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Challenges of Residual Minimization-based Model Order Reduction in Car Aerodynamics

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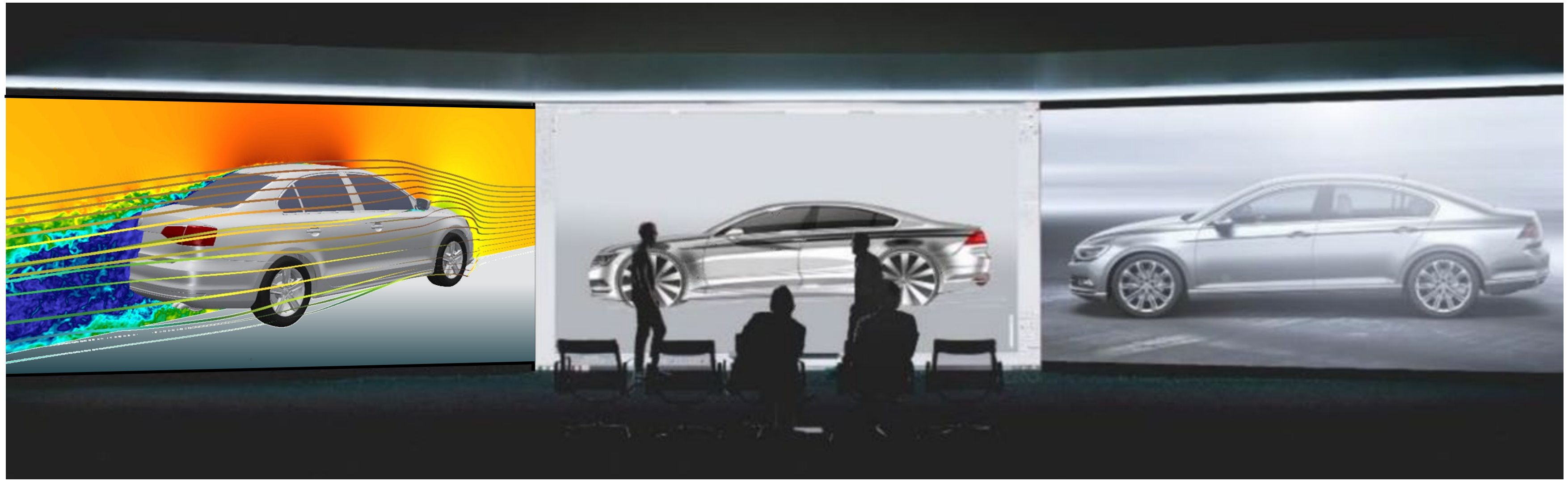
Agenda

1. Introduction
2. Steady-State Least Squares Petrov Galerkin Reduced Order Model
3. Applications
   - Backward Facing Step
   - Ahmed Body
4. Summary

Introduction

Interactive Aerodynamic Design

Problem: Iterative loop between design and aerodynamic computation: 12h up to several days
Solution: Real-time aerodynamic prediction for interactive aerodynamic design



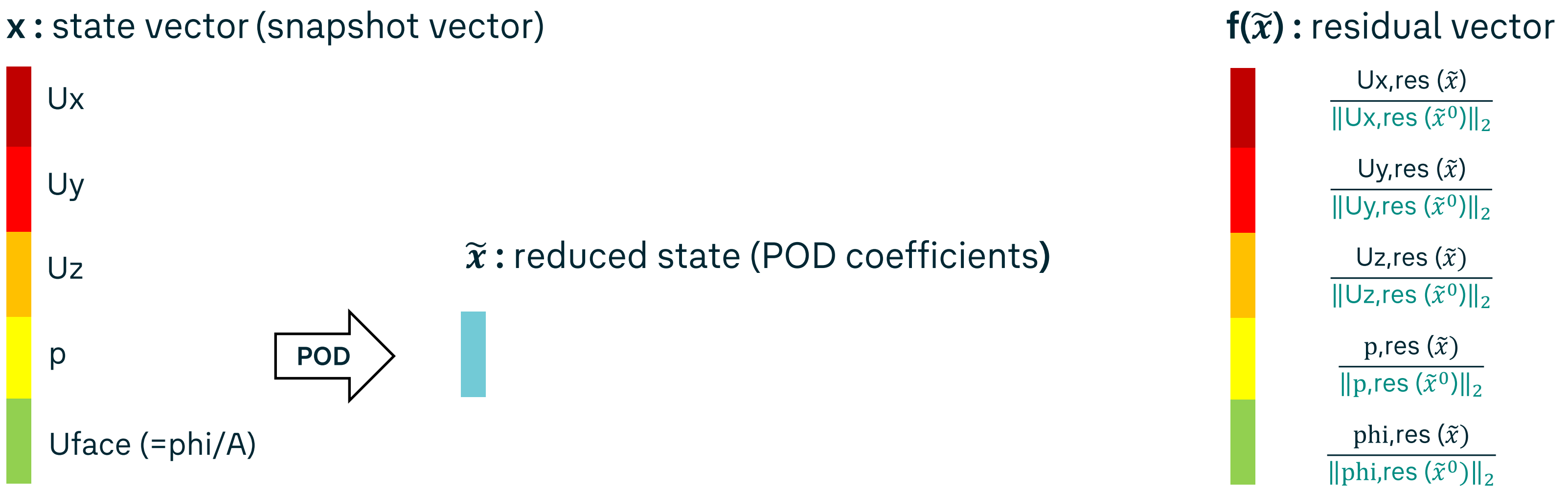
Required: Parametric geometry, Design of Experiments (DoE), Reduced Order Model (ROM)

