Full and Reduced Variational Multiscale Evolve and Filter approach for convection-dominated Navier-Stokes equations

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Addressing convection-dominated flows is a crucial task, with multiple applications in diverse fields, both in industry and science. However, under under-resolved regimes, numerical oscillations may arise, spoiling the accuracy of the numerical simulations.

To overcome this issue, numerical stabilizations and regularized strategies can be employed. Among them, the evolve-filter (EF) has notable success. EF is modular, simple, and effective both for the full-order models (FOMs) and the reduced-order models (ROMs). The EF technique consists of smoothing the oscillating behavior of the convection-dominated flow using a spatial filter. The filter action is determined by a parameter called the filter radius. However, large values of the filtering radius may yield inaccurate results, with overly diffusive behavior.

In this collaboration between Politecnico di Torino, Università Cattolica del Sacro Cuore, Universidad de Sevilla, and Virginia Tech, we propose novel strategies to alleviate the results of an over-diffusive filter. These approaches are based on the variational multiscale (VMS) framework. In this context, the resolved velocity field is decomposed into large and small resolved scales. Onto this decomposition, we build novel algorithms consisting of four steps:

1. We evolve the velocity through a time-stepping simulation of standard Navier-Stokes equations.
2. We decompose the evolved velocity.
3. We filter the small resolved scales for a fixed filter radius.
4. We correct the decomposed velocity, substituting the small resolved scales with the filtered resolved scales.

Numerical investigations [1] show that the novel strategies are more robust and more accurate with respect to standard filters for the same fixed filter radius.

Specifically, we propose two different approaches related to different ways of decomposing the evolved velocity field. One strategy relies on a first differential filter in the second step of the algorithm and is called VMS evolve-filter-filter-correct (VMS EFFC), while the other is based on a divergence-free projection in a larger functional space and is denoted as VMS evolve-project-filter-correct (VMS EPFC).

These strategies are tested over a flow past a cylinder at Reynolds 1000 both at the FOM and ROM levels. They are compared to standard results of EF and Galerkin projections (G-FOM and G-ROM). The conclusion is that VMS-based filters yield more accurate results and outperform EF, G-FOM, and G-ROM.

This project paves the way for more complicated applications of interest in the ARIA project context, such as stabilization, sensitivity, and control in industrial fields.

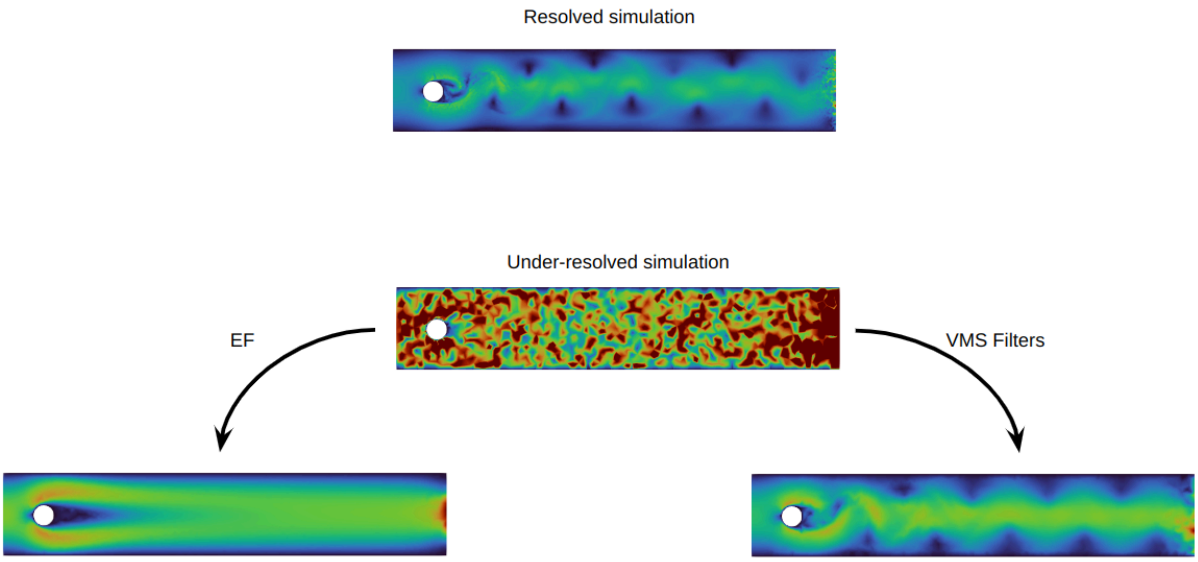

Figure 1: VMS Filters performances compared to EF strategy for convection-dominated Navier-Stokes equations.

[1] Ballarin, Berrone, Chacòn Rebollo, Iliescu and Strazzullo “Variational Multiscale Evolve and Filter strategies for convection-dominated flows”, *in preparation,* 2024.