ANTS workshop on helioseismology

December 3, 4, 5 2019 at Inria Bordeaux Sud-Ouest.

Abstract

The workshop takes place in the framework of the ANTS (Advanced Numerical meThods for helioSeismolog) project, which corresponds with the *associate team* between the Project-Team Magique 3D at Inria Bordeaux Sud-Ouest and the Max Planck Institute for Solar System Research at Göttingen (MPS). The workshop is dedicated to helioseismology and discuss the physical mechanism of the sun as well as the numerical methods for modeling and inversion. The workshop takes place at Inria Bordeaux Sud-Ouest Research Center, from December 3$^{\text{rd}}$ to December 5$^{\text{th}}$, 2019.

Contents

1 Schedule

The workshop will take place in the room **Ada Lovelace** at **Inria Bordeaux Sud-Ouest Research center, 200 avenue de la vieille Tour, 33400 Talence (France)**. The program is given below with the name of the speakers; we subsequently give the title and abstracts of the talks.

	Tuesday 3$^{\text{rd}}$	Wednesday 4$^{\text{th}}$	Thursday 5$^{\text{th}}$
AM 9.00– 9.30		L. Hägg	D. Fournier
AM 9.30–10.00		M. Berggren	T. Hohage
AM 10.00–10.30		M. Halla	D. Yang
AM 10.30–11.00		Coffee break	Coffee break
AM 11.00–11.30		H. Pham	F. Faucher
AM 11.30–12.00		Faucher/Fournier	*conclusion*
PM 12.00– 2.00		Lunch	Lunch
PM 2.00– 2.30	*Mini-course*	N. Rouxelin	
PM 2.30– 3.00	*on helioseismology*	J.-F. Mercier	
PM 3.00– 3.30	L. Gizon	C. Millet	
PM 3.30– 4.00		C. Hanson	
PM 4.00– 4.30	Coffee break	Coffee break	
PM 4.30– 5.00	ANTS *recent activities*	*Open discussions*	
PM 5.00– 5.30	J. Diaz		
PM 5.30– 6.00	J. Preuss		
PM 7.00		Conference diner	

2 List of abstracts

2.1 Tuesday, December 3rd

Mini-course on helioseismology.
Laurent Gizon — PM, 2.00 – 4.00

This mini-course introduces helioseismology, history and ongoing developments.

▷ **coffee break**

Recent activities of the associate team.
Damien Fournier and Ha Pham — PM, 4.30 – 5.00

We provide a short overview of the recent activities of the Associated Team Ants between Inria Magique 3D and the Max Planck Institute in order to introduce this three-days workshop.

Hybridizable Discontinuous Galerkin methods for elastodynamics.
Julien Diaz — PM, 5.00 – 5.30

Discontinuous Galerkin (DG) methods are widely used in geophysics for simulation of wave propagation in time domain. They can also be applied to harmonic domains but in this case, they may strongly increase the size of the linear system to be solved. Indeed, for a given mesh, these methods require more degrees of freedom than classical finite element methods. To solve this problem, we consider a new class of DG methods, the Hybridizable Discontinuous Galerkin methods. Their principle consists in expressing the unknowns of the initial problem as a function of the solution on the skeleton (the faces of the cells) of the mesh. Hence, the size of the matrix to be inverted only depends on the number of degrees of freedom on each face and on the number of faces while is depends on the number of degrees of freedom in each cell and on the number of cells for classical DG methods.

We have designed, implemented and analyzed a HDG method for the propagation of elastic waves in two and three dimensions. In order to tackle realistic geophysical problems, we have compared the performance of two linear solvers, a direct one (Mumps) and an hybrid one combining direct and iterative solvers thanks to an algebrical domain decomposition method (Maphys).

Learned infinite elements.
Janosch Preuss — PM, 5.30 – 6.00

The construction of accurate absorbing boundary conditions for the efficient numerical simulation of waves is very challenging. In this talk we present an approach for constructing absorbing boundary conditions for stratified media based on infinite elements. The novelty of our approach is that the infinite elements are learned from the background medium by means of an optimization procedure. We compare the performance of our approach with more established methods for toy problems. The method might be interesting for computational helioseismology, for example, it may be able to take the effect of sound speed variations above the photosphere into account without the need for a mesh in this region.