State of the art: Proper Orthogonal Decomposition + Interpolation (POD+I)

Goal: Residual-Minimization-based ROM for higher accuracy, especially in poorly sampled areas of the design space

Steady-State Least Squares Petrov Galerkin Reduced Order Model

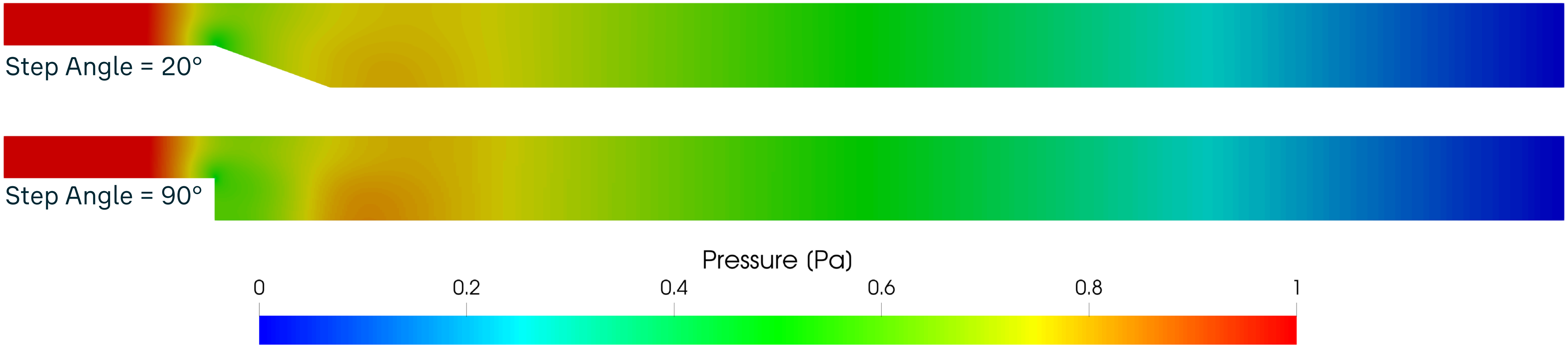
Overview



Objective Function = $\|A\, f(\tilde{x})\|_2$

- Discrete **residual scaling** via norm of residuals of initial solution (POD+I)
- **Nonlinear least-squares** solver from Python package scipy

Backward Facing Step - Varying Angle

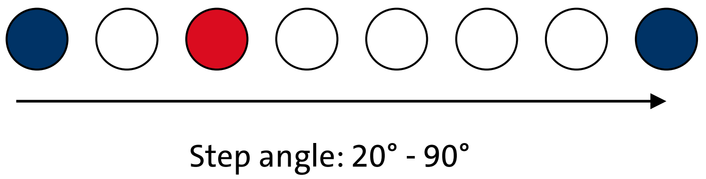


Full order model (FOM) :

