Test suite for reduced-order modelling of bifurcation phenomena in fluid mechanics

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We propose the fluidic pinball [4] as a benchmark configuration for reduced-order modelling of successive bifurcations [1] and complex dynamics of wake flows. This two-dimensional configuration comprises three parallel rotatable cylinders centred on an equilateral triangle and placed in a uniform flow. The simple geometry allows a fast and lightweight numerical study. Yet, the 4-dimensional parameter space (Reynolds number and three rotation speeds) can feature virtually any common bifurcation and type of nonlinear dynamics. The unforced flow alone exhibits steady, symmetry-breaking, periodic, quasi-periodic, and chaotic features with increasing Reynolds number [4]. In this talk, the numerical setup is introduced, then an overview of flow instabilities and bifurcation is given, e.g., the successive bifurcations of pitchfork and Hopf types associated with the Bénard-von Kármán instability [7] and symmetry-breaking instability [3], respectively. We aim at simplified, interpretable reduced-order models of fluid flow, which can provide a better understanding of the aerodynamic force [5], and facilitate estimation [8], dynamics prediction and effective control design [2]. Our results of the mean-field Galerkin model [4, 5] and hierarchical network model [6] for the transient and post-transient dynamics between multiple invariant sets are presented as examples.

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