Approximation of problems with parameter-dependent discontinuities in steady aerodynamics

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ARIA seminar
MOR test case initiative
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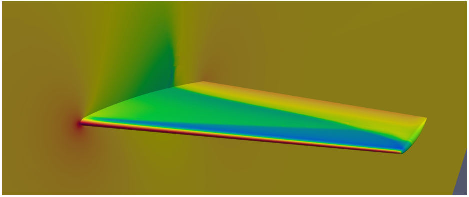
Motivation: parametrized aerodynamics

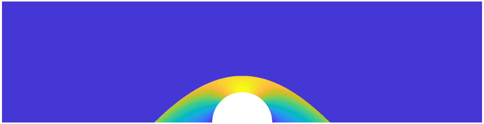
Examples: prediction of lift, drag, heat transfer, ...

- over range of operating conditions
- under geometry variations

with applications to parametric study, optimization, uncertainty quantification, ...



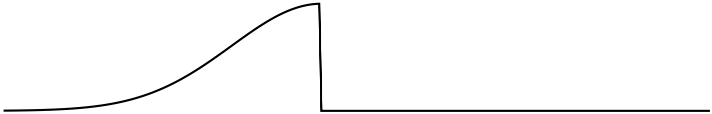
transonic RANS flow over ONERA M6



supersonic Euler flow over cylinder

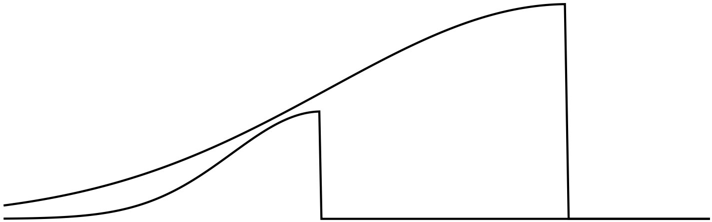
Challenge: transonic and supersonic flows exhibit *parameter-dependent shocks*

Reduction of parameter-dependent discontinuities: n-width barrier



<u>Fundamental issue</u>: linear subspace approximation ill-suited for advection-dominated features (slowly decay Kolmogorov n-width)

Reduction of parameter-dependent discontinuities: n-width barrier



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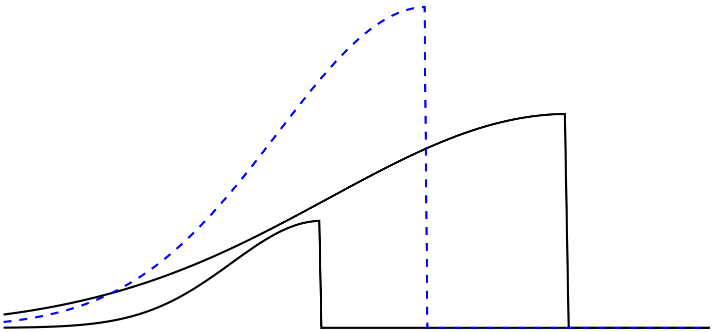

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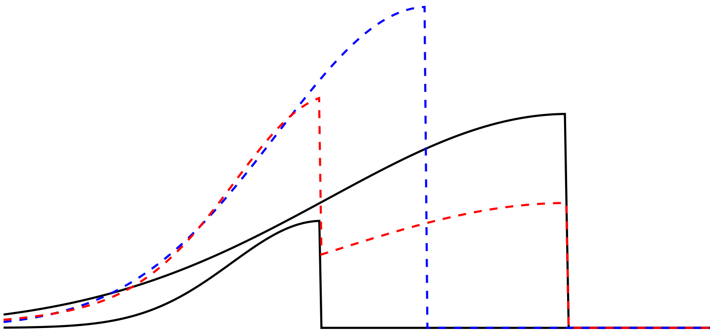


Fundamental issue: linear subspace approximation ill-suited for advection-dominated features (slowly decay Kolmogorov n-width)