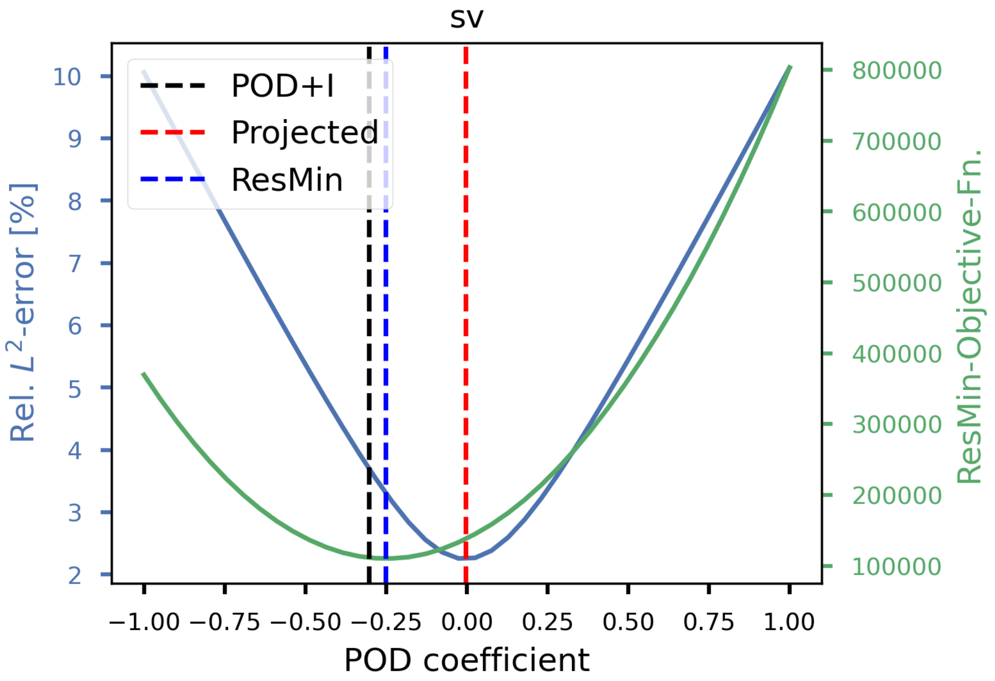
- Re=50
- SIMPLE algorithm from OpenFOAM®
- 2 training snapshots: 20° and 90°

Backward Facing Step - Varying Angle

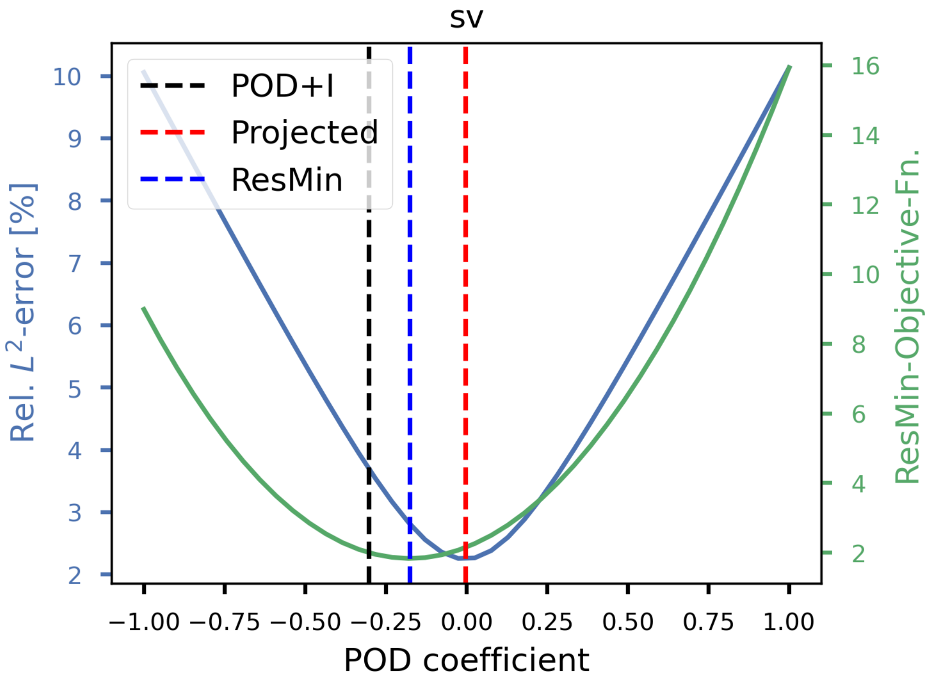
Challenge #1: The "Shift Problem"



