Private Sampling with Malicious Samplers

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Work supervised by Jan Ramon and improved by discussions with Andreas Peter (Univ. of Twente, Netherlands).

Outline

Introduction and Problem

Existent tools

Our solutions

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- Privacy Preserving Machine Learning
- Many parties with sensitive data
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A malicious P_j can poison $\mathcal{M}$ to

- decrease customers of other stores
- increase its own profit

Similar settings

Decentralized systems with untrusted participants

- financial systems [Ben Sasson et al., 2014]
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- Publish encrypted inputs (e.g. in a blockchain)
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In our ML setting

- Input remains private, but consistent
- If a party lies, it has to lie repeatedly
- This also holds in non-private ML: not possible to ensure truth on the input

Challenges

ML Computations

- Their domain is $\mathbb{R}$
- Involve transcendental functions (e.g. e^x, $\ln(x)$, ... for activation filters)
- Sample numbers from Gaussian, Laplacian distributions (e.g. for Differential Privacy [Dwork, 2006])

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Our contribution

We focus on sampling: prove that a private value x is sampled from a distribution $\mathcal{D}$.
- But we also contribute in transcendental computations.

Problem Statement

Let

- s malicious parties $P_1, \ldots, P_s$ that can tamper with the protocol.
- a well known distribution $\mathcal{D}$.

For some $i \in \{1, \ldots, s\}$, sample $x \in \mathbb{R}$ such that

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Example 2 Differentially Private Federated Learning

for t = 1 **to** T **do**
  At each party P_i**:** sample $\eta \sim \mathcal{D}$, compute $\Theta_u^t \leftarrow \textsc{LocalUpdate}(\Theta^{t-1}, \Theta_u^{t-1}) + \eta$
  Compute $\Theta^t \leftarrow \frac{1}{n} \sum_u \hat{\Theta}_u^t$
end for

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Commitments [Blum, 1983]

Commit to a value in $\mathbb{Z}_p$ while keeping it hidden

- Binding: the value cannot be changed once committed
- Similar to an encrypted value, but not neccesarily decryptable

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- $x_1, \ldots, x_n$ committed values and $C : \mathbb{Z}_p^m \to \mathbb{Z}_p^k$ circuit (only modular $+$ and $\times$)
  $\implies$ can prove $C(x_1, \ldots, x_n) = \bar{0}$

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This is not FHE: We are just proving relations, not computing over encryptions

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- Any polynomial relation in $\mathbb{Z}_p$
- If $x = A$ then S_1 else if $x = B$ then S_2:
  $(x - A = 0 \wedge S_1) \vee (x - B = 0 \wedge S_2)$
- ...

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Inversion method: $x \leftarrow CDF_{\mathcal{D}}^{-1}(u_1)$, if $CDF_{\mathcal{D}}^{-1}(x)$ is efficient

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- Box Müller:

$$\begin{cases} x_1 & \leftarrow \sqrt{-2\ln(u_1)}\sin(2\pi u_2) \\ x_2 & \leftarrow \sqrt{-2\ln(u_1)}\cos(2\pi u_2) \end{cases}$$

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- Polar Method: $u_1, u_2 \in (-1, 1)$

$$\begin{cases} \rho & = u_1^2 + u_2^2 \qquad \text{(if } \rho \geq 1 \text{ or } \rho = 0\text{, re-sample } u_1, u_2) \\ x_1 & \leftarrow u_1\sqrt{-2\ln(\rho)/\rho} \\ x_2 & \leftarrow u_2\sqrt{-2\ln(\rho)/\rho} \end{cases}$$

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Can amortize the generation of s uniforms with cost O(1) per party.

