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Test suite for Reduced-Order Modelling of bifurcation phenomena in fluid mechanics

ARIA Seminar on MOR test case initiative

27 Janaury 2023 | Online

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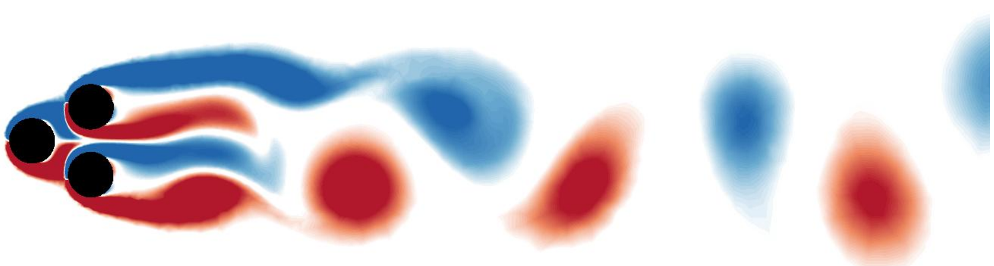


ROM benchmarks work for fluidic pinball

ROMs: Simplifications of high-dimenisonal, complex systems.

$$\partial_t \boldsymbol{u} + \nabla \cdot \boldsymbol{u} \otimes \boldsymbol{u} = \frac{1}{Re} \triangle \boldsymbol{u} - \nabla p, \quad \nabla \cdot \boldsymbol{u} = 0 \quad \Rightarrow \quad \begin{aligned} \dot{\boldsymbol{a}} &= f(\boldsymbol{a}, Re) \\ \boldsymbol{a}(t) &= (a_1, \cdots, a_N) \end{aligned}$$

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{u}_0(\boldsymbol{x}) + \sum_{i=1}^{N} a_i(t) \boldsymbol{u}_i(\boldsymbol{x}).$$

Following ROMs can be tested:

1. Identify the right modes u_i — **Deng et al. 2020 JFM**
2. Identify the dynamical system f(a) for single operating condition
3. Identify the dynamical system f(a,b,*Re*) for multiple operating conditions
4. Control-oriented dynamics
5. Control-oriented estimator s,b -> a — **Li et al. 2022 JFM**
6. Cost-related surrogate model a,b -> C_L,C_D — **Deng et al. 2021 JFM**

Summary first

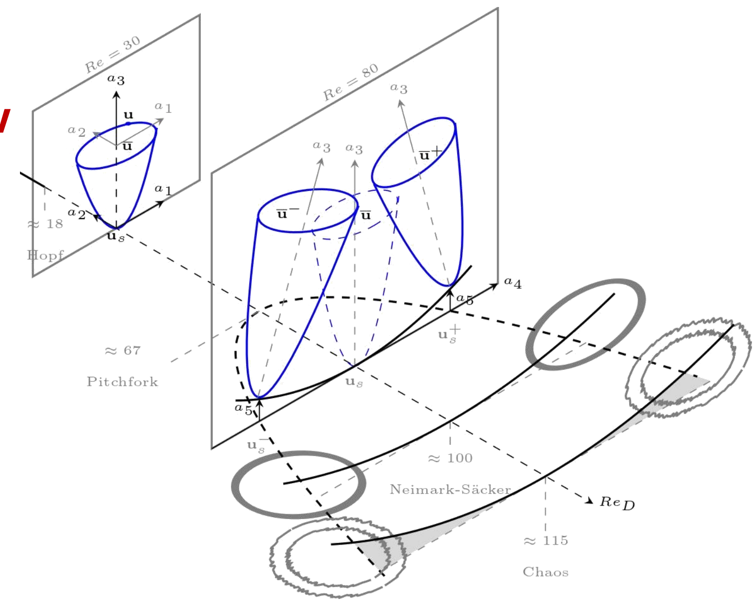
- Name of the test suite: **Fluidic pinball**
- Main referents: **N. Deng & B. R. Noack, R. Martinuzzi ...**
- Specific methodological challenges: **Bifurcation phenomena in fluid mechanics & Parameter-induced ROM.**
- Target engineering applications: **Prototype for multibody system & MIMO control**
- Hierarchy of test cases of increasing difficulty: **Detailed report** in Noack & Morzyński 2017 Tech. Rep. and Deng et al. 2020 JFM
- Computational speed and cost: **DNS, 48 minutes** for **100 convective times** on a workstation (Dell T7920 with 128Gb RAM, Xeon Gold 6258R, 2TB SSD) **2.8 MB/snapshot** (10 per time unit).
- List of quantities of interest: **Multiple inputs** (**Re**, b_1, b_2, b_3), **Multiple output** (velocity & pressure field, C_D, C_L, periods)

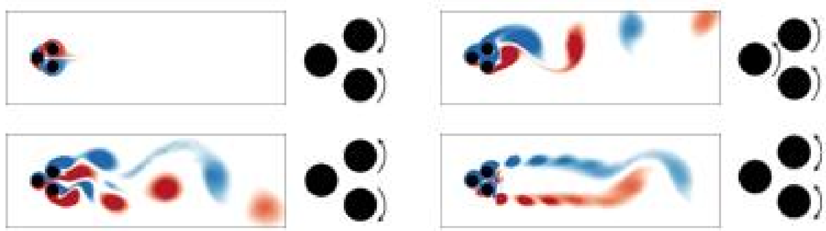
Outline

1. Benchmark configuration -- the fluidic pinball
2. Bifurcation & instabilities
3. Challenges in ROM of bifurcations
4. Test cases
5. Some examples

1 Low-cost benchmark for ROM and MLC

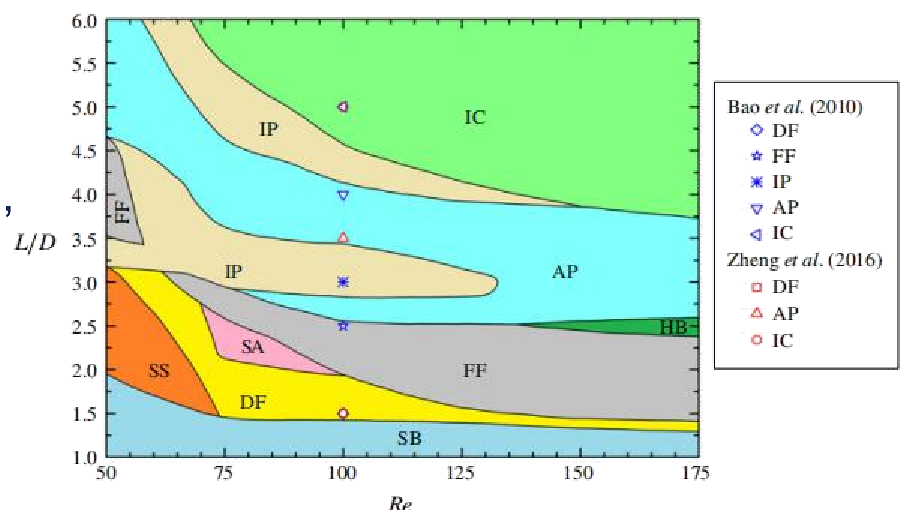
- **Nonlinear MIMO system**
  - Simple geometry to allow **fast simulation with low computational cost**.
  - **Successive bifurcations and many dynamical regimes** to allow testing **ROM** methods, **control** laws, and mimicking **experimental** flow.
  - **1 grid with 3+1 parameters** (Re, b_1, b_2, b_3)



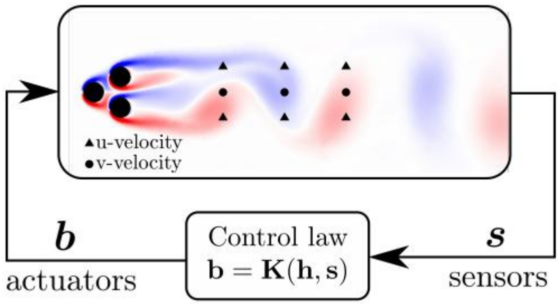


- **Prototype for multibody flow**
  - **Interaction between multiple structures**,
  - Complex dynamics: **9 flow regimes**

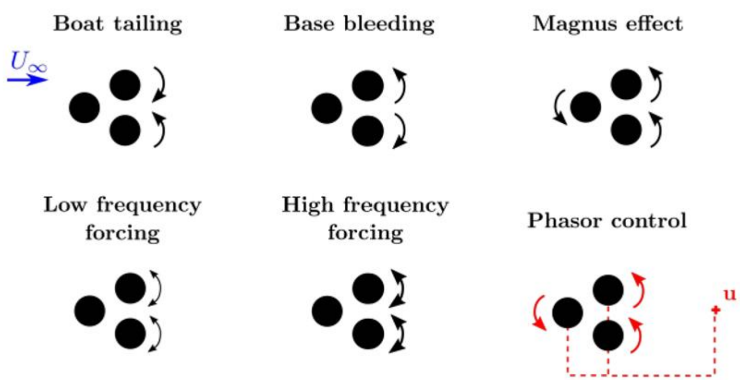
(**Chen et al. 2020 JFM**)

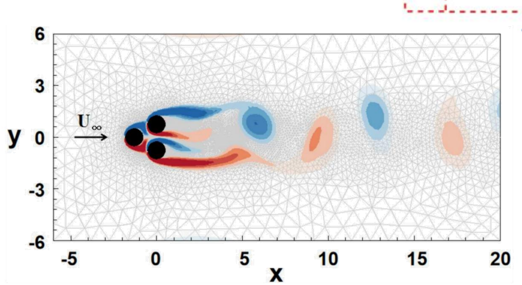


1.1 A MIMO control benchmark for MLC



h: time-dependent functions
s: sensor signals





- **Model predictive control** by S. Brunton (University of Washington)
- **Deep reinforcement learning control** by J. Rabault (University of Oslo) and T. Guégan & L. Cordier (Pprime Institute)
- **Experiments** in the University of Calgary lead by R. Martinuzzi and LISN/CNRS lead by F. Lusseyran
- **Myriad of regimes** (Chen *et al.*, 2020 *JFM*)

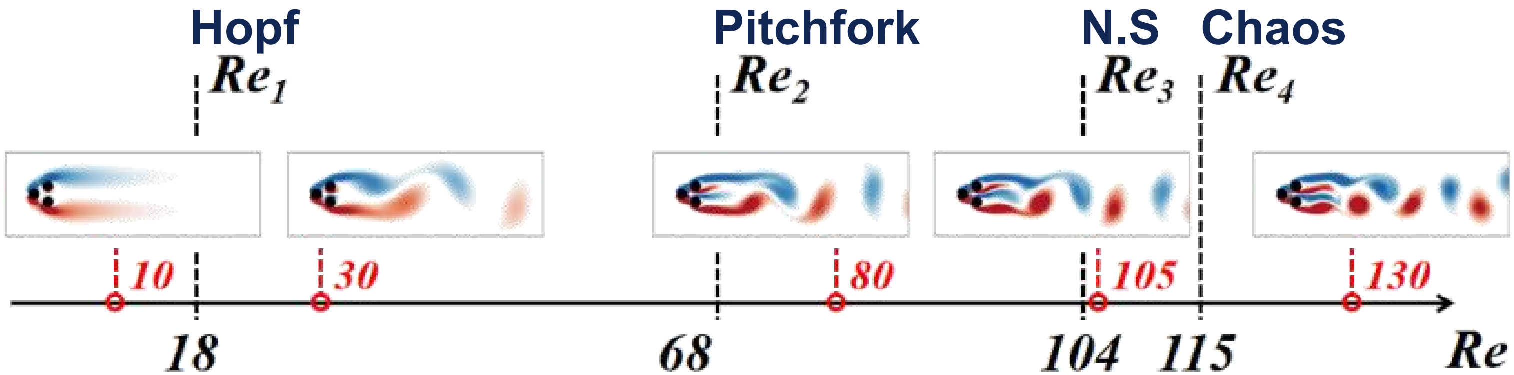
2023 PF Wang
2022 PF Castellanos
2022 AMS Blanchard
2021 JFM Cornejo Maceda
2020 EF Raibaudo [E]
2019 Arxiv Bieker
...

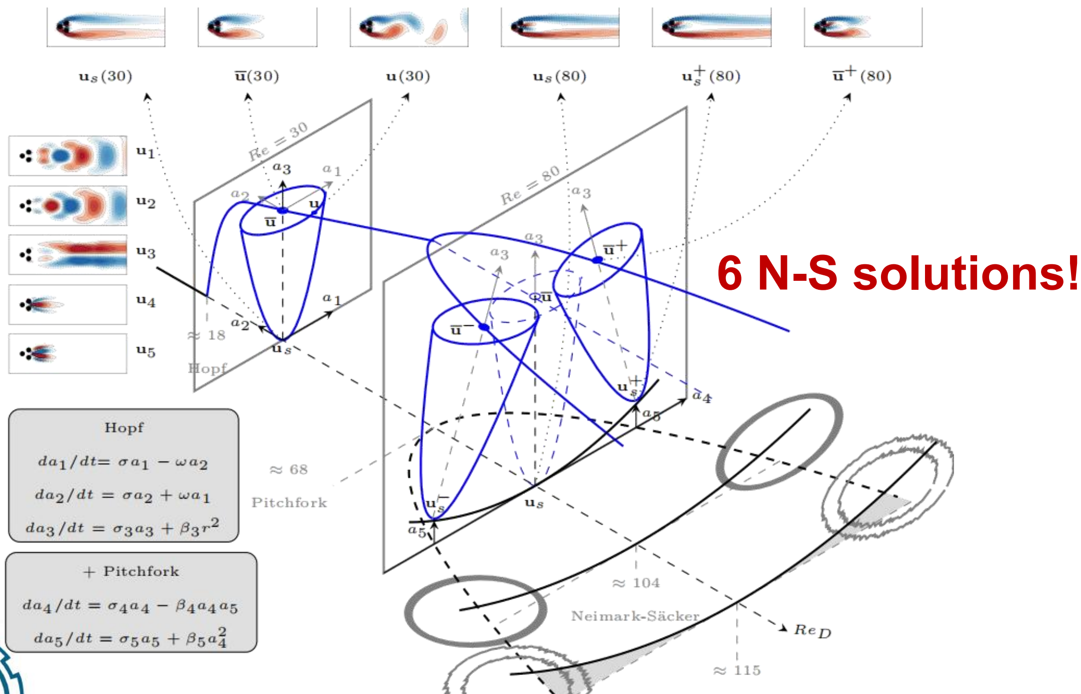
2022 JFM S. Li
2022 JFM Y. Li
2021 PRF Ghraieb
2021 PF Raibaudo [E]
2020 SIAM J. Peitz
2017 ETFS Bansal [E]

