
&- h(X^q_{t:t+\lambda_{qp}} \mid (X^{r_i}_{t-\Gamma_{pq|r_i}})_{1 \leq i \leq K}, X^p_{t-\gamma_{pq}-1:t-\gamma_{pq}+\lambda_{pq}}, X^q_{t-1}),
\end{aligned}$$

where $\Gamma_{pq|r_1}, \cdots, \Gamma_{pq|r_K}$ are the lags between $X^{\mathbf{R}}$ and X^q.

NBCB

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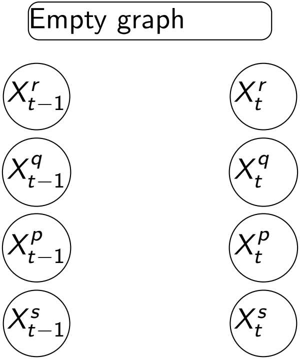

Figure: Running example: structured inferred by NBCB.

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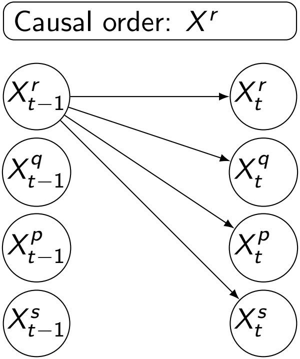

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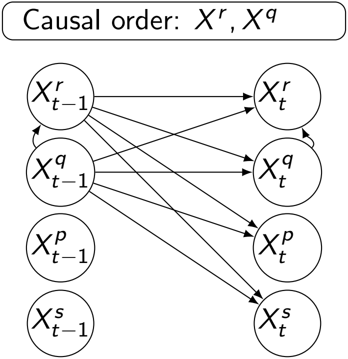

Figure: Running example: structured inferred by NBCB.

NBCB

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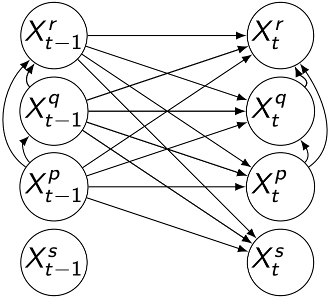
Causal order: X^r, X^q, X^p



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NBCB

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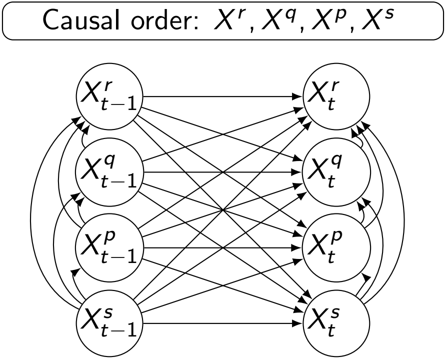


Figure: Running example: structured inferred by NBCB.

NBCB

A mix between noise-based and constraint-based approaches

Conditional independence using TCE

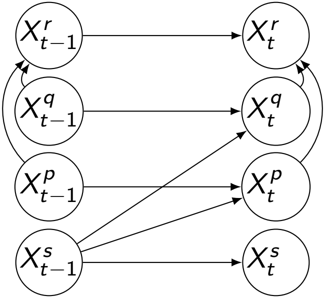
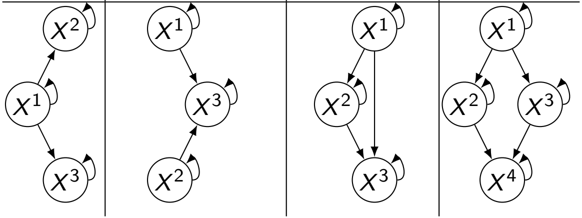


Figure: Running example: structured inferred by NBCB.

Experiments

Table: Structures of simulated data.

Fork	V-structure	Mediator	Diamond



Experiments

Simulated data 1

Table: Results obtained on the simulated data for the different structures with 1000 observations. We report the mean and the standard deviation of the F1 score. The best results are in bold.

	V-struct	Fork	Mediator	Diamond
GC	0.37 ± 0.25	0.44 ± 0.38	0.83 ± 0.22	0.66 ± 0.26
PCMCI	0.67 ± 0.37	0.78 ± 0.17	0.84 ± 0.09	0.82 ± 0.16
VarLiNGAM	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.03 ± 0.08
TiMINo	0.65 ± 0.37	0.52 ± 0.44	0.80 ± 0.19	0.60 ± 0.25
NBCB	0.67 ± 0.28	0.67 ± 0.38	0.66 ± 0.32	0.71 ± 0.16

Table: Results obtained on the unfaithful simulated data for the different structures with 1000 observations. We report the mean and the standard deviation of the F1 score. The best results are in bold.

	unfaith. Mediator	unfaith. Diamond
GC	0.12 ± 0.27	0.14 ± 0.23
PCMCI	0.05 ± 0.15	0.20 ± 0.22
VarLiNGAM	0.0 ± 0.0	0.02 ± 0.06
TiMINo	$\mathbf{0.64} \pm 0.08$	0.49 ± 0.03
NBCB	0.56 ± 0.26	$\mathbf{0.5} \pm 0.31$

Experiments

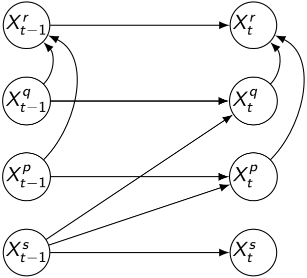
Real datasets

Table: Results for real datasets. We report the mean and the standard deviation of the F1 score.