W/o Residual Scaling

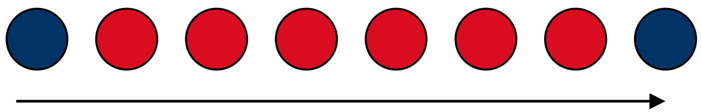


With Residual Scaling

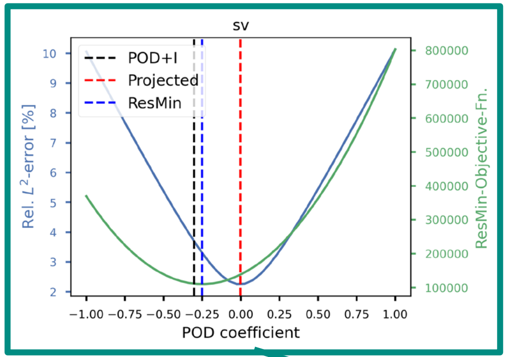


Backward Facing Step - Varying Angle

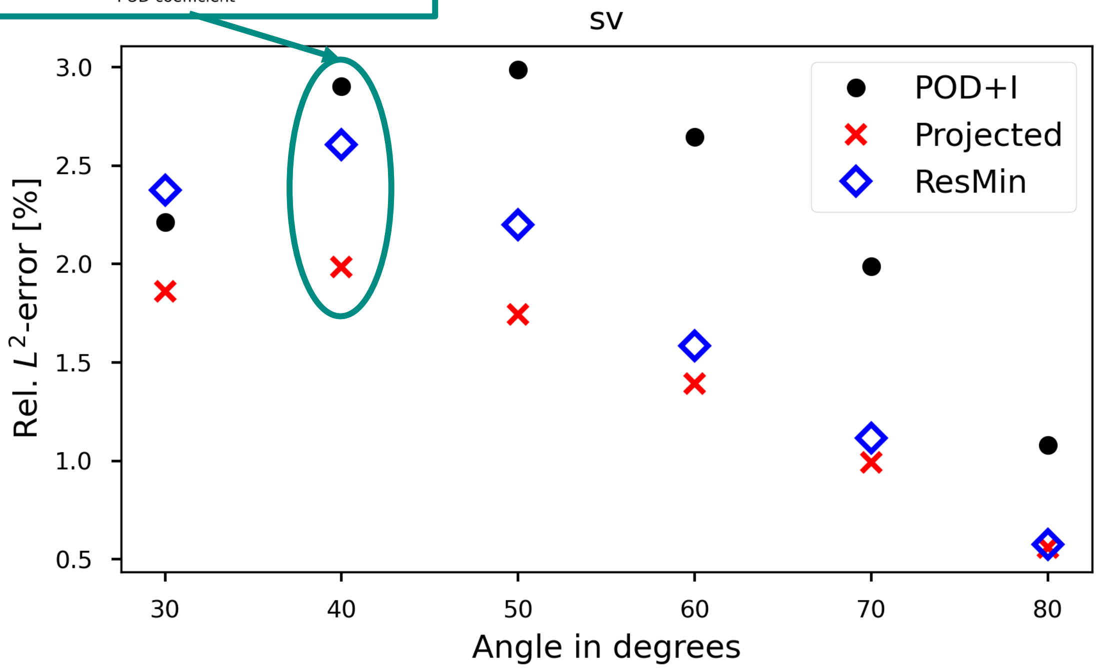
Train snapshots
Test snapshots

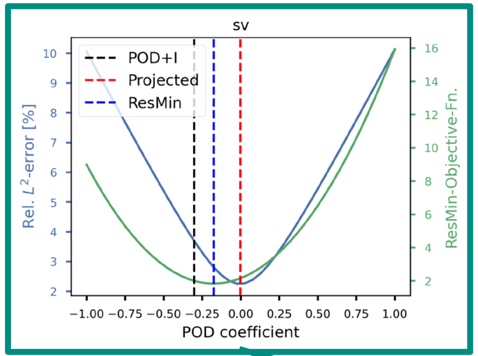