Noack & Morzyński 2017 Tech. Rep.
http://berndnoack.com/

1.2 A challenging benchmark for ROM

- ***Re*-dependent flow & successive bifurcations:**

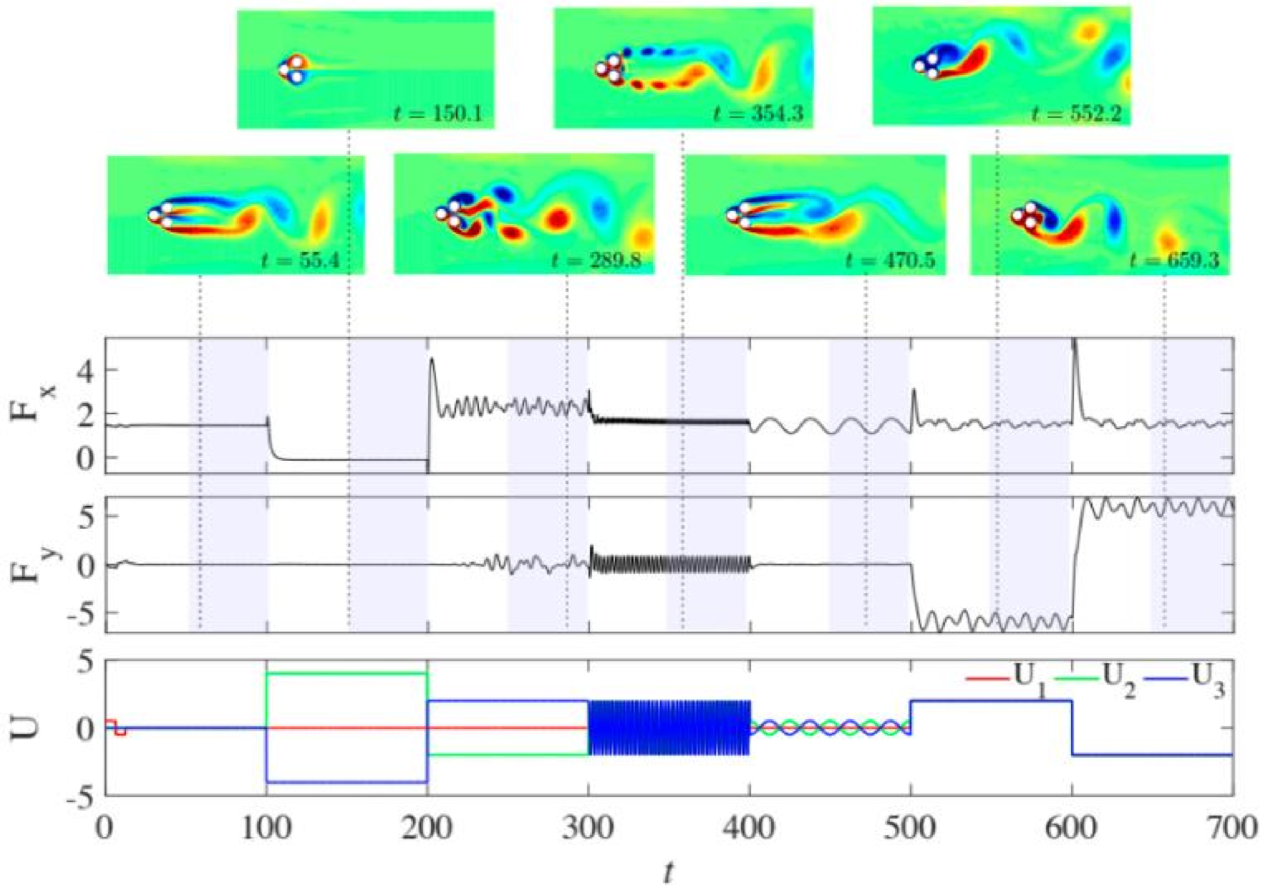




2023 JFM Farzamnik
2022 JFM Deng
2022 Arxiv Guemes
2022 ETFS Chen
2021 ETFS Cortina-Fernández
2021 JFM Deng
2021 EPL Deng
2021 Euromech[C] Menier
2021 PIV[C] Raiola
2020 JFM Deng
2019 JFM Ishar
2016 JFM Noack
...

1.2 A challenging benchmark for ROM

- (b_1, b_2, b_3) - dependent flow (🗎 Ishar et al. 2019 JFM)

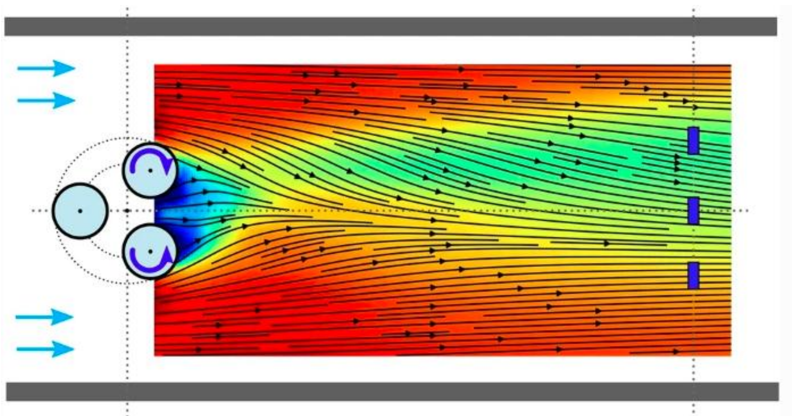


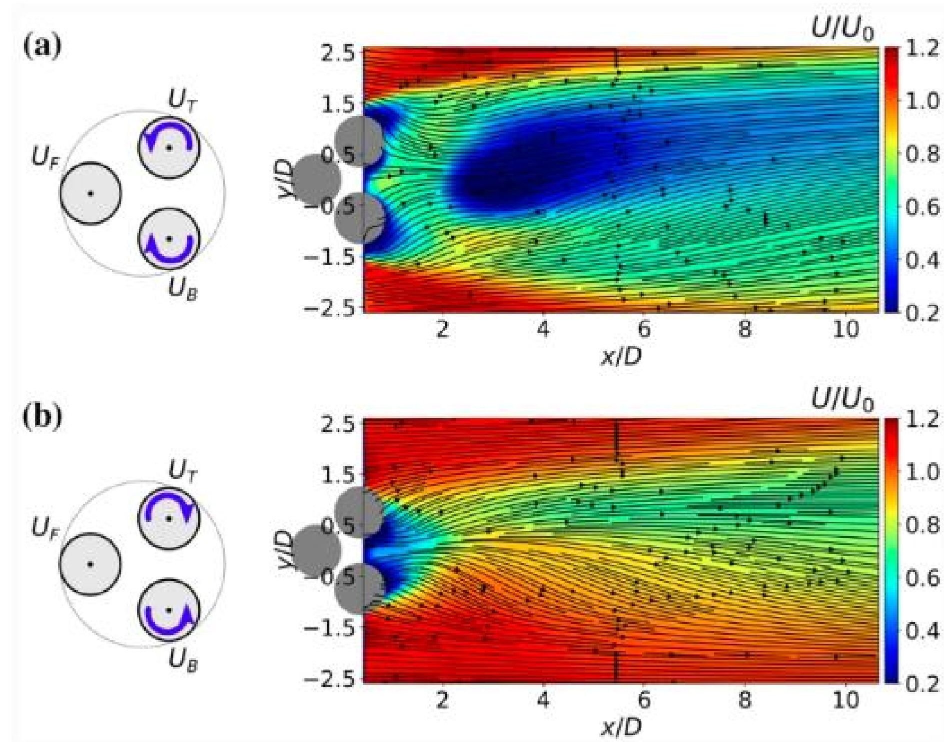
A very challenging parametric ROM test case with 1+3 parameters (Re, b1, b2, b3) !

1.3 Experimental study

- **Experimental study (R. Martinuzzi at University of Calgary)**

Raibaudo et al. 2021 EF





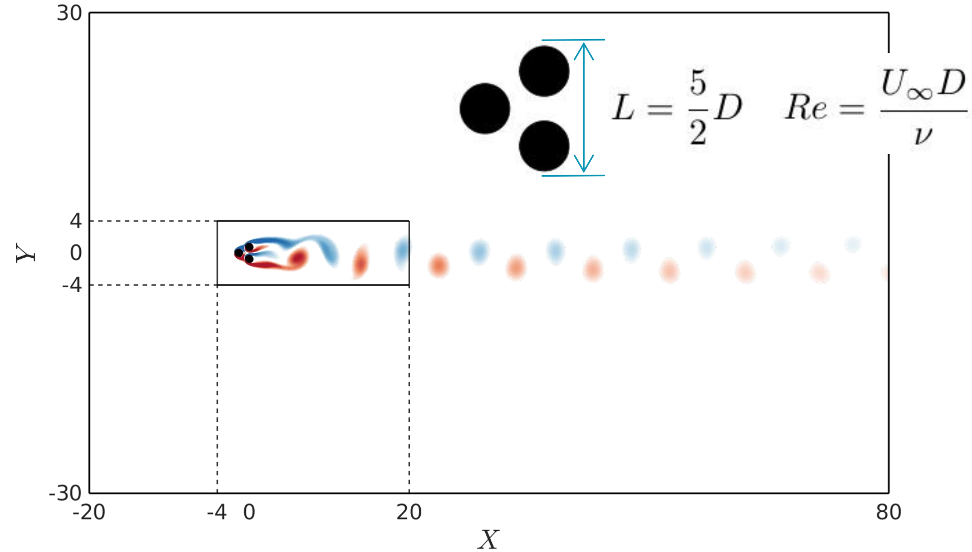
Lusseyran at LISN/CNRS

1.4 Full-order model from Fast DNS

2D incompressible non-dimensionalized N-S Equations scaled with the cylinder diameter ***D***, the oncoming velocity ***U***, the time scale ***D/U***, and the density ***ρ*** of the fluid.

$$\partial_t \boldsymbol{u} + \nabla \cdot \boldsymbol{u} \otimes \boldsymbol{u} = \frac{1}{Re} \Delta \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$



- **DNS** code provided by **Marek Morzyński.**
- Time integration with **3rd-order accuracy.**
- FEM discretization with **15258** triangles and **30826** vertices in **[−20, 80]** × **[−30, 30]** with **2nd-order Taylor-Hood finite (T6 triangular) elements.**
- **No-slip BC**: $U_r = 0$ on the cylinders, $U_X = 1$ at the inlet and the side wall;

  Stress-free BC: the outlet.
- **2.8 MB/snapshot, with plain text of (U,V,P) on grid.**

Noack & Morzyński 2017 Tech. Rep.

http://berndnoack.com/

1.4 Steady solver

Multiple steady solutions by solving the steady Navier-Stokes equations with a Newton-Raphson iteration method.