2.2 Wednesday, December 4th

On the well-posedness of Galbrun's equation
Linus Hägg — AM, 9.00 – 9.30

Galbrun's equation, which governs the evolution of a so-called Lagrangian displacement, has been used to model acoustics in background flows as well as perturbations of astrophysical flows. Given the Lagrangian displacement, there are formulae to calculate the physically relevant Eulerian perturbations. We take the opposite view on Galbrun's equation and define the Lagrangian displacement from the Eulerian perturbations. More precisely, given a solution to linearized Euler's equations, which govern the evolution of the Eulerian perturbations, we define the Lagrangian displacement as the solution to an initial-boundary value problem for a first order partial differential equation, which is driven by the Eulerian velocity perturbation. If the so-called "no resonance" assumption is fulfilled, our Lagrangian displacement is a solution to Galbrun's equation.

We establish well-posedness of an initial-boundary value problem for linearized Euler's equations for steady background flows that are tangential to the domain boundary. For such background flows, we demonstrate that our Lagrangian displacement is well-defined and that its initial datum can be chosen to enforce the "no resonance" assumption. We also demonstrate that (sufficiently regular) solutions to Galbrun's equation satisfy a classical energy estimate.

Bilinear forms for first-order systems
Martin Berggren
joint work with Linus Hägg — AM, 9.30 – 10.00

A finite-element approximation of a boundary-value problem is usually carried out according to a standard protocol: (i) reformulation, and (ii) discretization. The *reformulation* collects the partial differential equation, the boundary conditions, and possibly other side conditions into a common expression. For linear boundary-value problem, the result is

$$\begin{aligned} &\text{find } u \in V \text{ such that} \\ &a(v,u) = l(v) \quad \forall v \in L, \end{aligned} \tag{1}$$

where l and a are continuous linear and bilinear forms on Hilbert spaces L and $L \times V$. Indeed, the solution theory for boundary-value problems is often based on such variational formulations. The *discretization* step involves picking finite-dimensional spaces V_h and L_h approximating V and L as well as, sometimes, approximations of a and l.

A notable exception to this scheme is the treatment of first-order systems such as advection problems and the first-order formulation of the acoustic and Maxwell equations. A standard and successful finite element method for such problems is the discontinuous Galerkin method, where linear and bilinear forms are introduced *directly* on the finite-dimensional spaces, element by element. Regarding well-posedness of the problem before discretization, if discussed at all, authors typically refers to the theory of Friedrichs systems, which is not of the type (1).

Based on a reformulation of the theory of Friedrichs systems due to Ern, Guermond, and Caplain [1], we show a number of examples of how to incorporate the generalized Friedrichs systems in the standard variational framework (1). The critical step here concerns the fulfillment of characteristic boundary conditions, which Ern et al. [1] impose strongly in the definition of the solution space. We choose a weak enforcement in line with the practice in discontinuous Galerkin methods. With a proper choice of solution space V, test space L, and forms a and l, we show well-posedness of a few example first-order systems directly for the problem in the form (1). The variational formulations we present are not identical to the ones typical in discontinuous Galerkin methods and might constitute the first step towards a new class of numerical schemes for first-order systems.

[1] A. Ern, J.-L. Guermond, and G. Caplain. An intrinsic criterion for the bijectivity of Hilbert operators related to Friedrichs' systems. Comm. Partial Differential Equations, **32**:317341, 2007.

On the well-posedness of the vectorial equations of stellar oscillations.
Martin Halla — AM, 10.00 – 10.30

We consider vectorial equations of stellar oscillations in the frequency domain. We neglect rotational, gravitational and magnetic terms, but allow the presence of background flows. We choose a bounded domain and a homogeneous essential boundary condition to study the problem. We propose to model the damping of waves by a different expression. The new model has favorable properties for background flows of smaller magnitude than the sound speed. We discuss why our analysis fails for the convenient damping model. At last, we consider some ideas for the construction of reliable computational methods.