Proving Transcendental computations

Cryptographic Primitives for $\mathbb{R}$ (fixed-precision)

- Encode reals in $\mathbb{Z}$ (up to a certain fixed precision)
- Use integer proofs to implement computer operations: $+, \times$, bit-shift $(>>)$, $\div$
- requires dealing with rounding issues

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Use numerical approximations

From computer operations can compute

- $\sin, \cos, \log, e^x, \sqrt{x}$ with CORDIC algorithm [Walther, 1971] (mostly requires $+$ and $>>$)
- Gaussian $CDF^{-1}(x)$ with rational functions and Taylor polynomials

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We prove their correct execution

Preliminar results

Group Exponentiations (GExp) are the dominant computations

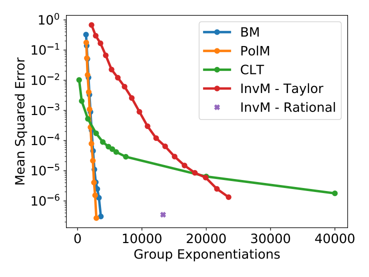
- Prove $\sin$, $\cos$, $\log$, e^x, $\sqrt{x}$ with n bits of precision with $O(n^2)$ GExp
  (Of independent interest in ML)

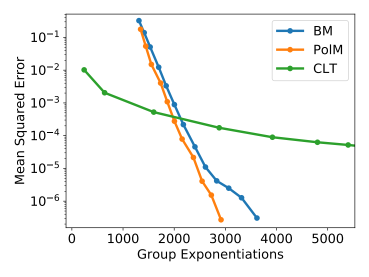
Simulated Gaussian sampling proofs

- Central Limit Theorem Approach (CLT)
- Box Muller (BM) and Polar Method (PolM)
- Inversion Method (InvM) with Taylor and rational approximations

Experiments

Measured MSE wrt to a quality Gaussian [1] over 10^7 samples per method





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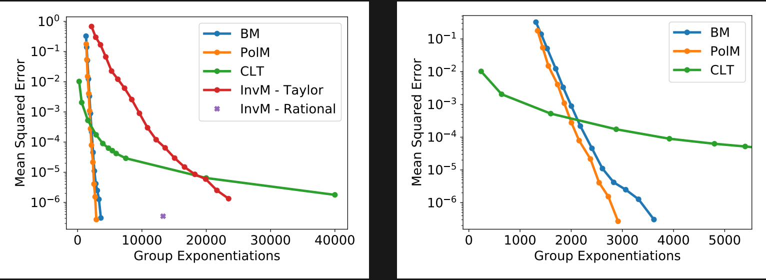

A quality sample requires < 3000 GExp

- ~ 0.17 seconds in an Intel Core i7 [2] (but largely optimizable)

[1] Implemented with C++ boost library

[2] With the implementation by [Franck and Großschädl, 2017]

Arbitrary Distributions: a sketch

Set Membership: Can prove $x \in S$ for private x and public $S \subset \mathbb{Z}_p$ [Camenisch et al., 2008]

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Inversion Method with *table lookups* (in the clear)

- Sample from $\mathcal{D}$ distribution from 2^n equiprobable bins
- Precompute 2^n points $t_1, \ldots, t_{2^n}$ of CDF^{-1} in $(0, 1)$
- Sample uniformly $u \in \{1, \ldots, 2^n\}$ and return the u-th point

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Private table lookups

- Let $D = \{\mathrm{enc}(1, t_1), \ldots, \mathrm{enc}(2^n, t_{2^n})\}$ for some integer encoding enc
- $u \sim \mathcal{U}\{1, \ldots, 2^n\} \wedge \mathrm{enc}(u, x) \in D \implies x \sim \mathcal{D}$

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$O(1)$ GExp per sample but $O(2^n)$ GExp of preprocessing (describing D)

Conclusion

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- computationally tractable proofs of transcendental relations and statisticals distributions

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Future Work

- Optimize numerical algorithms for cryptographic primitives
- Try other ZKP frameworks: compare prover work - verifier work - communication trade offs
- Plug our methods to Multiparty Computation frameworks (e.g ABY3 [Mohassel and Rindal, 2018])

Thank you!

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