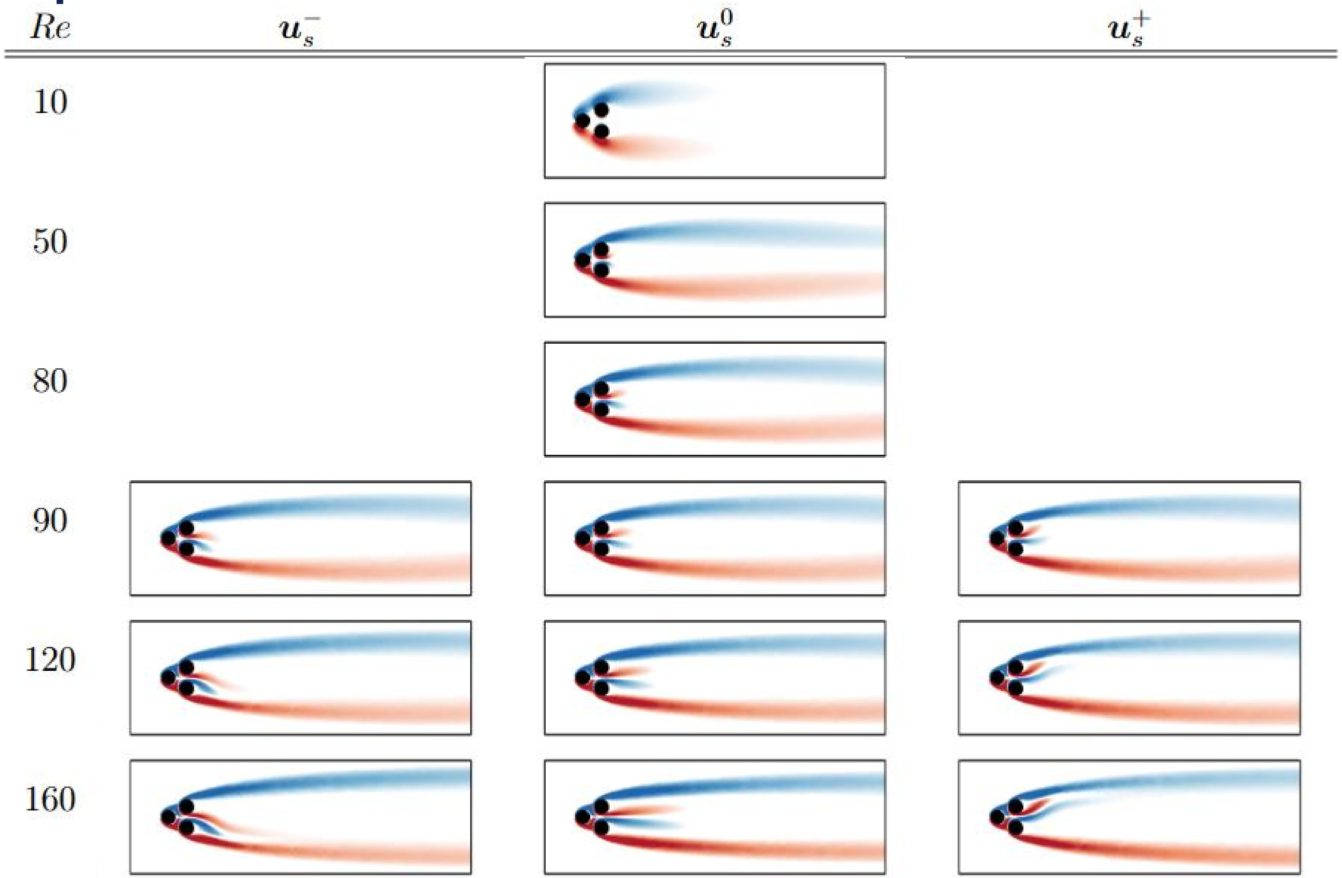


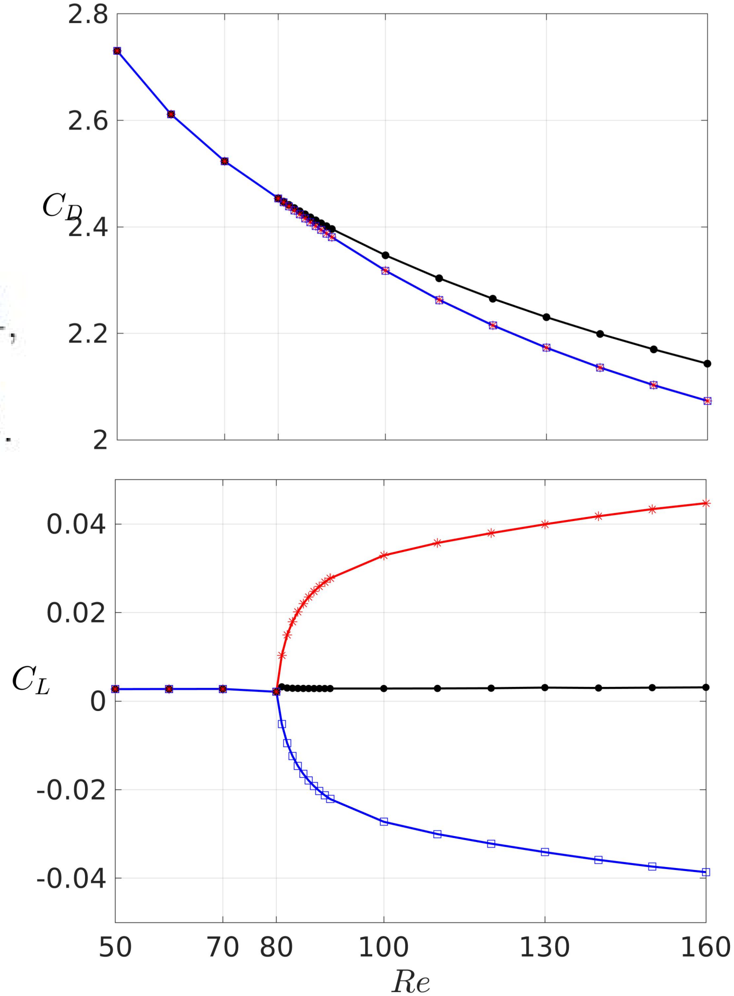
Figure 3: Steady solutions at different values of the Reynolds numbers. Vorticity fields are plotted in colour with the range $[-1.5, 1.5]$.

1.4 Steady solver

Symmetry-breaking bifurcation of the steady solution

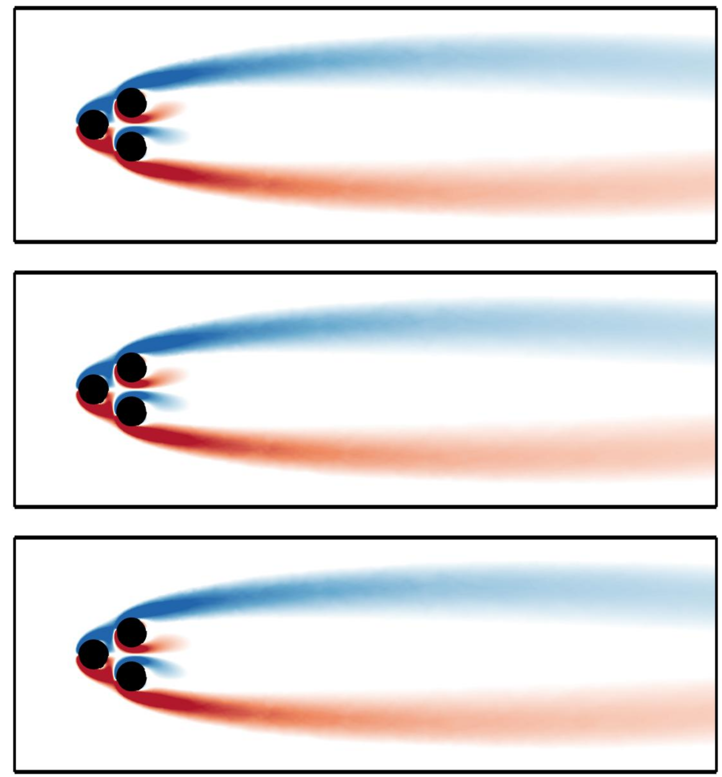
$$C_D(t) = \frac{2F_D(t)}{\rho U^2},$$

$$C_L(t) = \frac{2F_L(t)}{\rho U^2}.$$



Crawford & Knobloch 1991 ARFM

Bifurcating point: *Re=81*

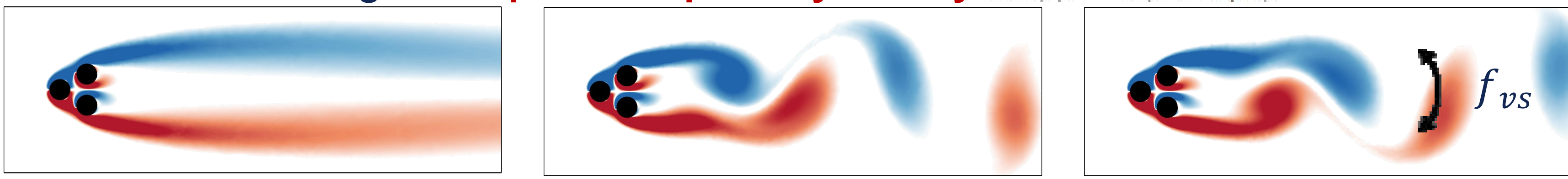


1.5 Review of unsteady flow regimes

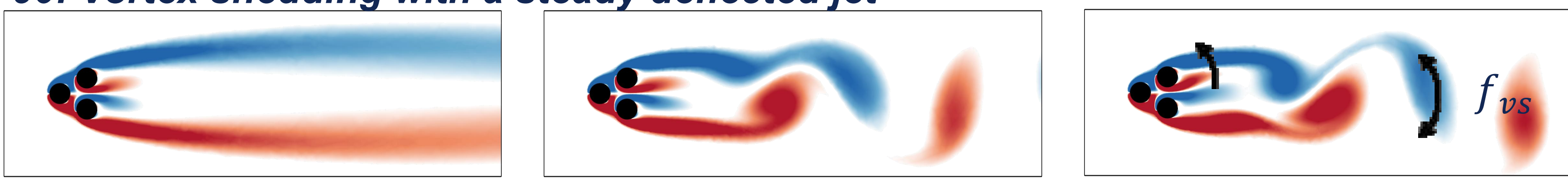
DNS starting close to the symmetric steady solution:

t1=0 t2=700 t3=1500

***Re* = 60: Vortex shedding with** **Spatio-temporal symmetry** $R\mathbf{u}(t) = \mathbf{u}(t + T/2)$

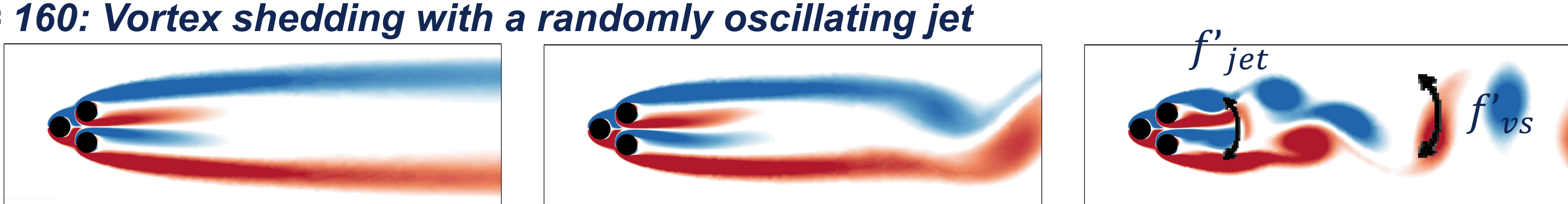


***Re* = 90: Vortex shedding with a steady deflected jet**



***Re* = 120: Vortex shedding with an oscillating deflected jet**

***Re* = 160: Vortex shedding with a randomly oscillating jet**



Reflectional symmetry $R(u, v, p, \omega)(x, y) \equiv (u, -v, p, -\omega)(x, -y)$

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Linearized N-S Equation around the base flow:

$$\partial_t \boldsymbol{u}' = -\nabla \cdot (\boldsymbol{u}' \otimes \boldsymbol{U}_b + \boldsymbol{U}_b \otimes \boldsymbol{u}') + \nu \triangle \boldsymbol{u}' - \nabla p',$$

$$0 = \nabla \cdot \boldsymbol{u}',$$ **+ homogeneous BC.**

Tuckerman & Barkley 2000 Springer book

Barkley & Henderson

Linear stability of the steady solution

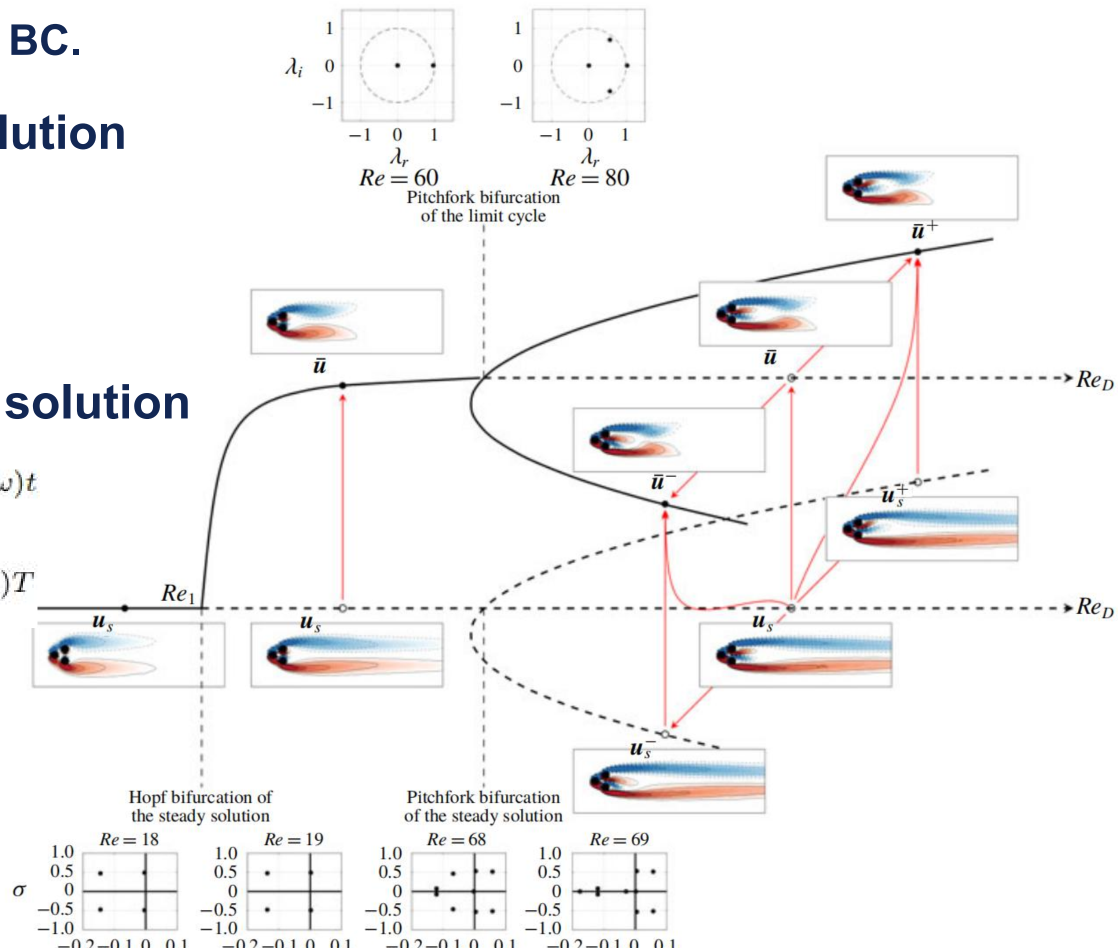
$$\partial_t \boldsymbol{q}' = \mathcal{L}_{\boldsymbol{U}_s} \boldsymbol{q}' \quad \boldsymbol{q}' = (\boldsymbol{u}', p')$$

$$(\sigma + i\omega)\, \hat{\boldsymbol{q}} = \mathcal{L}_{\boldsymbol{U}_s} \hat{\boldsymbol{q}}$$

Floquet stability of the periodic solution

$$\partial_t \boldsymbol{q}' = \mathcal{L}_{\boldsymbol{U}_p(t)} \boldsymbol{q}' \quad \boldsymbol{q}'(\boldsymbol{x}, t) = \hat{\boldsymbol{q}}(\boldsymbol{x}, t) e^{(\sigma + i\omega)t}$$

$$A_F = \exp\left(\int_{t_0}^{t_0+T} \mathcal{L}_{\boldsymbol{U}_p(t)} dt\right) \quad \lambda_F = e^{(\sigma + i\omega)T}$$