▷ **coffee break**

Outgoing solution and radiation conditions for scalar wave equation in helio-seismology without flow
Ha Pham,
joint work with H. Barucq, L. Gizon, F. Faucher and D. Fournier — AM, 11.00 – 11.30

In this talk, we consider the question of how to define physical solution for the scalar wave equation in helioseismology without flow. With the Louiville change of variable, the equation is rewritten as a Schrodinger equation. Assumptions such as ideal atmospheric pressure leads to a Coulomb-like potential with mild singularity at the origin.

Using the scattering theory of Ikebe-Saito for long-range potential, one can define physical solutions for this equation, which decay in the absence of attenuation and behave like expanding spherical wave at infinity. In this way, the physical solution of the original problem are exactly defined in terms of the physical solutions of the conjugate problem. This is the theoretical contribution of the work.

On the practical side, we obtain the exact Dirichlet-to-Neumann map in the atmosphere with ideal atmospheric assumption which is used to evaluate radiation boundary conditions. In addition, we provide explicit expression of the 3D kernel and its harmonic expansion in terms of Whittaker functions in the toy case where the ideal atmospheric assumption is extended to the whole domain. These are used as references to evaluate the accuracy of discretization schemes.

In the second part, we show numerical comparisons of radiation boundary conditions constructed from the conjugate equation under ideal atmospheric assumption with the existing ones in literature.

Fast and accurate computation of Green's functions for the scalar wave equation in helioseismology

Florian Faucher & Damien Fournier

joint work with H. Barucq, L. Gizon and H. Pham AM, 11.30 – 12.00

We provide a new approach to compute the Green's function with minimal computations in the scalar case.

▹ **lunch break**

On the equivalence between Galbrun's Equation and the Linearized Euler's Equations

Nathan Rouxelin PM, 2.00 – 2.30

Galbrun's equation in the frequency domain is usually used to describe the propagation of acoustic waves in the Sun. However, it was shown in (Chabassier & Duruflé, 2018) that the discretization of this equation with high-order discontinuous Galerkin methods leads to poor numerical results for solar-like background flows. The authors also noted that better numerical results could be obtained by solving the Linearized Euler's Equations instead of Galbrun's Equation. In this talk we investigate the equivalence between these two equations in the following way: the solution of Galbrun's equation is reconstructed by first solving the Linearized Euler's Equations and then a vectorial transport equation. We will discuss the difficulties arising from the boundary conditions and the geometry of the background flow. Indeed it turns out that both equations are equivalent in absence of boundary conditions. It is also interesting to note that this equivalence questions were also used in (Hägg & Berggren, 2019) to obtain theoretical results on Galbrun's Equation in time domain.

Acoustics in recirculating flows
Jean-François Mercier PM, 2.30 – 3.00

We study the propagation of acoustic perturbations induced by a time-harmonic source in an imposed mean flow. Instead of the usual compressible Linearized Euler Equations, we consider the equivalent Goldstein equations, better adapted to develop a theoretical framework. The problem is proved to be of Fredholm type for a large class of flows of small vorticity. This theoretical approach fails for recirculating flows. We focus on a circular vortex flow and we show that outside a spectrum of singular frequencies, the acoustic problem is well-posed. To go further, combining Fourier modal expansions, the limit absorption principle and the Fuchs theorem (Frobenius method), we determine explicitely the modal solutions. We prove that at some frequencies called singular, the limit of a modal solution exits standard Sobolev spaces and becomes singular on some streamlines, tending to delta and principal value distributions.

Reduced order modeling of wave propagation in randomly stratified media
Christophe Millet PM, 3.00 – 3.30

Despite the continuing increase of computing power, which allows numerical models to be run with ever-higher resolution, the multiscale nature of geophysical and astrophysical fluid dynamics implies that important subgrid-scale physical processes (e.g., turbulence) are still resolved through stochastic parameterizations. Recently, due to the proliferation of data obtained through observation, numerical simulation and laboratory experimentation, there has been an outgrowth of research activity directed at improving community understanding, modeling and parameterization of acoustic-gravity wave propagation in density-stratified fluids. However, as I will discuss in this talk, wave propagation can be extremely sensitive to the statistical information to specify the waveguide parameters and because of the high dimensionality of the underlying random processes, some form of data-driven reduction is necessary. In this presentation, I will discuss how stochastic reduced order models (SROMs) can be obtained through a combined approach based on asymptotic analysis and rescaling and shifting data distributions. Algorithmic, physical and data-related challenges will be discussed toward the end of achieving robust SROMs in the large deviation regime. I will conclude with a brief outlook on the use of SROMs as turbulence parameterizations in the field of climate simulation.