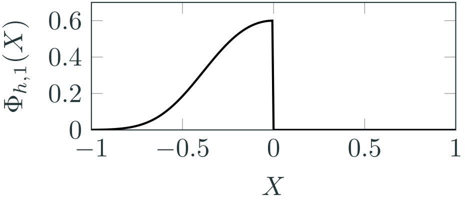
One potential approach: mapped snapshots

Idea:

- apply parameter-dependent domain mapping to align features
- use linear subspace in reference domain to reduce dimension
- push forward to physical domain

Reference basis

$$\Phi_{h,1} : X \mapsto a \exp\left[-\left(\frac{x}{b}\right)^2\right] H(-x)$$

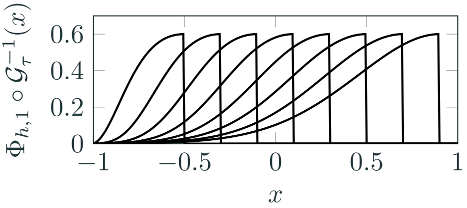


Domain mapping

$$\mathcal{G}_\tau : X \mapsto X + \tau(1 - X^2)$$

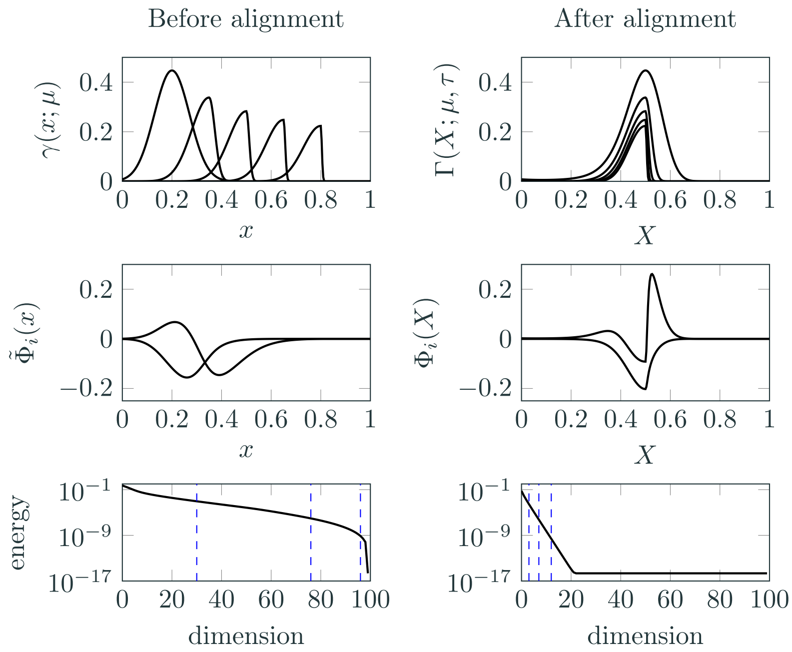
Physical basis

$$\phi_{h,1} : x \mapsto \Phi_{h,1}(\mathcal{G}_\tau^{-1}(x))$$



Note: the idea readily extends to 2D and 3D.

Feature alignment improves compressibility of snapshots



Snapshots (*top*), first two POD modes (*middle*), and decay of energy (*bottom*)

Goal: explore and assess different methods in the simplest setting

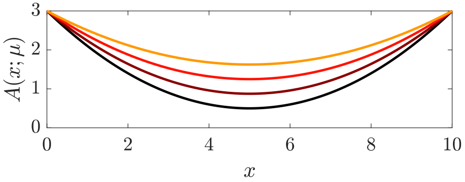
Configuration: steady, inviscid flow of an ideal gas through a parametrized nozzle, modeled as a one-dimensional conservation law