Deng et al. 2021 EPL

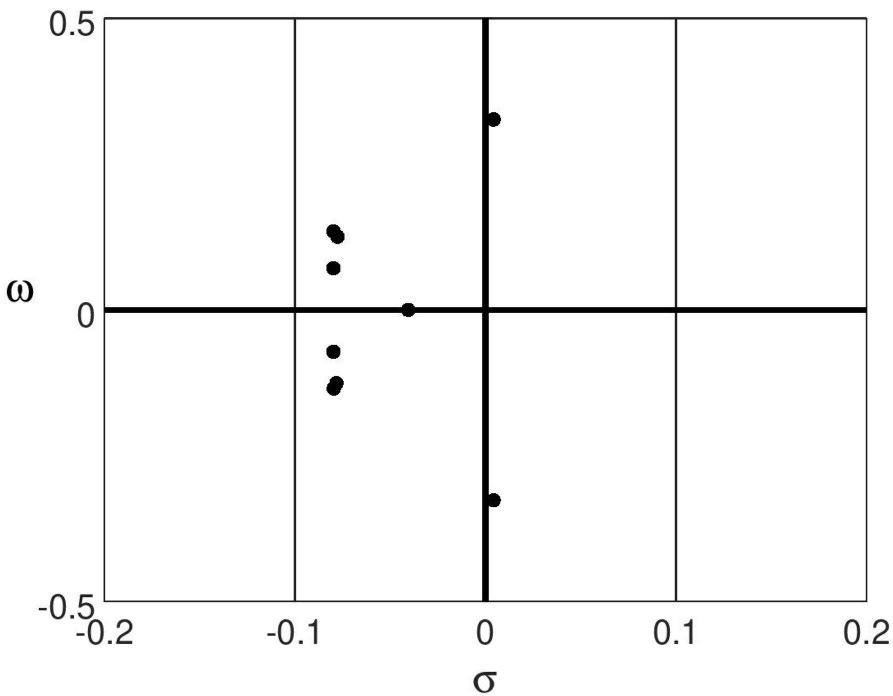
2.2 Hopf bifurcation

Linear stability analysis of the symmetric steady solution at *Re=20:*

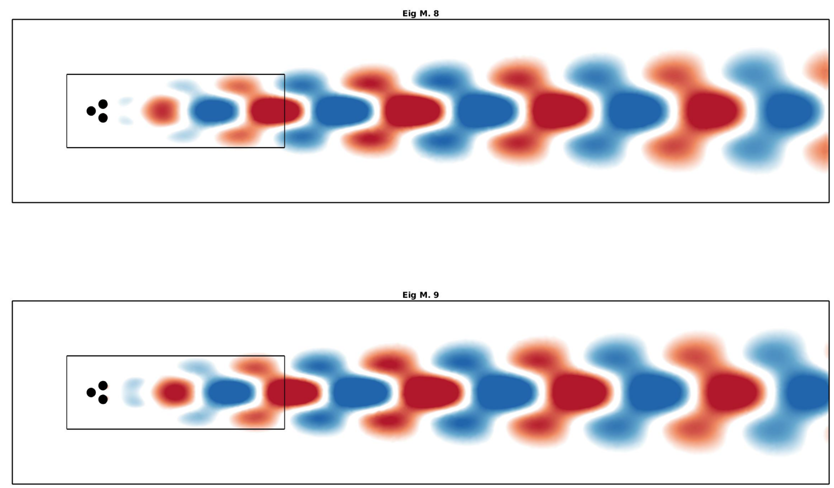
<u>a steady state changes to an oscillatory state</u>

Dušek *et al.* 1994 JFM

Eigenvalues:



Unstable eigenmodes:

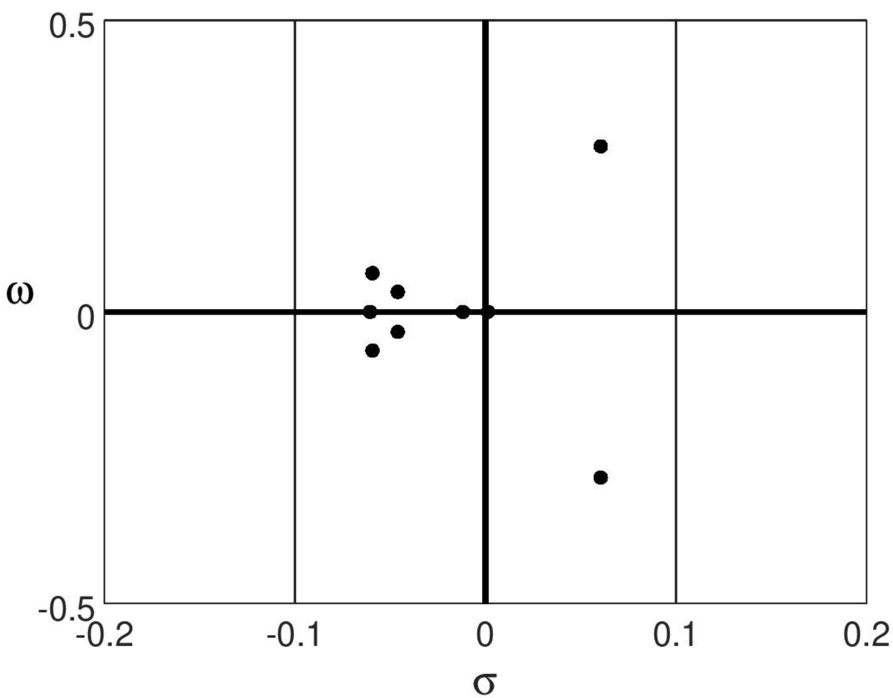


2.3 Pitchfork bifurcation

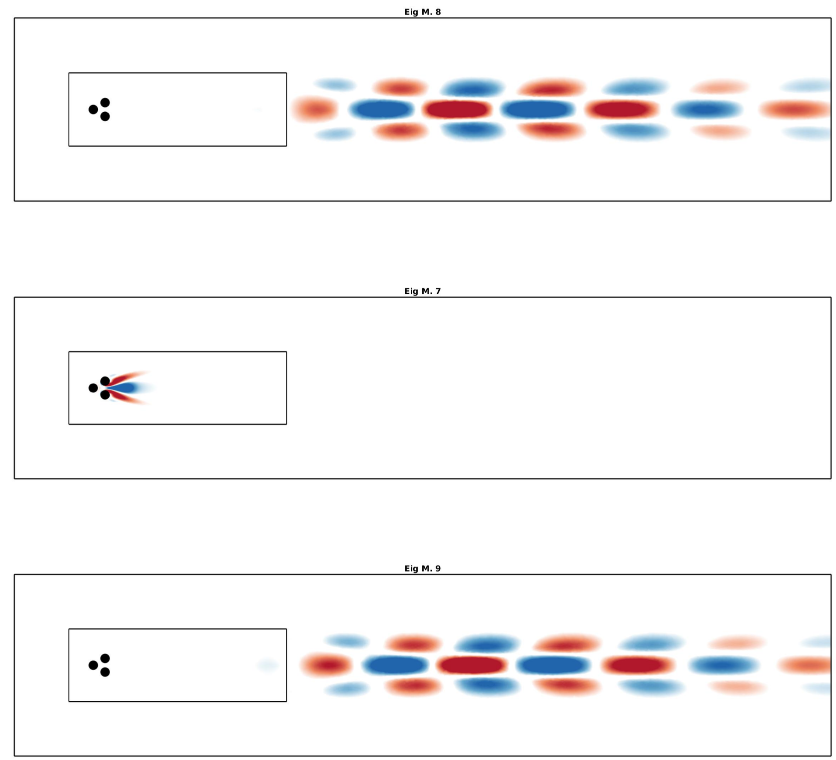
Linear stability analysis of the symmetric steady solution at *Re=81*

the reflectional symmetry is broken in a pitchfork bifurcation

Eigenvalues:



Unstable eigenmodes:



Outline

1. Benchmark configuration -- the fluidic pinball
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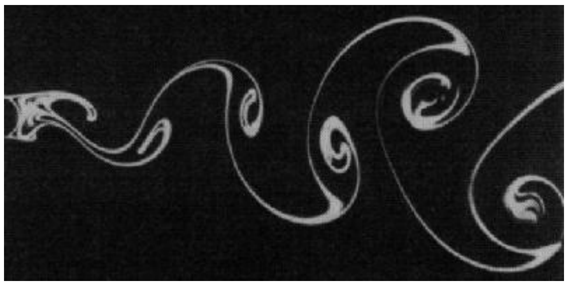
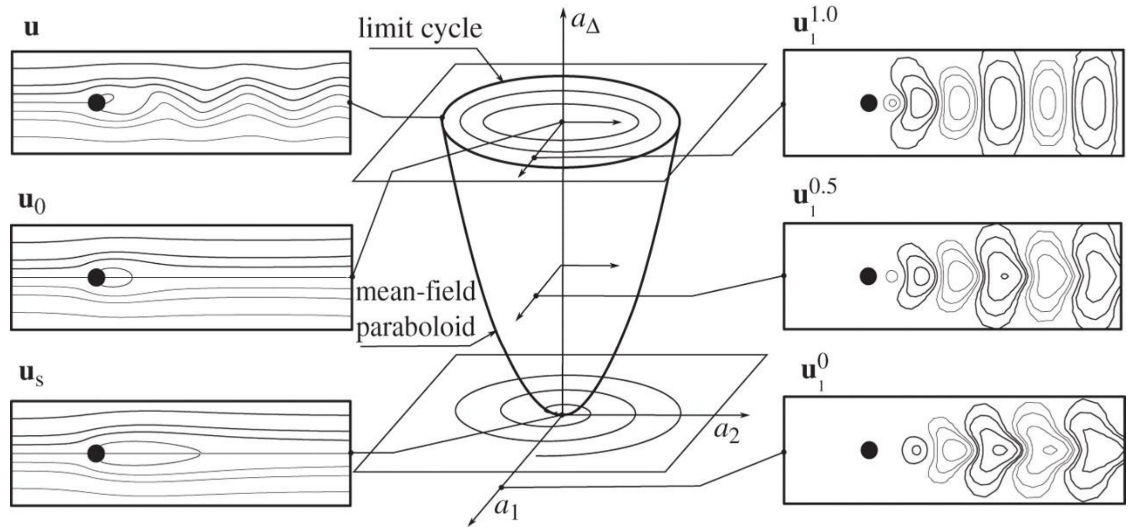
3.1 Transient and post-transient dynamics

Nonlinear saturation from the steady solution to the asymptotic state

Benard-Von Karman vortex street (Van Dyke 1982) :





Mean flow variations

Mode deformation

Stuart–Landau equation :

$$\frac{dA}{dt} = \sigma A + \alpha |A|^2 A$$

$$\mathbf{u} = \underbrace{\mathbf{u}_s + a_\Delta \mathbf{u}_\Delta}_{\mathbf{u}^B} + \underbrace{a_1 \mathbf{u}_1 + a_2 \mathbf{u}_2}_{\mathbf{u}^C} + \underbrace{\mathbf{u}^S}_{\text{higher harmonics}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} a_1 = \sigma a_1 - \omega a_2,$$

$$\frac{\mathrm{d}}{\mathrm{d}t} a_2 = \sigma a_2 + \omega a_1$$

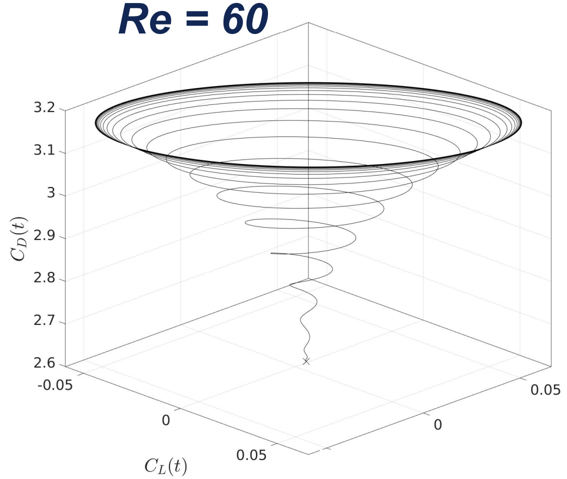
$$\frac{\mathrm{d}}{\mathrm{d}t} a_\Delta = -\sigma_\Delta a_\Delta + c(a_1^2 + a_2^2)$$

Noack et al. 2003 JFM **Tadmor et al. 2011 PTRSA**

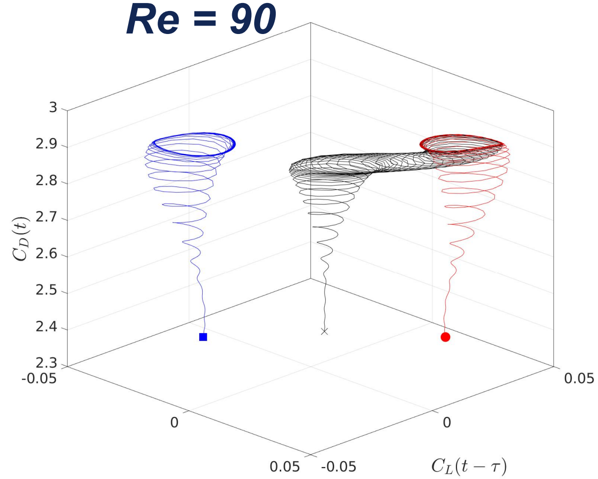
3.2 Multiple invariant sets in the state space

Phase space with C_L and C_D (DNS starting with different steady solutions)

Re = 60

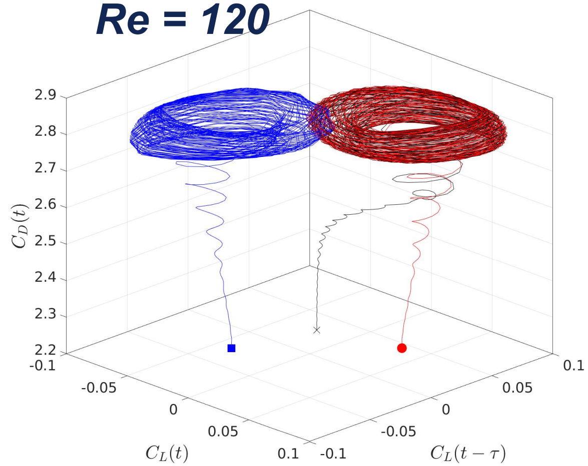


Re = 90

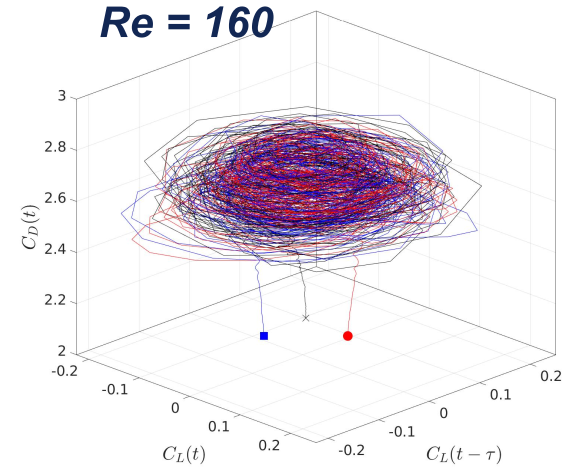


3 steady solutions,
3 limit cycles,
2 torus,
1 chaotic attractor.