Step angle: 20° - 90°

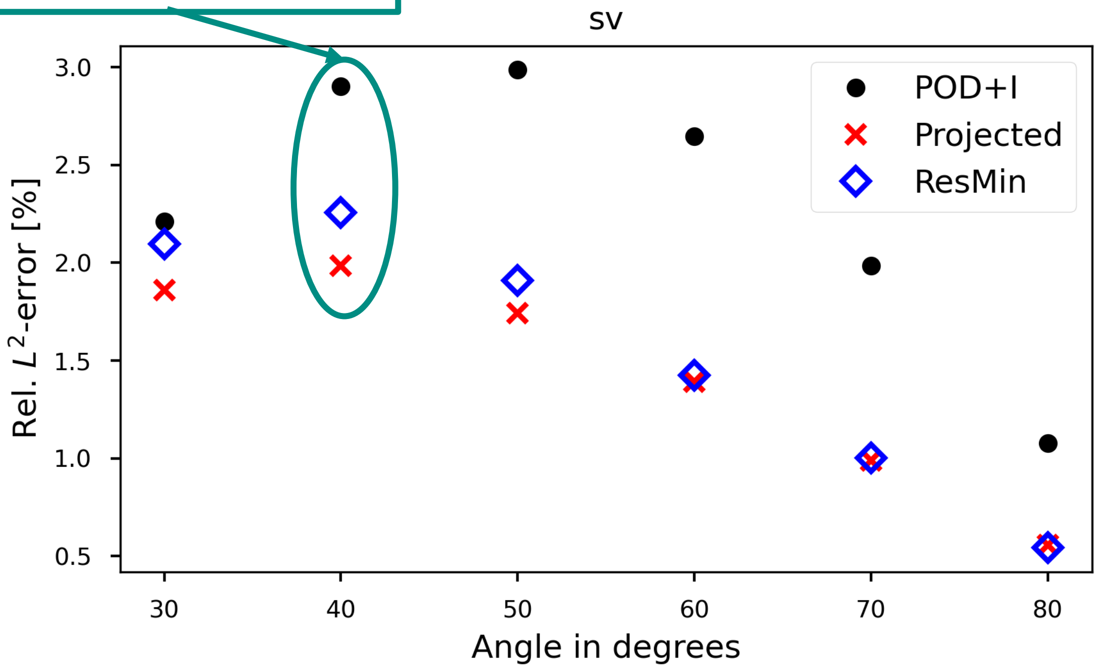


W/o Residual Scaling





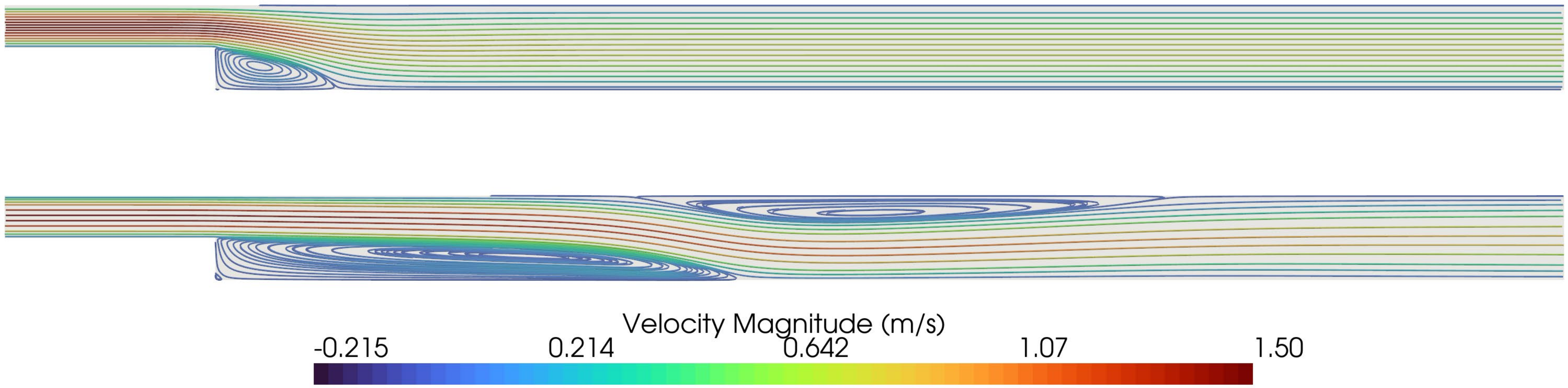
With Residual Scaling



Backward Facing Step - Varying Re

Re 50

Re 450

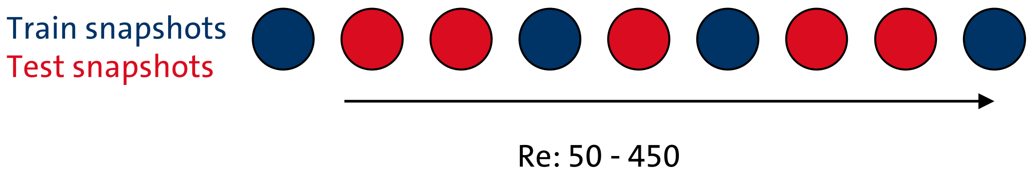


Full order model (FOM) :

