Optimized evolve-filter-relax regularization algorithms for convection-dominated flows

Finite Element (FE) simulations are often characterized by spurious oscillations and poor accuracy when dealing with convection-dominated flows and coarse mesh resolutions. When these phenomena manifest, numerical stabilization techniques turn out to be useful to alleviate the spurious oscillations and improve the results of Full Order Model (FOM) simulations.

In particular, one wide-spread technique is the Evolve-Filter-Relax (EFR) regularization, which consists at each time step t_n in three simple steps:

- the *evolve* step, which is exactly the discretized solution of the Navier-Stokes (NS) equations. The result is a not-filtered velocity $w(t_{n+1})$;
- the *filter* step, which is used to smooth and regularize the evolved velocity, to obtain a filtered velocity, namely $w_f(t_{n+1})$. The entity of the filter depends on the so-called filtering parameter δ;
- the *relax* step, which consists in a convex combination of the filtered and non-filtered velocity. So, the final velocity would be: $u(t_{n+1}) = \chi * w_f(t_{n+1}) + (1 - \chi) * w(t_{n+1})$, where $\chi \in [0, 1]$ is the relaxation parameter.

Now, in this formulation it is crucial to tune both the filtering and the relaxation parameters choosing the optimal values for δ and χ, respectively. This parameters' optimization was one of the topics of the collaboration between SISSA (Italy), Politecnico di Torino (Italy) and Virginia Tech (US), within the context of the ARIA project.

In the literature of academic benchmarks there exist common choices for the parameters' values, corresponding to the Kolmogorov n-width for δ, and to $k\,\Delta t$ for χ, where k is a positive integer. These choices have been used in previous works such as [1], [2]. However, the common choice is not always the optimal one. In particular, we noticed that, in well-known academic benchmarks such as the flow past a cylinder at $Re = 1000$, it is more efficient to have time-dependent parameters: $\delta(t)$ and $\chi(t)$.

But how can we now find the optimal value at each time step?
We just solve a simple gradient-based optimization. In this algorithm, the objective function to minimize is the discrepancy between the EFR velocity result on a coarse mesh and a "reference" velocity, obtained directly solving NS with the FE method on a refined mesh.

More in detail, we investigated:

I. The time-dependent optimization of $\chi(t)$, at δ fixed and equal to the Kolmogorov n-width. The results of this optimization (Figure 1) suggest that the optimal relaxation parameter is equal to 1 almost at each time step. It means that no relaxation is needed, and we can just consider a EF regularization.
II. The time-dependent optimization of $\delta(t)$, in the EF algorithm. In this case, we obtained a chaotic behavior for $\delta(t)$ (Figure 2), but the optimized-EF velocity field is much more accurate than the standard-EF velocity, as can be seen in Figure 3.

SISSA, PoliTo and VT are preparing a manuscript on this topic and plan an extension of this project also at the reduced order level and including innovative machine learning techniques.

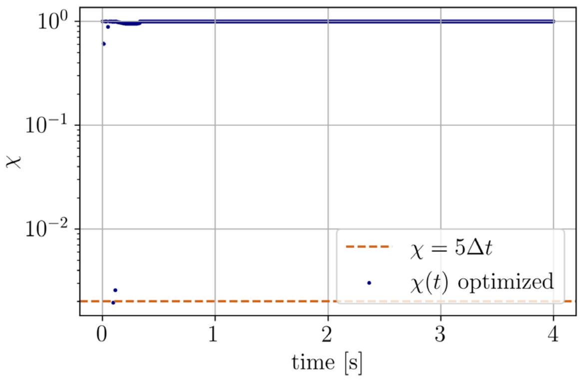

Figure 1: The optimized relaxation parameter.

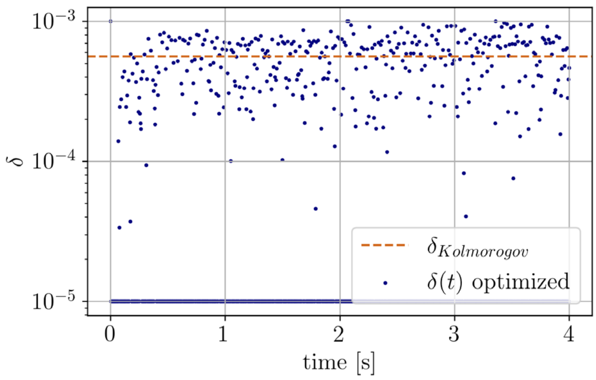

Figure 2: The optimized filtering parameter.

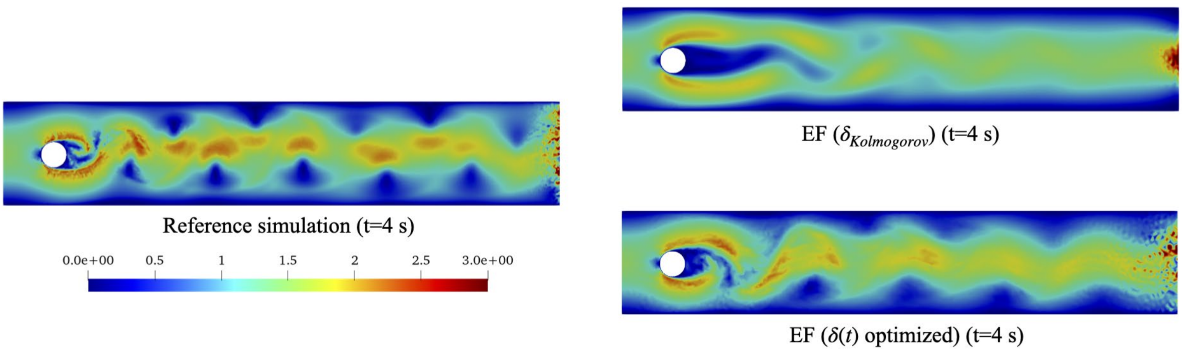

Figure 3: The velocity fields for the reference simulation on refined mesh and for different values of the filtering parameter.

[1] Strazzullo, M., Girfoglio, M., Ballarin, F., Iliescu, T., & Rozza, G. (2022). Consistency of the full and reduced order models for evolve-filter-relax regularization of convection-dominated, marginally-resolved flows. *International Journal for Numerical Methods in Engineering*, *123*(14), 3148-3178.

[2] Strazzullo, M., Ballarin, F., Iliescu, T., & Canuto, C. (2023). New Feedback Control and Adaptive Evolve-Filter-Relax Regularization for the Navier-Stokes Equations in the Convection-Dominated Regime. *arXiv preprint arXiv:2307.00675*.