Re = 120



Re = 160



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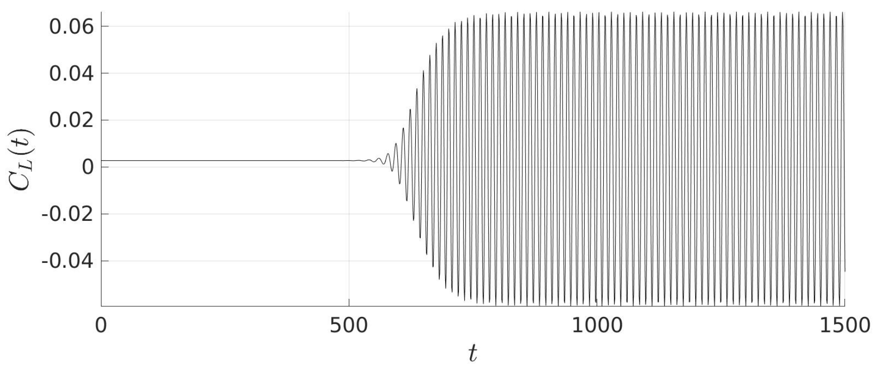
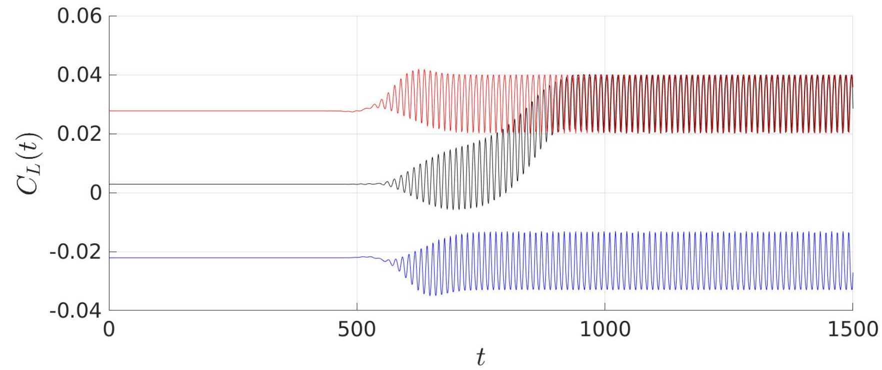
4 Hierarchy of test cases of increasing difficulty

ROM for the unforced flow exhibiting steady, periodic, symmetry-breaking, quasi-periodic, and chaotic features at different Reynolds numbers

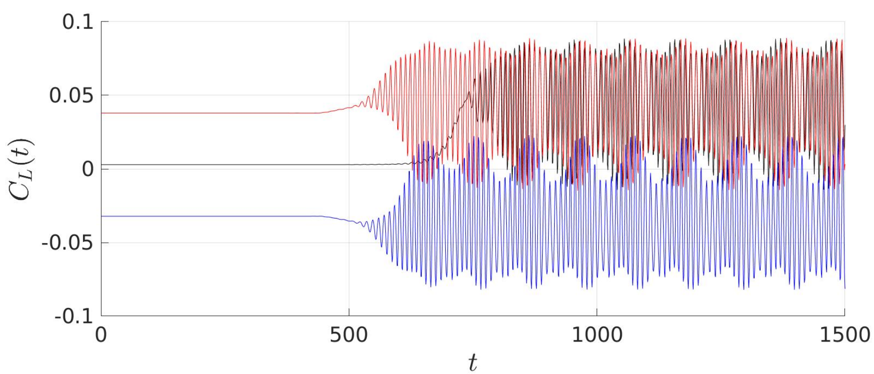
Re = 60



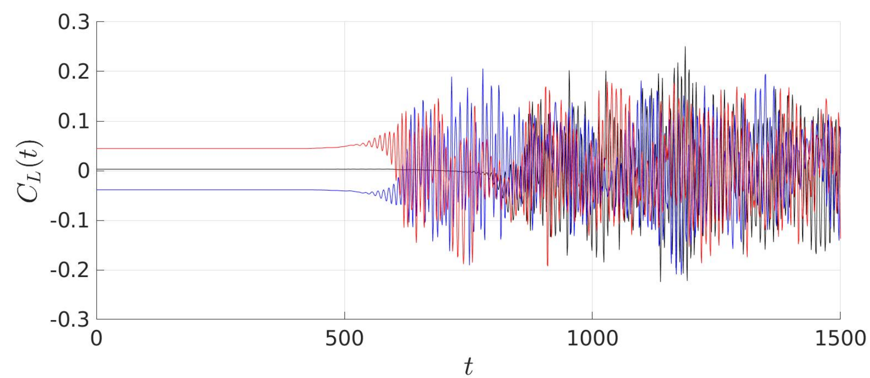
Re = 90



Re = 120



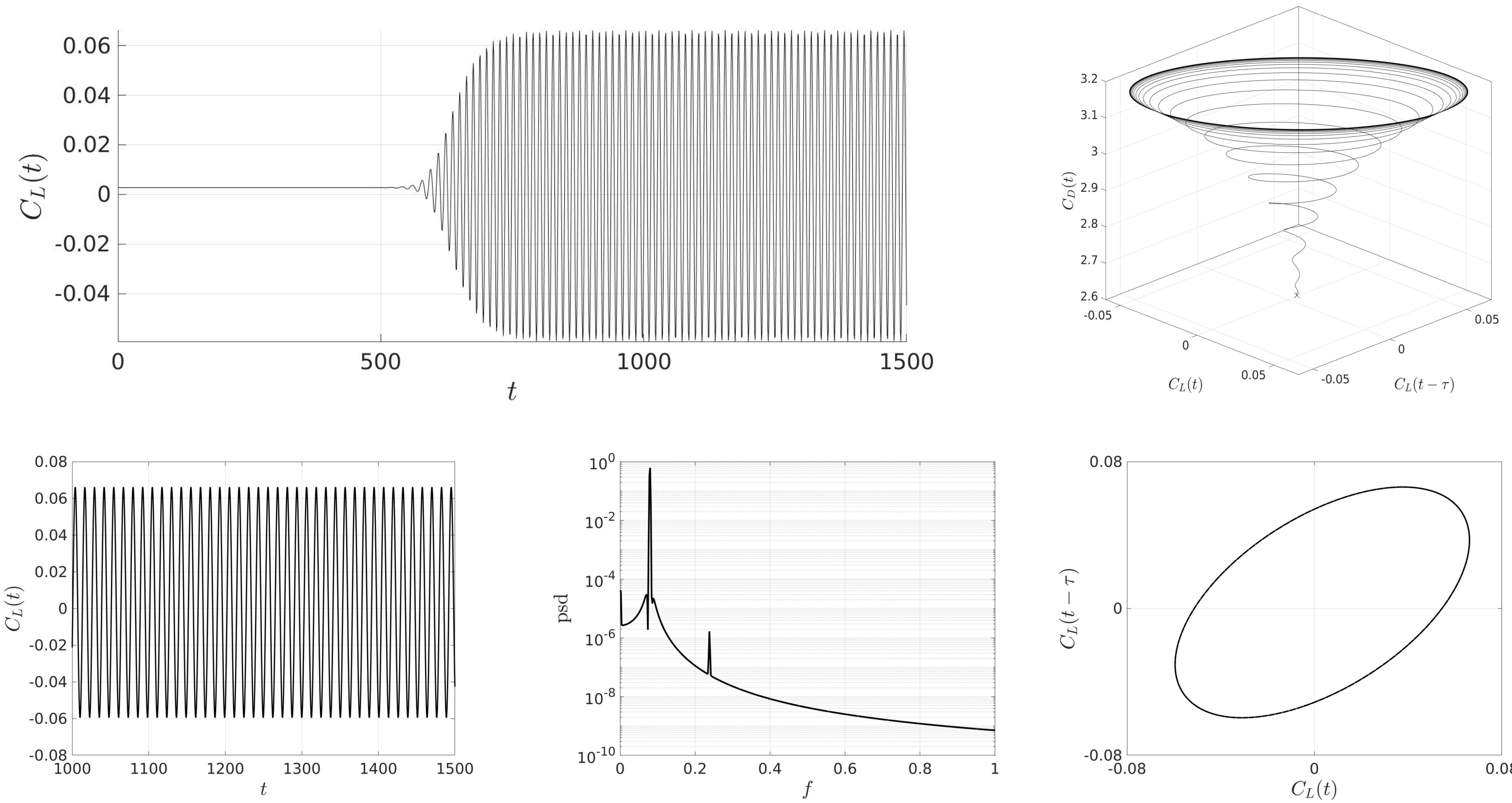
Re = 160



4.1 Periodic vortex shedding

MOR for the transient and post-transient dynamics at *Re = 60:*

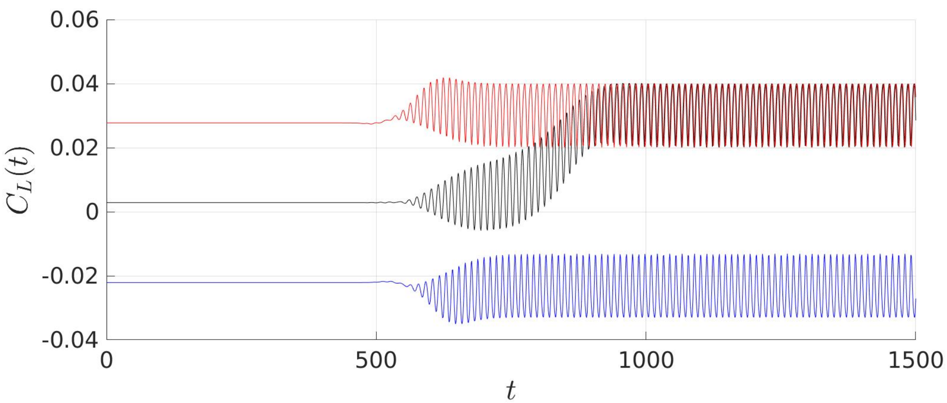
Single transient from 1 steady solution to 1 periodic solution.

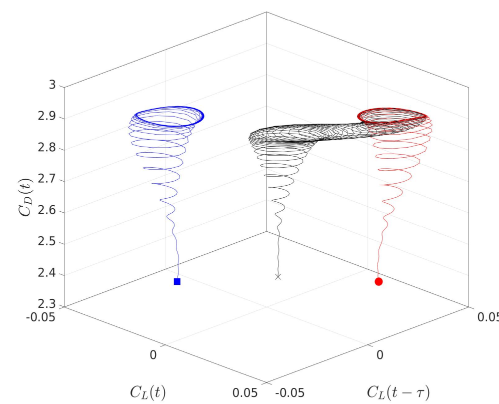


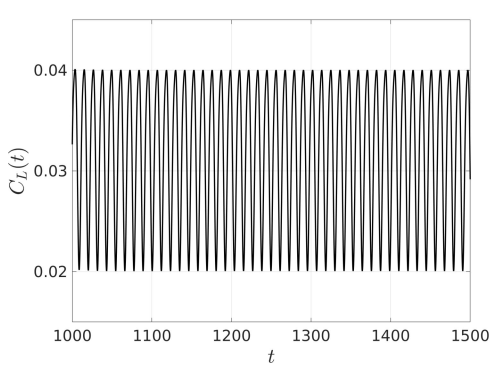
4.2 Periodic vortex shedding with a deflected jet

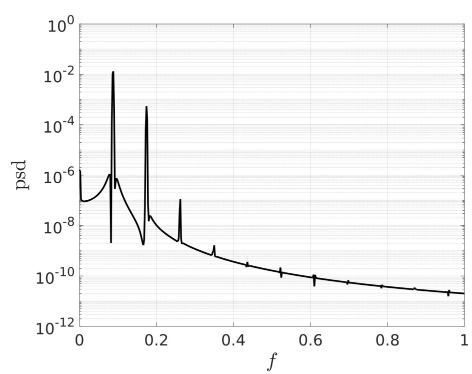
MOR for the transient and post-transient dynamics at *Re = 90:*

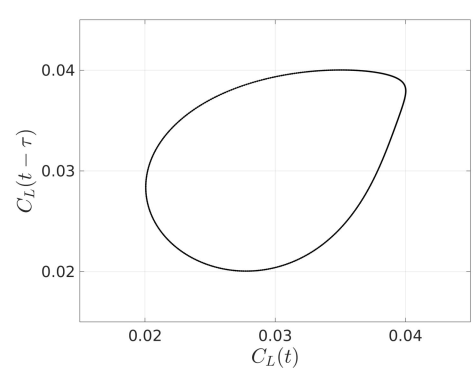
Four transients from 3 steady solutions to 3 periodic solutions.