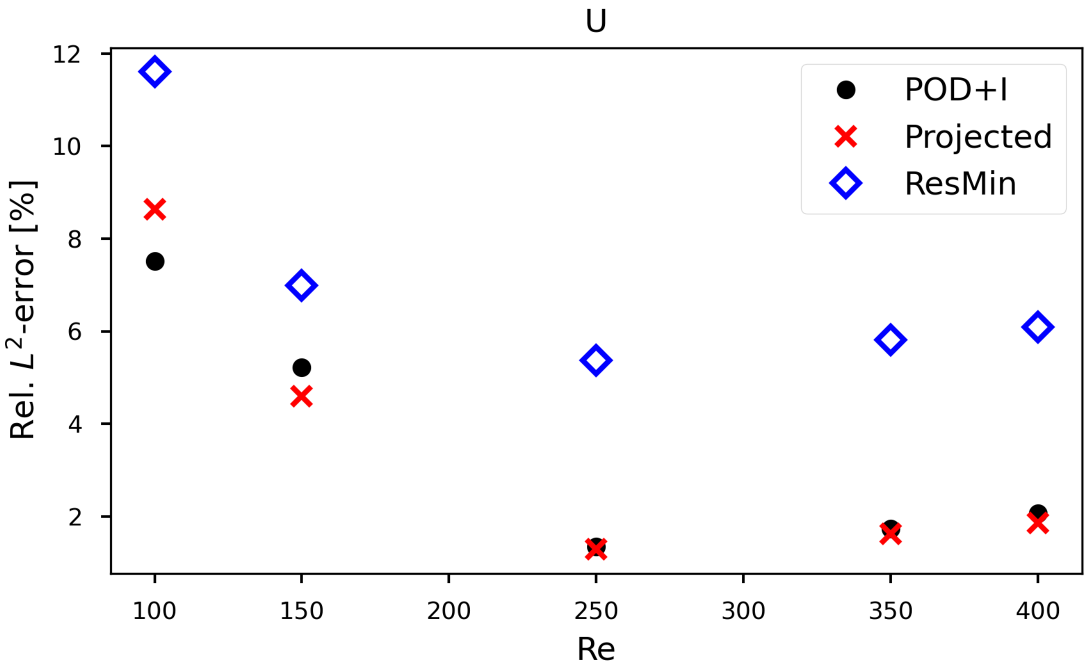
- SIMPLE algorithm from OpenFOAM®
- Re = 50 - 450

Backward Facing Step - Varying Re

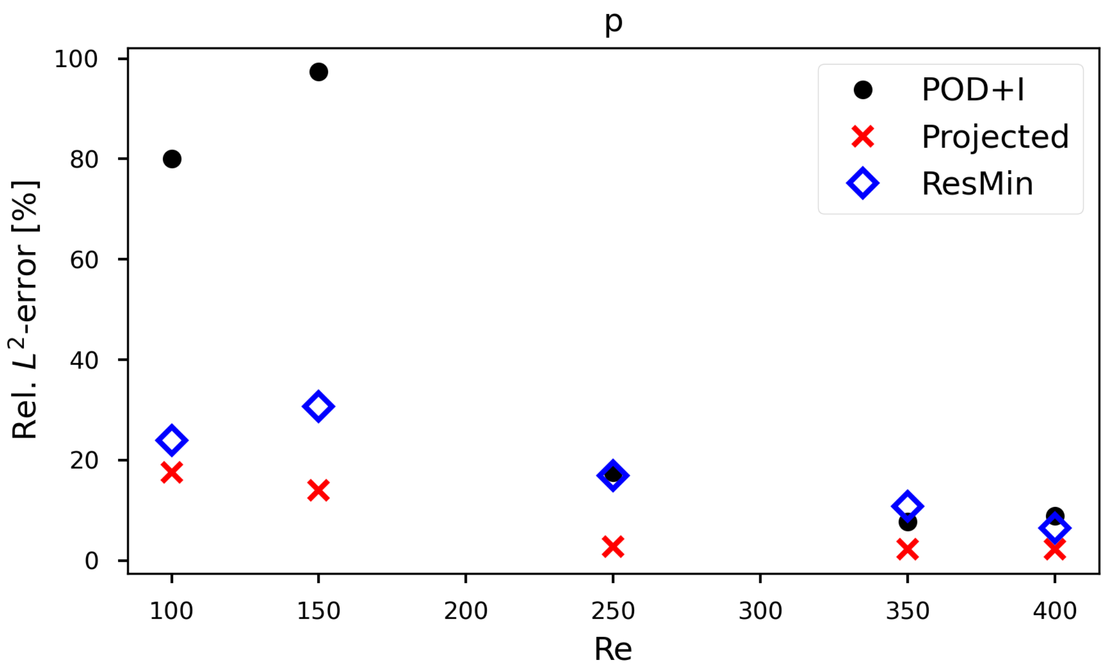
All Residuals



Error Velocity (Ux and Uy)