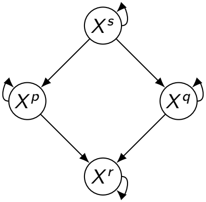
	Temperature	Diary	FMRI
GC	0.66	0.33	0.24 ± 0.18
PCMCI	1	0.5	0.22 ± 0.18
VarLiNGAM	0	0.0	0.49 ± 0.28
TiMINo	0	0.0	0.32 ± 0.11
NBCB	1	0.8	0.40 ± 0.21

Outline

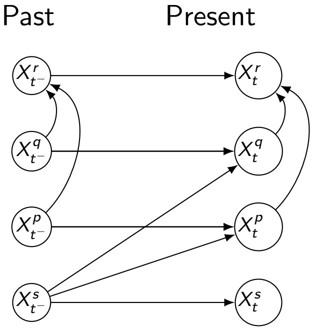
PCGCE[2]: discover an extended summary causal graph



Window Causal Graph



Summary Causal Graph



Extended Summary Causal Graph

[2] UAI 2022

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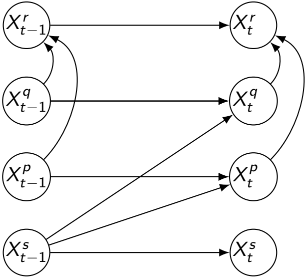

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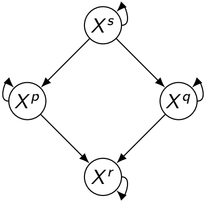

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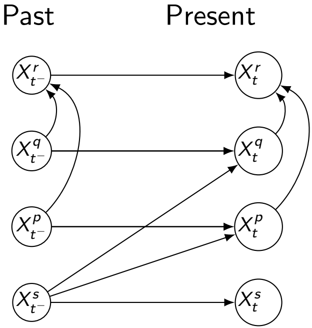

Extended Summary Causal Graph

Window causal graph

Difficult to be validated and analyzed by experts

Computationally expensive

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[2] UAI 2022

PCGCE: discover an extended summary causal graph

Assumptions

- Causal Markov condition
- Faithfulness
- Causal sufficiency for PCGCE (but extension to FCIGCE)

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- γ: *maximum gap between a cause and its effect*
- X_{t-}^p do not cause X_t^q iff there exists
  - $X^{\mathbf{Pr}} = \{X_t^{Pr_1}, \cdots, X_t^{Pr_m}\}$ and $X^{\mathbf{Pa}} = \{X_{t-}^{Pa_1}, \cdots, X_{t-}^{Pa_l}\}$ s.t.
  - $\mathrm{GCE}(X^p \to X^q | X^{\mathbf{Pa}}, X^{\mathbf{Pr}}) = 0 \qquad (1)$
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Estimation and testing

kNN estimator and local permutation test

PCGCE: discover an extended summary causal graph

Running example

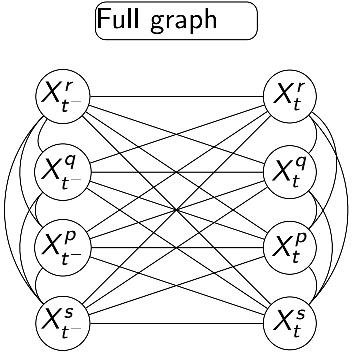
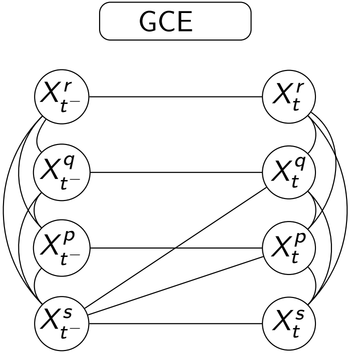


Figure: Running example: structure inferred by PCGCE with instantaneous relations.

PCGCE: discover an extended summary causal graph

Running example



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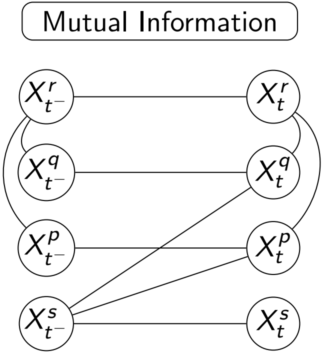

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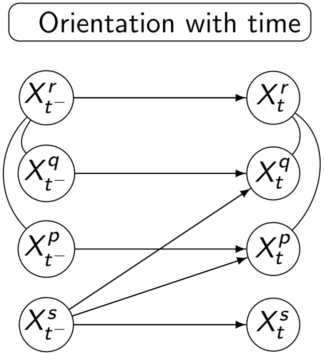

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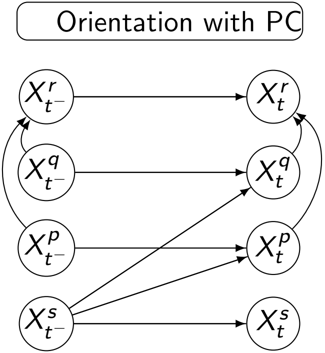

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PCGCE

Experiments and conclusion

- Several experiments to understand the gain
- Performances comparable to PCMCI, but algorithm much faster

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- PCGCE and FCIGCE can loose performance for high maximum time lags, compared to window-approaches
- We can think of reducing the dimension of the past slice (using autoencoders?)

Outline

Conclusion and perspectives

- Many families to discover causal graph for time series (more than the one introduced here: score-based, logic-based, topology-based, difference-based)
- Hybrid methods can take benefit of several worlds
- An important question is: which causal graph do we want to infer?
- The representation of time series is essential (windows - lags)

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- Some ad: we are organizing a trimester on causality (between Paris, Grenoble and Saclay) in April, May, June 2023!

References

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