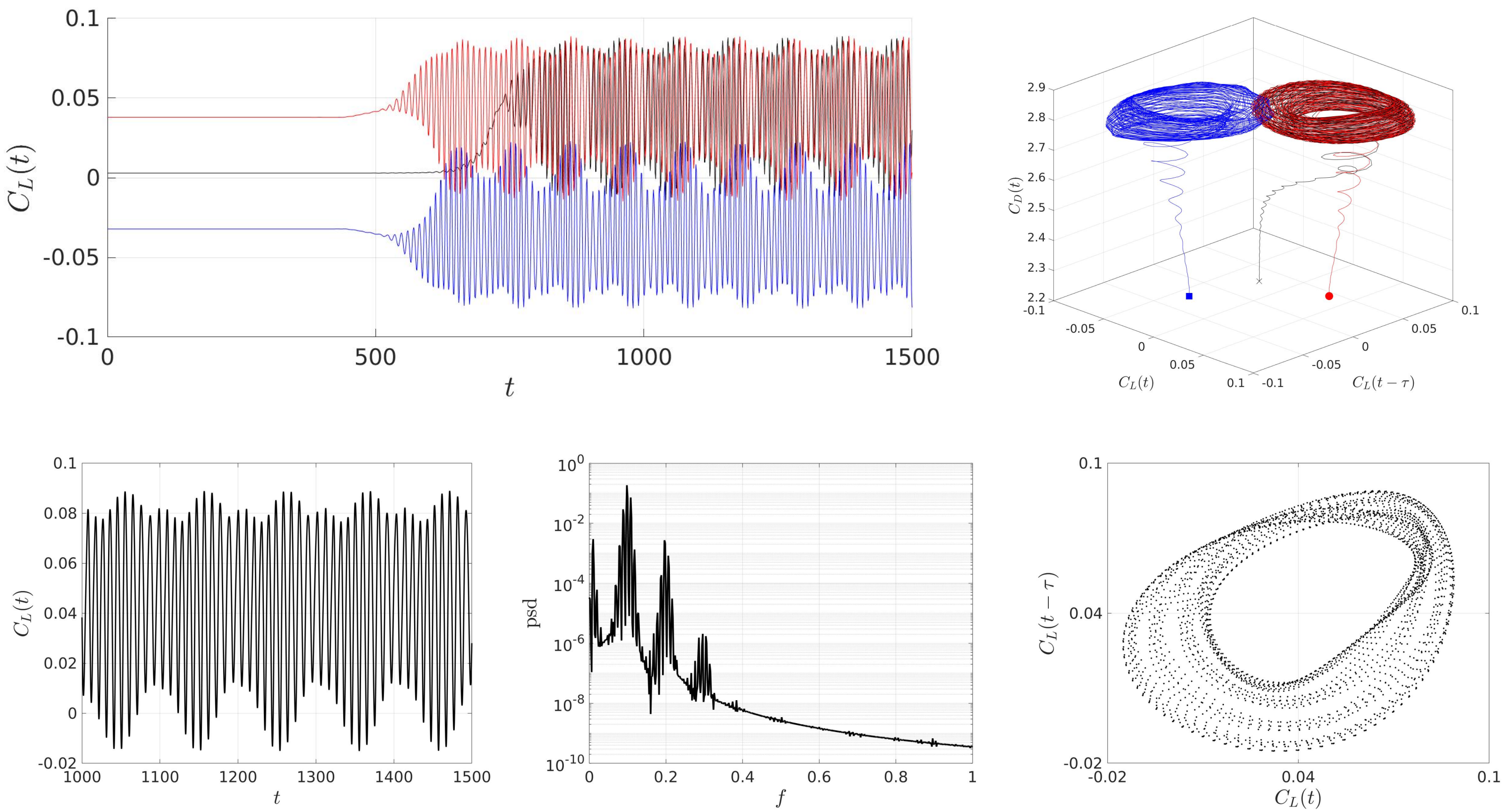




4.3 Quasi-periodic vortex shedding

MOR for the transient and post-transient dynamics at *Re = 120:*

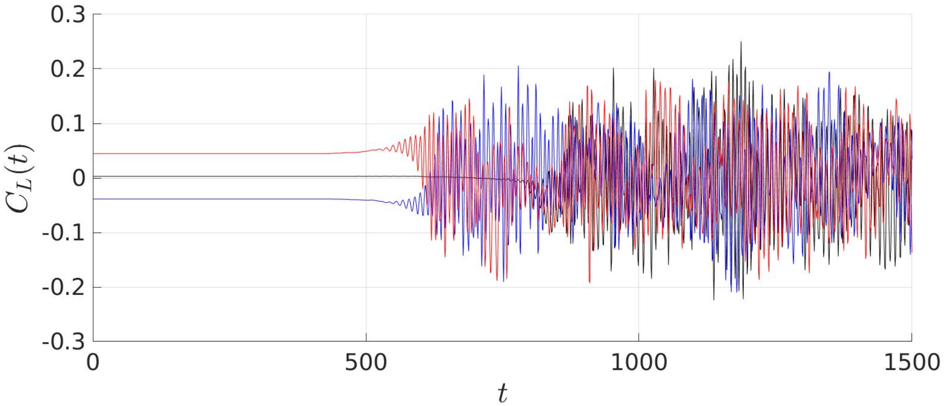
Four transients from 3 steady solutions to 2 quasi-periodic solutions.

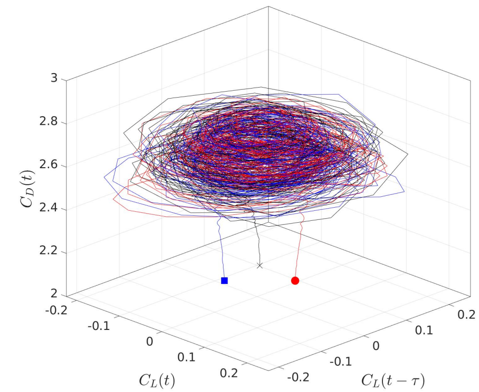


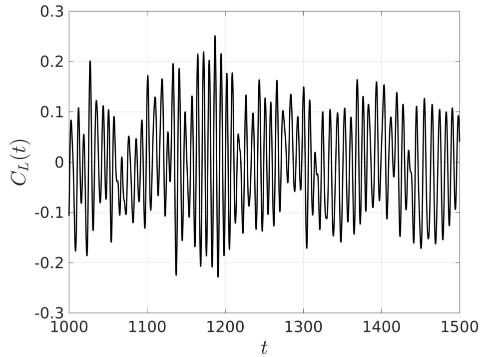
4.4 Chaotic vortex shedding

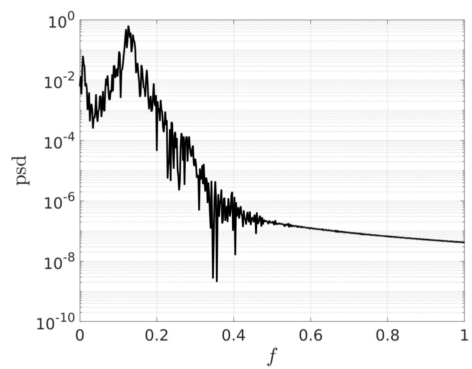
MOR for the transient and post-transient dynamics at *Re = 160:*

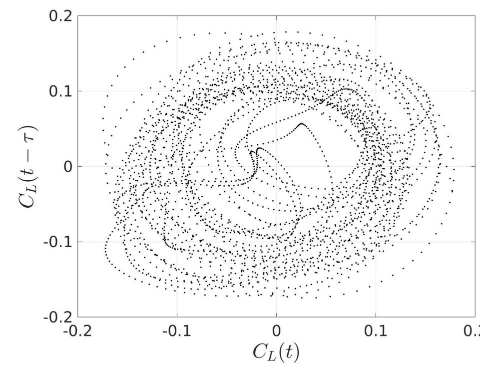
Four transients from 3 steady solutions to 1 chaotic solutions.











4.5 Assessment metric for ROM methods

To obtain interpretable ROMs, the proposed methods should

1. Represent the bifurcation phenomena in fluid and solid mechanics:
   - resolve correct all invariant sets (fixed points, limit cycles and so on)
   - identify dynamical systems for the bifurcations

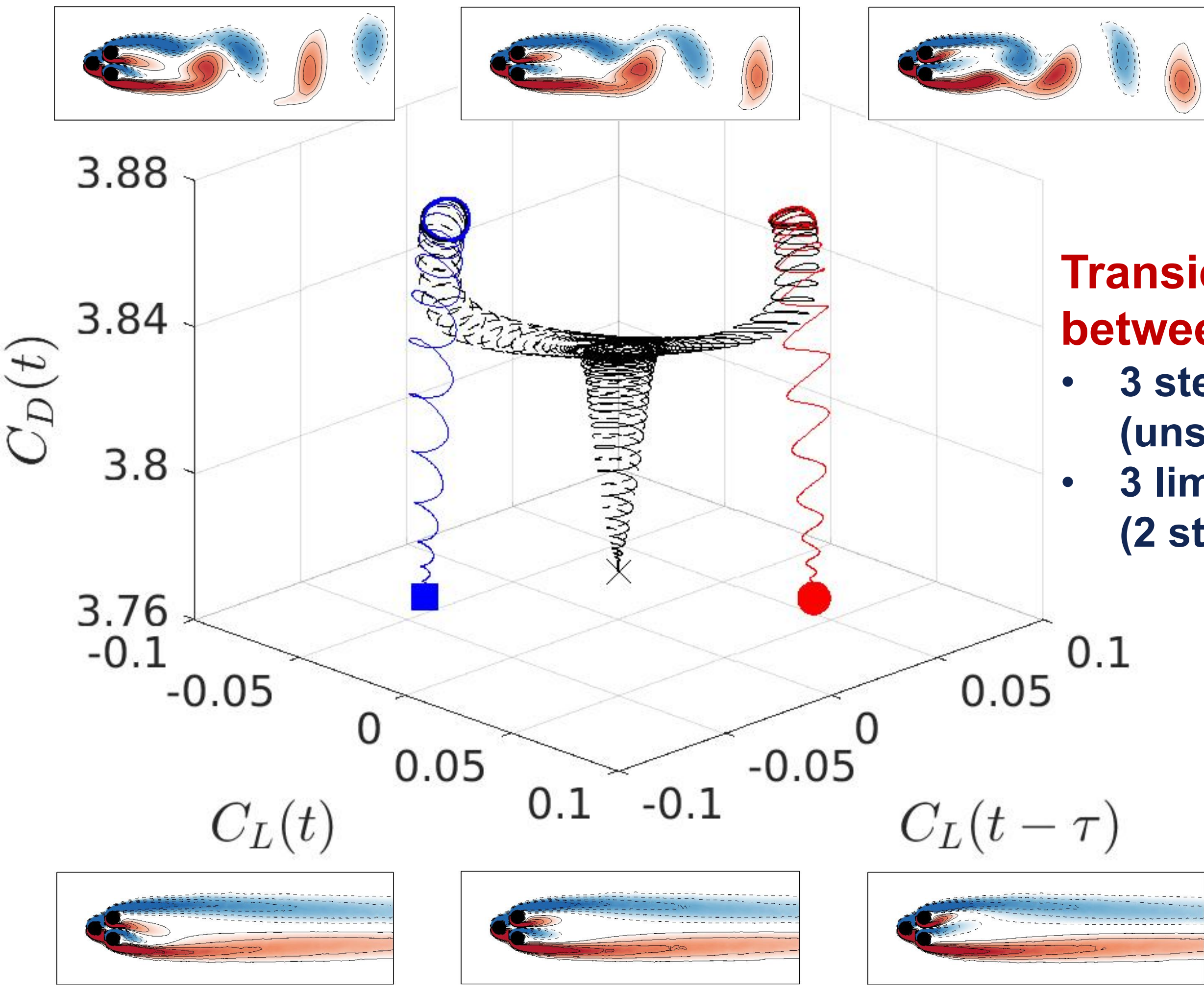
2. Reproduce the main features of transient and post-transient dynamics:
   - linear instability of the unstable fixed point (growth rate, initial frequency)
   - nonlinear saturation to the periodic, quai-periodic state (magnitude and frequency in the asymptotic regime)

Outline

1. Benchmark configuration -- the fluidic pinball
2. Bifurcation & instabilities
3. Challenges in ROM of bifurcations
4. Test cases
5. Some examples

5.1 Least-order MFM based on bifurcations

Multiple Navier-Stokes solutions and invariant sets (*Re=80*)



Transient dynamics between 6 N-S solutions:

- **3 steady solutions (unstable)**
- **3 limit cycles (2 stable & 1 unstable)**

Deng et al. 2020 JFM

5.1 Least-order MFM based on bifurcations

A model **compatible with the Navier-Stokes equations** to describe dynamics based on the **bifurcations.**

The normal forms of the supercritical Hopf & Pitchfork bifurcations are **cubic**:

Hopf: $\dot{A} = \sigma A - \beta |A|^2 A, \; A \in \mathbb{C}$

Pitchfork: $\dot{B} = \sigma_4 B - \beta_4 B^3, \; B \in \mathbb{R}$

The non-linearities of the N.S.E are **quadartic**:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\, \boldsymbol{u} = -\nabla p + Re^{-1} \Delta \boldsymbol{u} + \boldsymbol{f}$$