Utilizing Machine Learning for Helioseismic Inversions.
Chris S. Hanson PM, 3.30 – 4.00

Imaging the hidden internal structure of the sun is only achievable through exploiting the acoustic oscillation signal observed at the surface. Many inverse problems exist in the field of local helioseismology, but are non-trivial due to the low signal-to-noise ratio. However, via forward modeling we understand the range in which the solution exists.

Through machine learning we can exploit this prior on the solution and develop data-driven solutions, that satisfy the observations and physical constraints.

▹ **coffee break**

Open Discussions
PM, 4.30 – 6.00

In this afternoon session, we shall discuss some open problems of helioseismology, as well as the current difficulties of computational methods for the physics of Sun. Three main topics have been highlighted prior to the workshop:

1. Including the gravity term in the numerical simulations;
2. Accounting for the flows in the numerical simulations;
3. Representation of the attenuation in the Sun.

Each topics will be briefly introduced before opening the discussions. However, additional discussions from questions raised during the talks will enrich the planning.

▹ **The conference dinner will take place at the restaurant "*La Belle Epoque*" located 2 Allée d'Orléans, 33000 Bordeaux.**

2.3 Thursday, December 5th

From travel-time tomography to full waveform inversions	
Damien Fournier	AM, 9.00 – 9.30

Travel-time tomography is often used in helioseismology as the measurements can be understood intuitively. The travel times are extracted from the cross-covariance function but other parameters could be measured or the full waveform could be used.

I will first present some recent results of the inference of the meridional flow in the solar interior using 22 years of travel-time measurements at the solar surface. Then, I will show advantages of using the full cross-covariance function instead of the travel-time measurements.

Helioseismic holography	
Dan Yang	AM, 9.30 – 10.00

Helioseismic holography numerically focuses acoustic waves observed on the Sun's visible surface to its interior/far-side to study the structure and dynamics therein. Accurate Green's functions and proper strategies are of particular importance for the high-quality imaging of helioseismic holography. In this talk, I will give an example on how these two aspects improve the imaging of active regions on the solar far-side.

Uniqueness results for passive imaging problems in helioseismology
Thorsten Hohage AM, 10.00 – 10.30

Under certain conditions the correlations of observed solar oscillations on the surface correspond the the imaginary part of the Green function of the differential operator.

In this talk we address the question if such idealized, noise-free data at one or a few frequencies uniquely determine certain coefficients of the differential operator such as density or sound speed. We study two situations: For the full three-dimensional case we derive local uniqueness results, and for the radially symmetric case we show a global uniqueness result if measurements at two heights are available.

▷ **coffee break**

Reciprocity-based inversion enabling sparse acquisitions
Florian Faucher
joint work with G. Alessandrini, H. Barucq, M. V. de Hoop, R. Gaburro and E. Sincich
AM, 11.00 – 11.30

We perform quantitative reconstruction with the *Full Reciprocity-gap Waveform Inversion* (*FRgWI*) method. The reconstruction relies on iterative minimization of a misfit functional which is specifically designed for multi-component data. The use of reciprocity-based misfit functional provides additional features compared to the more traditional least-squares approaches with, in particular, that the observational and computational acquisitions can be different. Therefore, the source positions and wavelets that generate the measurements are not needed for the reconstruction procedure and, in fact, the numerical acquisition (for the simulations) can be arbitrarily chosen.

We illustrate our method with three-dimensional experiments, and investigate arbitrary numerical acquisitions with two configurations: firstly, when sparse measurements are given, we use a dense numerical acquisition (compared to the observational one). Secondly, with a dense observational acquisition, we employ a sparse computational acquisition to reduce the numerical cost.

Closing the workshop
AM, 11.30 – 12.00

Perspectives for 2020.

▷ **lunch break**

3 List of participants

Name	Affiliation
Alain Bachelot	University of Bordeaux
Agnès Bachelot	University of Bordeaux
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Martin Berggren	Umeå Universitet
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Juliette Chabassier	Inria Magique-3D
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