$$f(q)_{,x} = s(q) \qquad \text{in} \;\; (0, 10),$$

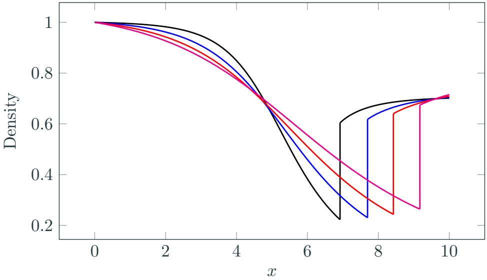
with

$$q = \begin{bmatrix} A\rho \\ A\rho v \\ A\rho E \end{bmatrix}, \qquad f(q) = \begin{bmatrix} A\rho v \\ A(\rho v^2 + P(q)) \\ Av(\rho E + P(q)) \end{bmatrix}, \qquad s(q) = \begin{bmatrix} 0 \\ P(q)A_{,x} \\ 0 \end{bmatrix},$$

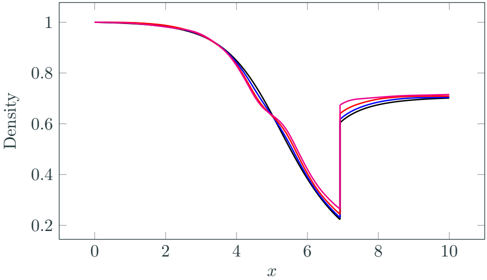
where ρ, v, E is the density, velocity, and total energy of the fluid, respectively, and pressure is given by $P(q) = (\gamma - 1)(\rho E - \rho v^2/2)$, and $\gamma = 1.4$.



Test Case I Example: original vs mapped snapshots

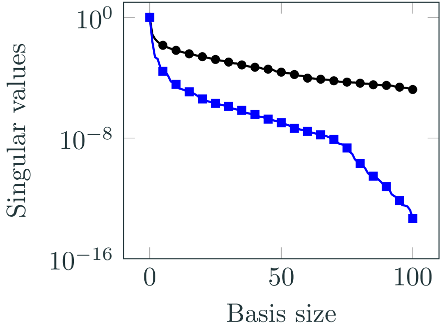


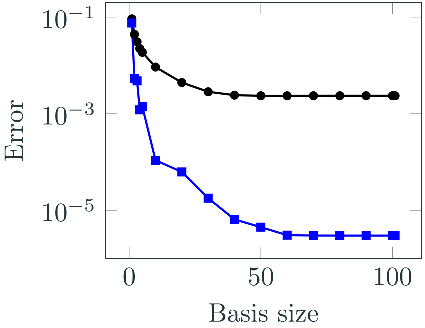
Original density snapshot at $\mu \in \{0.5, 0.875, 1.25, 1.625\}$



Mapped density snapshot at $\mu \in \{0.5, 0.875, 1.25, 1.625\}$

Test Case I Example: rapid error decay with nonlinear manifold





Left: Convergence of the singular values of the non-aligned (—●—) and aligned (—■—) snapshot matrices (101 training parameters).

Right: Convergence of the maximum relative $L^1(\Omega_0)$ error over the training set for the fixed-domain ROM (—●—) and ROM-IFT (—■—).

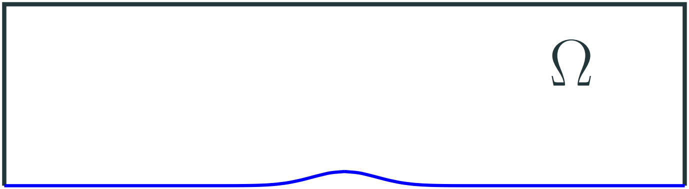
Goal: explore and assess different approaches in this simplest setting

Test Case II: transonic Gaussian bump (I)

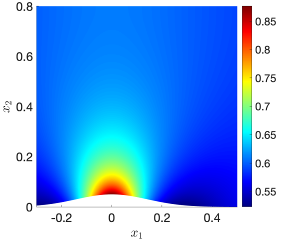
Goal: assess methods for transonic flows with shocks in parameterized geometries

Configuration:

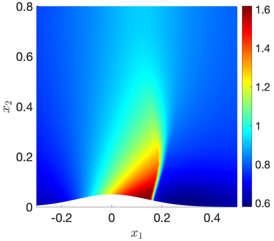
- Domain $\Omega(h) = \{x = (x_1, x_2) : |x_1| < 1.5,\ he^{-25x_1^2} < x_2 < 0.8\}$
- Compressible Euler equations for ideal gases
- Parametrized by Mach: $M_\infty \in [0.58, 0.78]$, bump height $h \in [0.05, 0.065]$



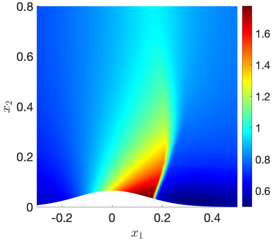
Visualization: behavior of the Mach number for several parameter values.



(a) $\mu = (0.05, 0.58)$



(b) $\mu = (0.05, 0.78)$



(c) $\mu = (0.065, 0.78)$

Challenges: shock developed for $M_\infty \gtrsim 0.65$; shock location and shape sensitive to parameter variations.

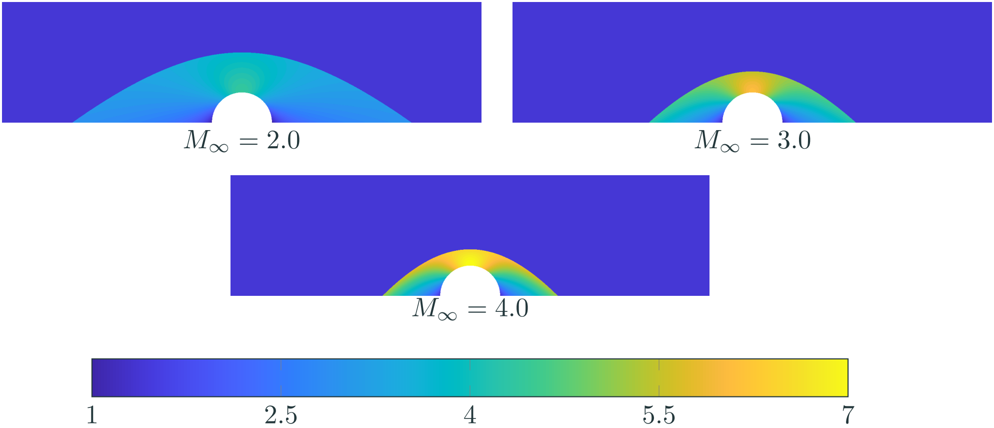
Simplifications: topology of the shock is constant for all parameters; exact geometry parameterization available for this problem.

Shock topology changes and complex geometry parameterizations will be considered in separate test cases.

Goal: assess methods for bow shocks in supersonic flows

Configuration:

- Compressible Euler equations
- Parametrized by Mach: $M_\infty \in [2, 4]$



Density

Assessment metrics

Accuracy: as a function of ROM "size" N and N_{snapshot}

- L^2 norm (for problems with known solution)
- Violation of conserved quantities (e.g., global enthalpy)
- Output error (e.g., lift, drag, average temperature on surface)

Offline training cost:

- Cost to construct a ROM of "size" N given N_{snapshot} snapshots (e.g., POD, NN training, hyperreduction)
- *Motivation*: In aerodynamic design applications, turnaround times and computational budgets make offline cost and N_{snapshot} important considerations

Online evaluation cost:

- ROM evaluation cost vs "size" N

Summary

Motivation: efficient MOR for transonic/supersonic aerodynamic flows

Challenge: parameter-dependent shocks $\Rightarrow$ slow n-width decay

Goal: develop, assess, and compare various *nonlinear* MOR techniques

Test cases:

1. transonic quasi-1d nozzle
2. transonic Gaussian bump
3. supersonic cylinder

Assessment metrics: accuracy, offline training cost, online evaluation cost