Hence, the least order model must have **five (3+2)** degrees of freedom (**Stuart 1958**):

$$\boldsymbol{u}(\boldsymbol{x},t) \approx \underbrace{\boldsymbol{u}_s(\boldsymbol{x})}_{Steady\ solution} + \underbrace{a_1(t)\,\boldsymbol{u}_1(\boldsymbol{x}) + a_2(t)\,\boldsymbol{u}_2(\boldsymbol{x}) + a_3(t)\,\boldsymbol{u}_3(\boldsymbol{x})}_{Hopf\ bifurcation} + \underbrace{a_4(t)\,\boldsymbol{u}_4(\boldsymbol{x}) + a_5(t)\,\boldsymbol{u}_5(\boldsymbol{x})}_{Pitchfork\ bifurcation}$$

with $\mathbf{a}_3$ being slaved to $\mathbf{a}_{1,2}$, $\mathbf{a}_5$ being slaved to $\mathbf{a}_4$:

$$A = a_1 + i a_2$$
$$B = a_4$$

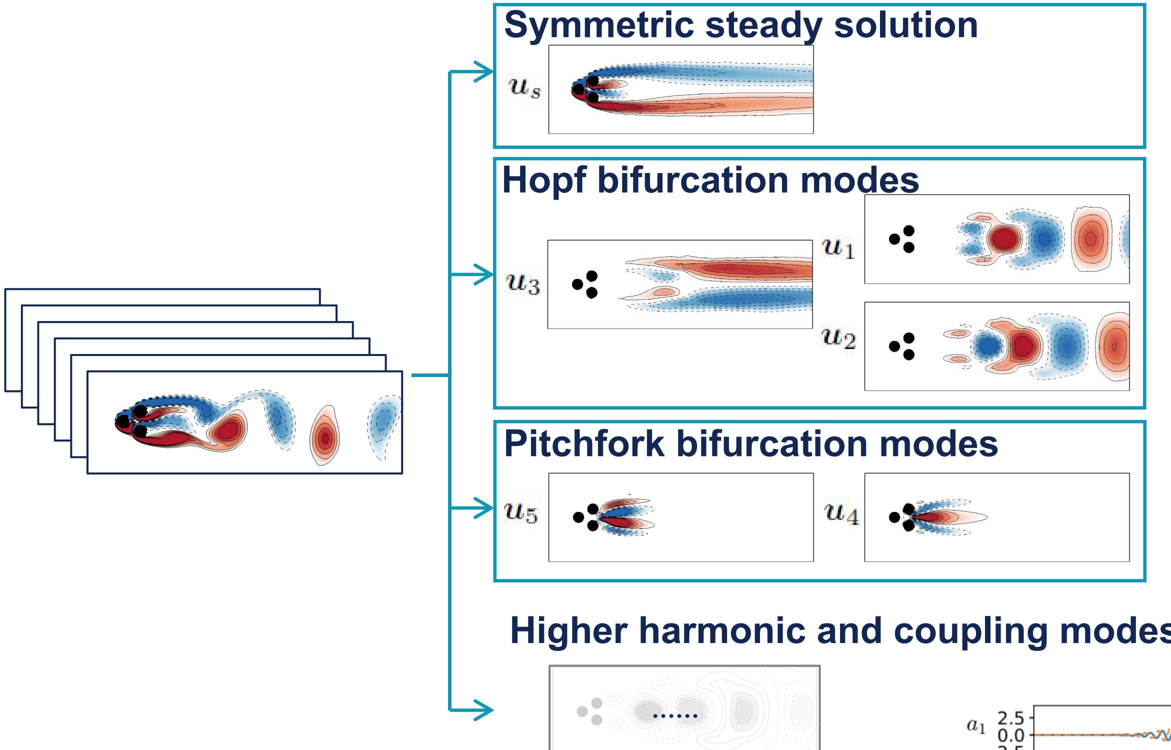
$$\begin{cases} \dot{a}_1 = (\sigma - \beta a_3)\, a_1 - (\omega + \gamma a_3)\, a_2 \\ \dot{a}_2 = (\sigma - \beta a_3)\, a_2 + (\omega + \gamma a_3)\, a_1 \\ \dot{a}_3 = -\lambda \left(a_3 - \kappa_3 \left(a_1^2 + a_2^2\right)\right) \\ \dot{a}_4 = (\sigma_4 - \beta_4 a_5)\, a_4 \\ \dot{a}_5 = -\mu \left(a_5 - \kappa_5 a_4^2\right) \end{cases}$$

+ cross terms

, with $|\lambda| \gg 1, |\mu| \gg 1$

5.1 Mean-field Galerkin + sparse identification

- Optimal modes for the bifurcations



Hopf:

$$\dot{a}_1 = \sigma(a_3)a_1 - \omega(a_3)a_2,$$
$$\dot{a}_2 = \sigma(a_3)a_2 + \omega(a_3)a_1,$$
$$\dot{a}_3 = \sigma_3 a_3 + \beta_3(a_1^2 + a_2^2),$$

PF:

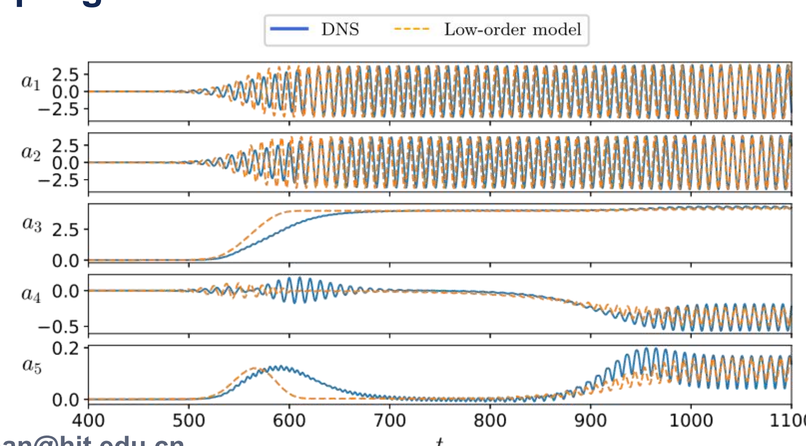
$$\dot{a}_4 = \sigma_4 a_4 - \beta_4 a_4 a_5,$$
$$\dot{a}_5 = \sigma_5 a_5 + \beta_5 a_4^2,$$

Cross terms

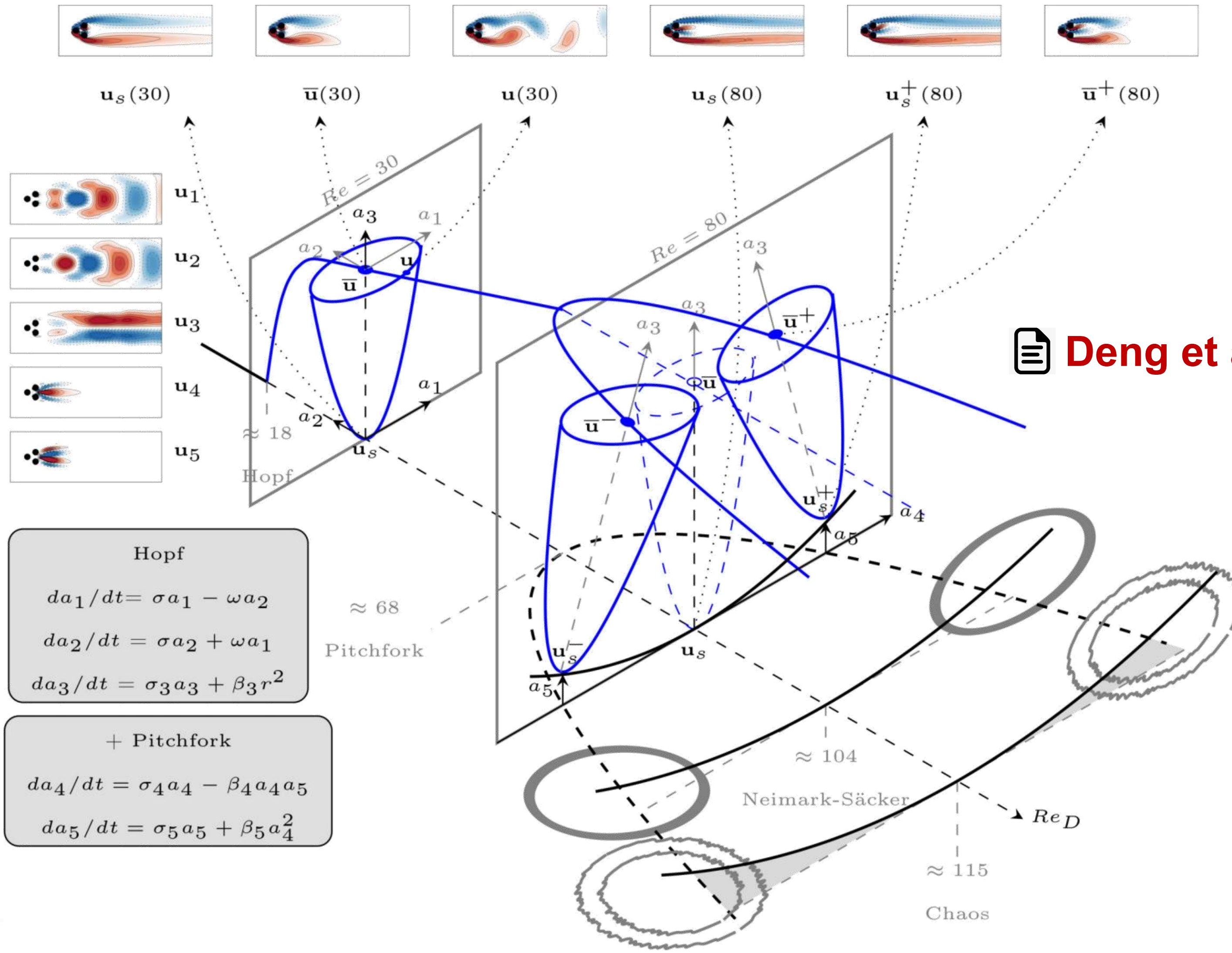
- Identify the coefficients by **SINDy**

(**Brunton et al. 2016 PNAS**)

Sparse identification with physical constraints (geometrical symmetry, frequency, slaving relation ...)



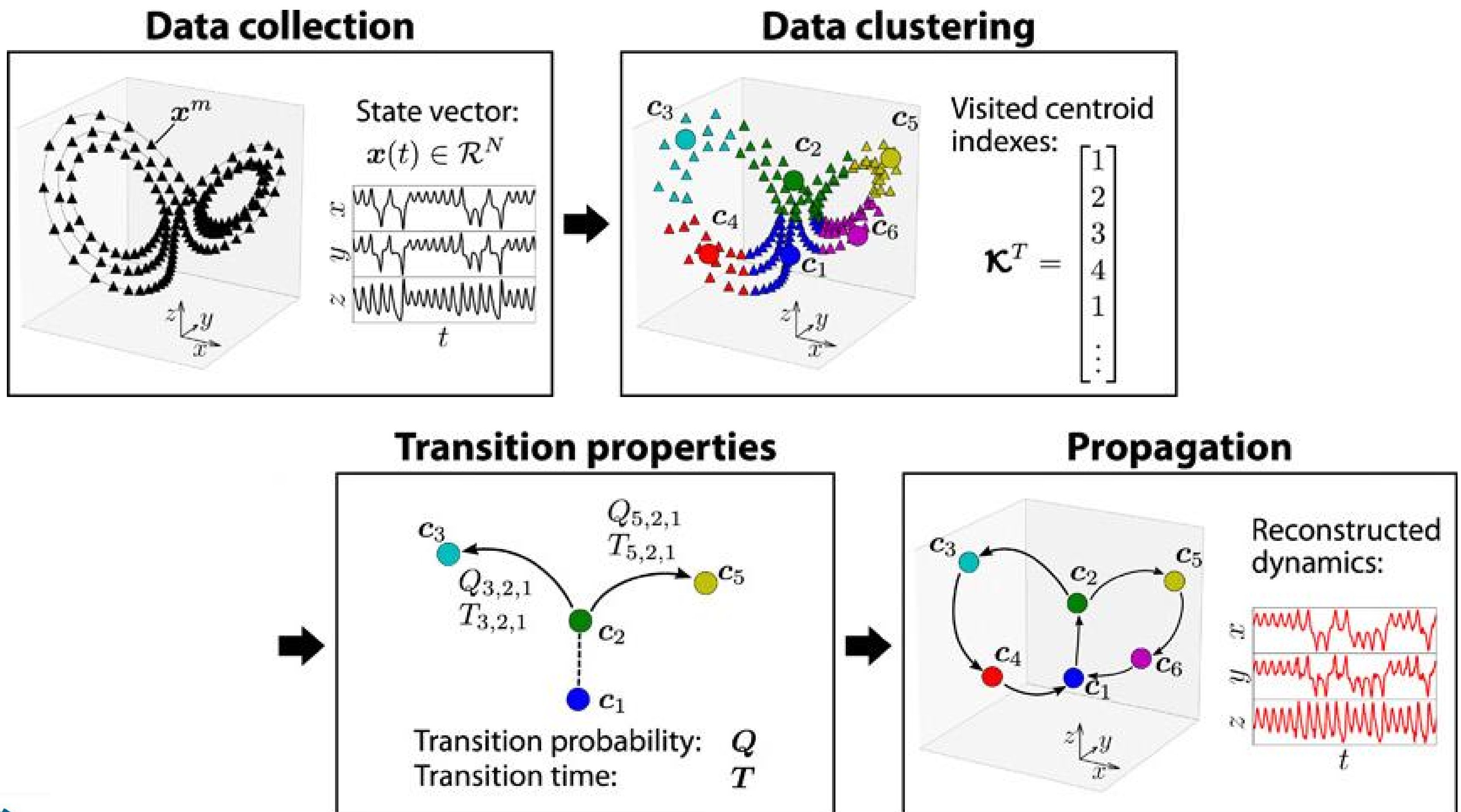
5.1 Modelling for the first two successive bifurcations



Deng et al. 2020 JFM

5.2 Cluster-based network model (CNM)

CNM overview (**Fernex et al. 2021 Science Advances & Li et al. 2021 JFM**)



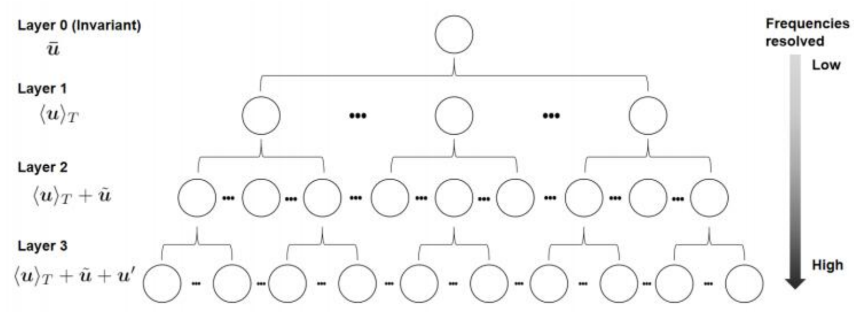
5.2 Hierarchical CNM (HiCNM)