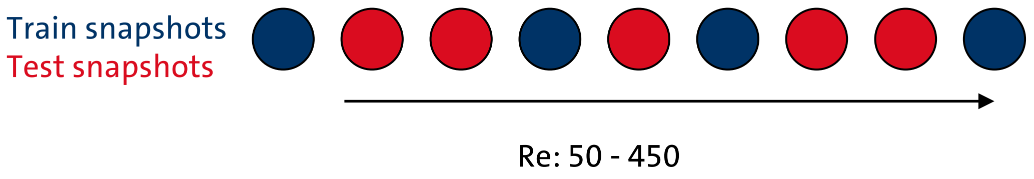


Error Pressure

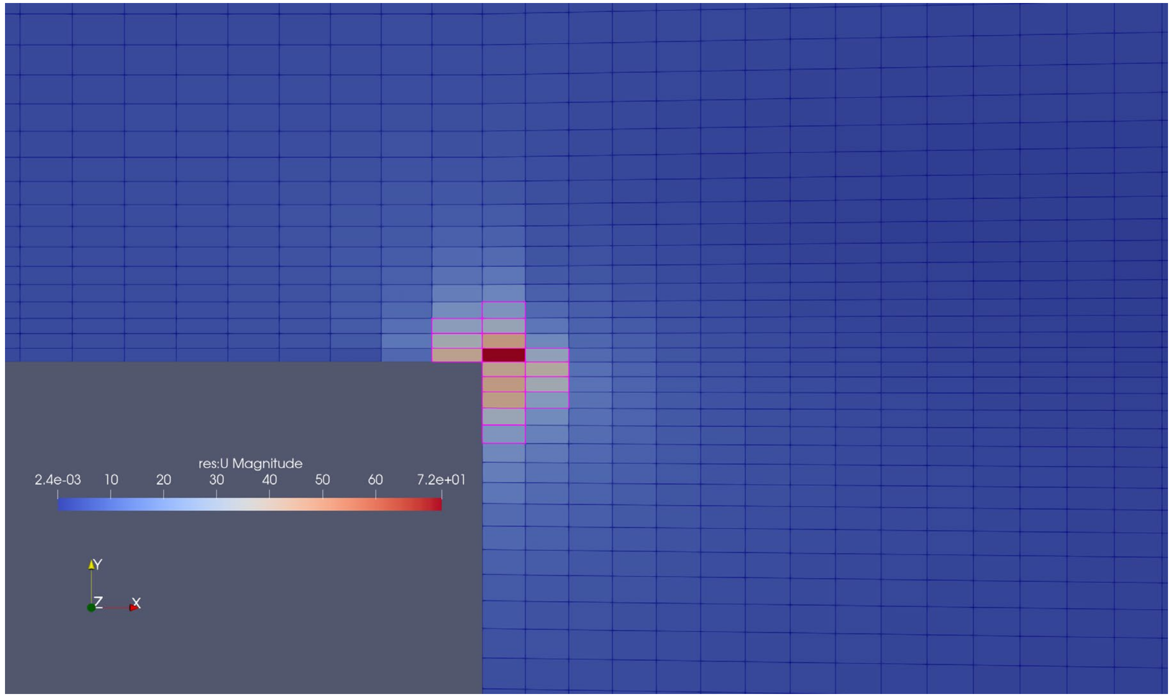


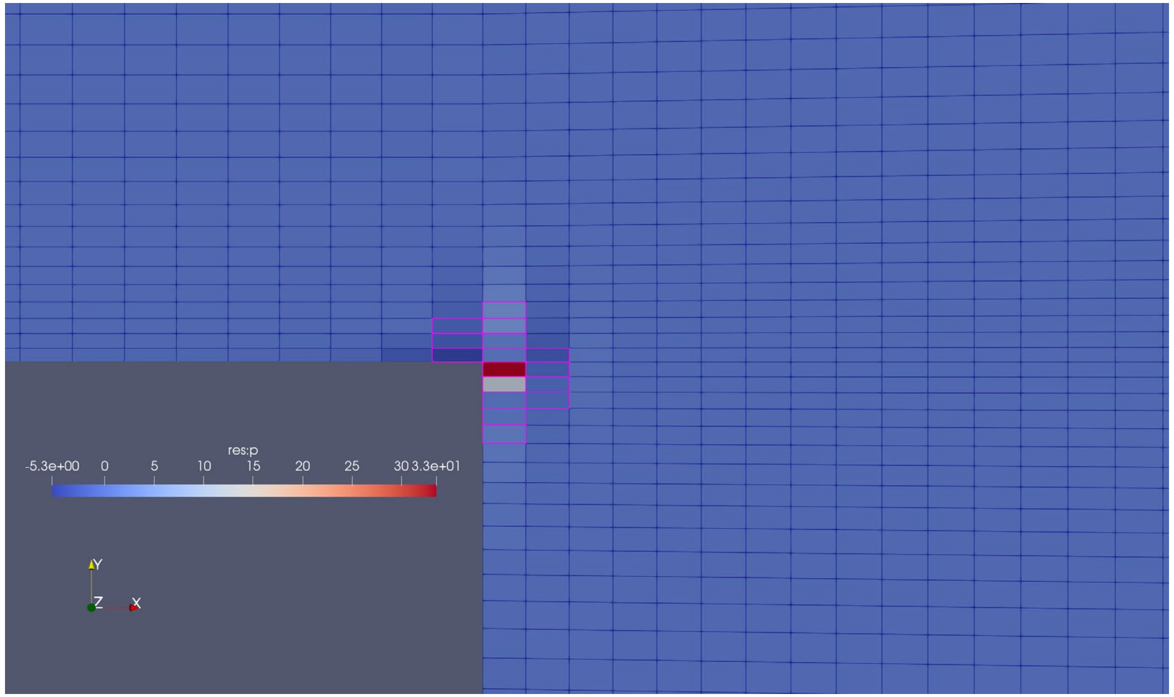
Backward Facing Step - Varying Re

Mask for High Residuals near Boundary



- Residuals of initial solution for test snap @ Re=100