Knowledge-embedded CNM: HiCNM

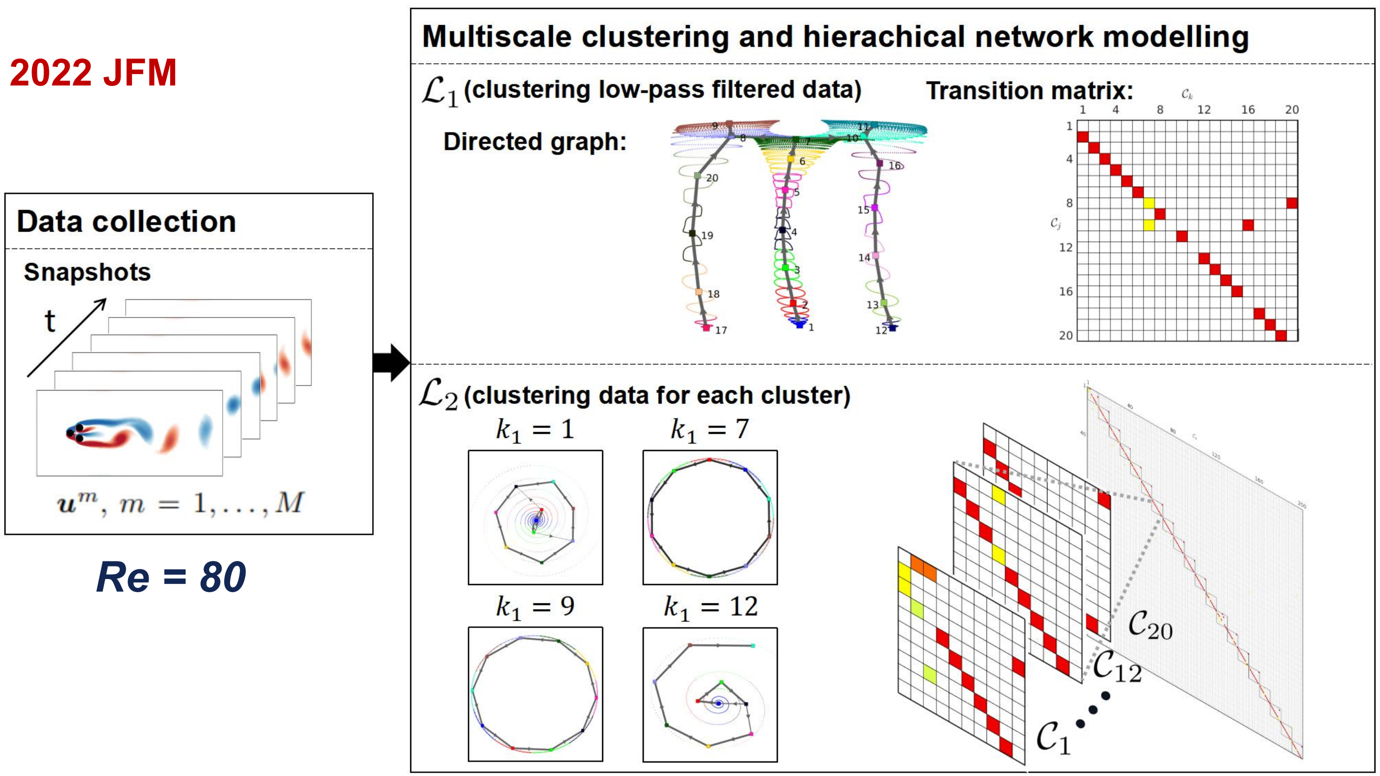
- Multiscale decomposition:

$$\boldsymbol{u}(\boldsymbol{x},t) = \underbrace{\langle \boldsymbol{u}(\boldsymbol{x},t)\rangle_T}_{\omega \ll \omega_c} + \underbrace{\tilde{\boldsymbol{u}}(\boldsymbol{x},t)}_{\omega \sim \omega_c} + \underbrace{\boldsymbol{u}'(\boldsymbol{x},t)}_{\omega \gg \omega_c}$$

- Hierarchical structure of clusters:



Deng et al. 2022 JFM



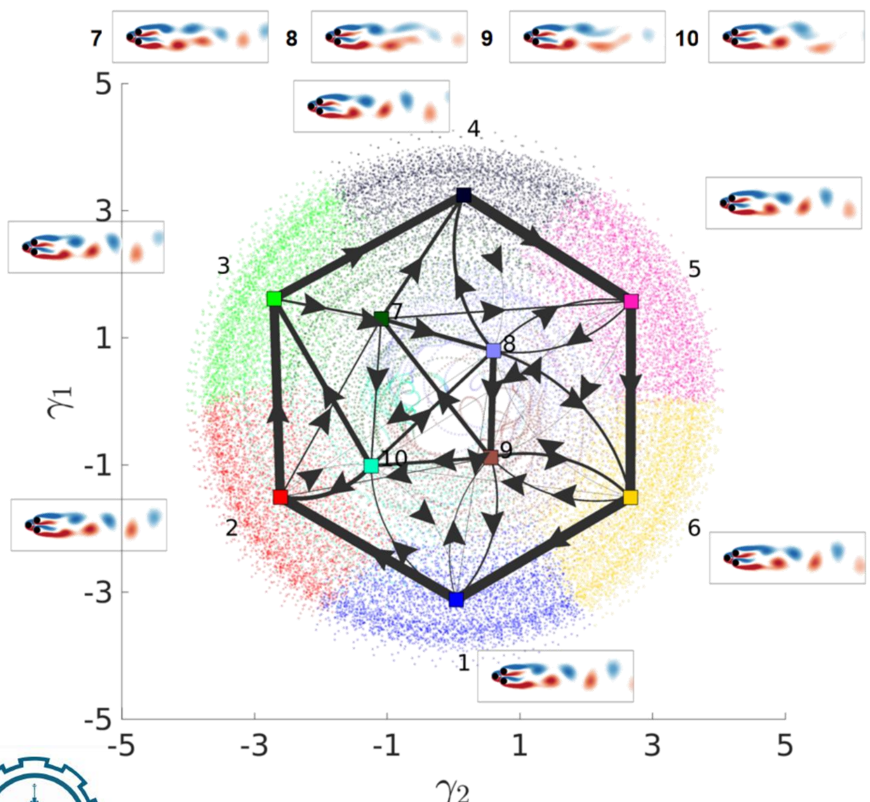
Re = 80

5.2 Hierarchical CNM (HiCNM)

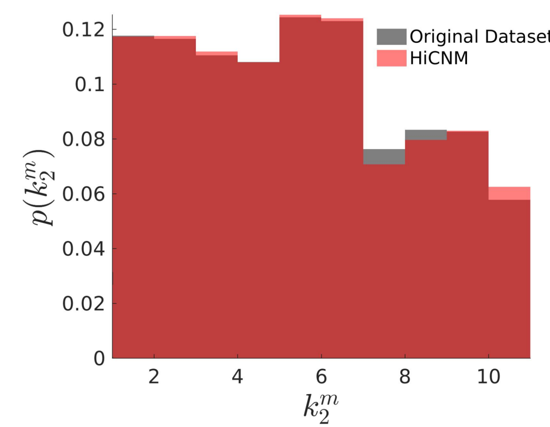
HiCNM can **automatically** and **systematically** identify the **local** and **global** dynamics in the transient from the unstable sets to chaotic set

➢ Graph of Chaotic dynamics and its reconstruction:

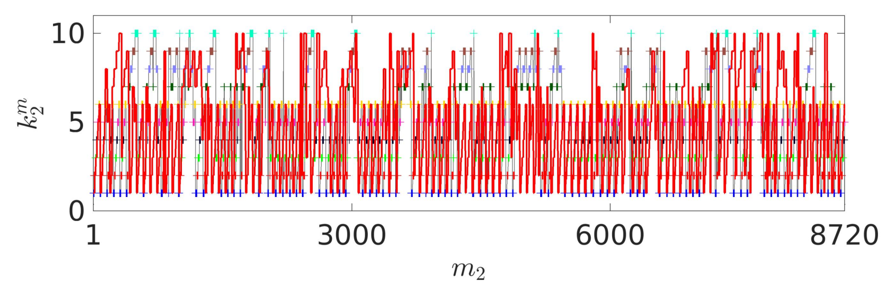
Graph of transitions



Probability distribution



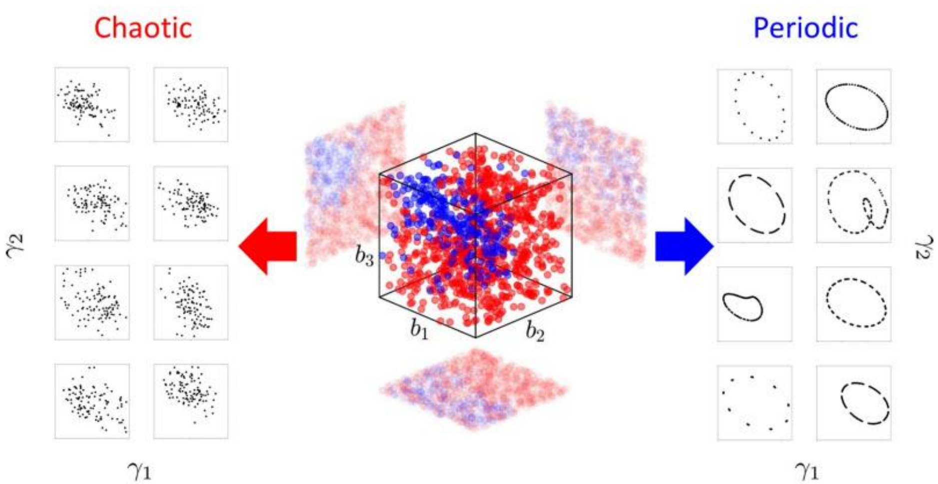
Time evolution of the cluster index

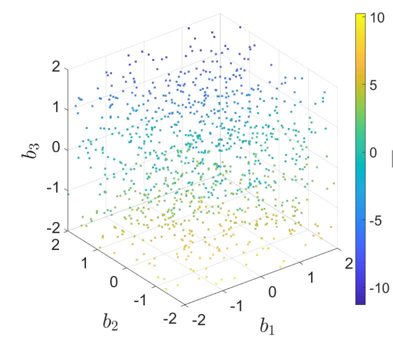


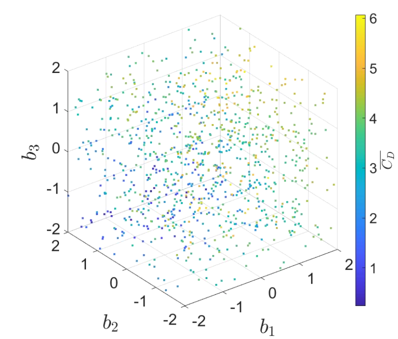
Perspectives

- Least-order MFM for the **subcritical bifurcation, quasi-periodic regime and chaotic regime**
- **Nonlinear correction with higher harmonics** to the least-order MFM
- **Automated ROM: knowledge- and data-driven ROM** for complex systems in flow mechanics and industrial applications.
- **Parameter-induced dynamics changes** in fluidic pinball: (b_1, b_2, b_3) - dependent flow

Dynamics and mean forces of 1000 DNSs with LHS in b-space







Li et al. 2022 JFM

Thank you !

Any questions?

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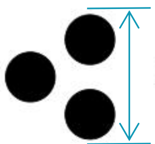
A1. Info for Pinball in Noack & Morzyński 2017 Tech. Rep.

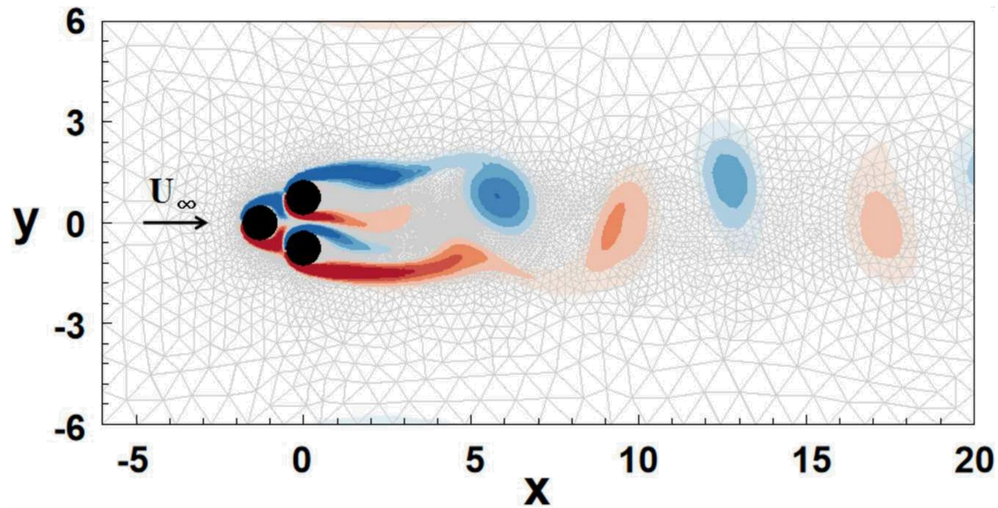
Incompressible non-dimensionalized NS scaled with the cylinder diameter ***D***, the oncoming velocity ***U***, the time scale ***D/U***, and the density ***ρ*** of the fluid.

$$\partial_t \boldsymbol{u} + \nabla \cdot \boldsymbol{u} \otimes \boldsymbol{u} = \frac{1}{Re} \Delta \boldsymbol{u} - \nabla p$$

$$\nabla \cdot \boldsymbol{u} = 0$$



$$L = \frac{5}{2} D \qquad Re = \frac{U_\infty D}{\nu}$$



- **2D DNS** code by **Marek Morzyński.**
- Time integration with **3rd-order accuracy.**
- FEM discretization with **4 225** triangles and **8 633** vertices in **[−6, 20]** × **[−6, 6]** with **2nd-order Taylor-Hood finite (T6 triangular) elements.**
- **No-slip BC**: $U_r = 0$ on the cylinders, $U_X = 1$ at the inlet and the side wall;

  Stress-free BC: the outlet.
- **750 MB / 1000 snapshots.**

Noack & Morzyński 2017 Tech. Rep.

http://berndnoack.com/

A1. Info for Pinball in Noack & Morzyński 2017 Tech. Rep.

- Main referents: **B. Noack & M. Morzyński (DNS), R. Martinuzzi (EXP)**
- Specific methodological challenges: **Bifurcation phenomena in fluid mechanics & Parameter-induced ROM.**
- Target engineering applications：**Prototype for multibody system & wake control**
- Hierarchy of test cases of increasing difficulty: **Detailed report** in **Noack & Morzyński 2017 Tech. Rep.** and **Deng et al. 2020 JFM**
- Assessment metrics. **26 minutes** for **100 convective times** on a Laptop of 2016 (Elitebook 820 G3 16Gb RAM, Core i7, proG6, 512GB SSD) & **750 MB/1000 snapshots** (10 per time unit).
- List of quantities of interest：**Multiple inputs** (Re, b_1, b_2, b_3), **Multiple output** (Velocity & pressure field, C_D, C_L, periods)