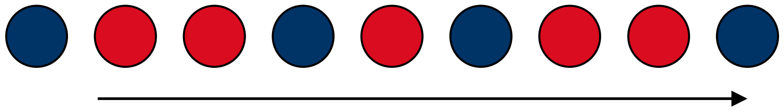
- Magnitude(res:U) >= 20 used as a criterion to find 16 cells

Backward Facing Step - Varying Re

Only Boundary Residuals

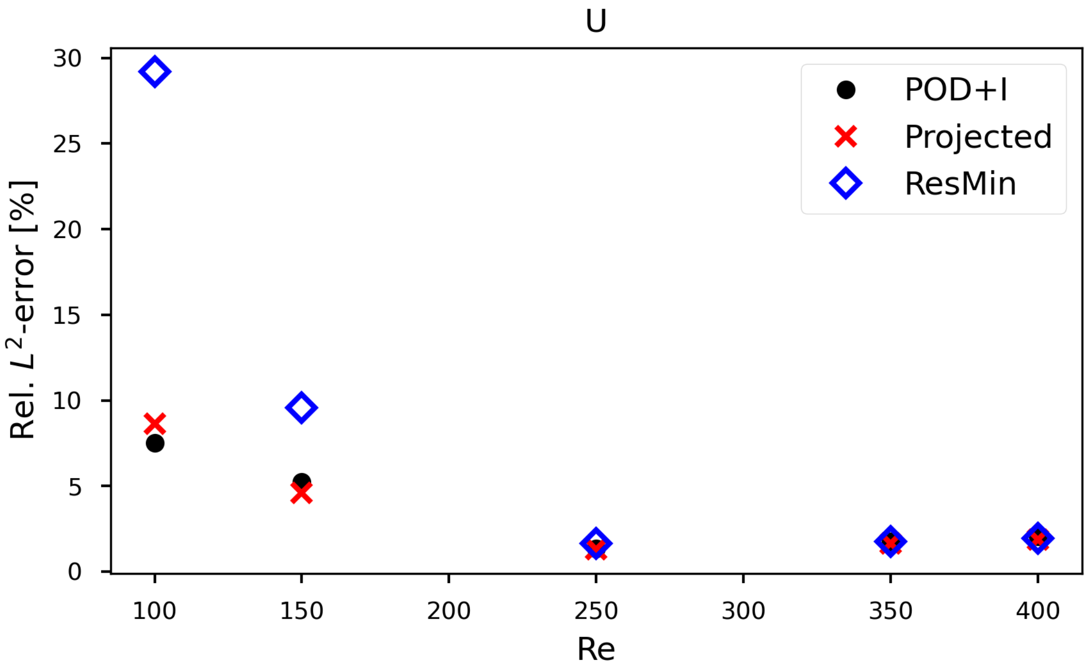
„Shift“ problem mitigated by
1. Residual scaling
2. Masking of residuals

Train snapshots
Test snapshots

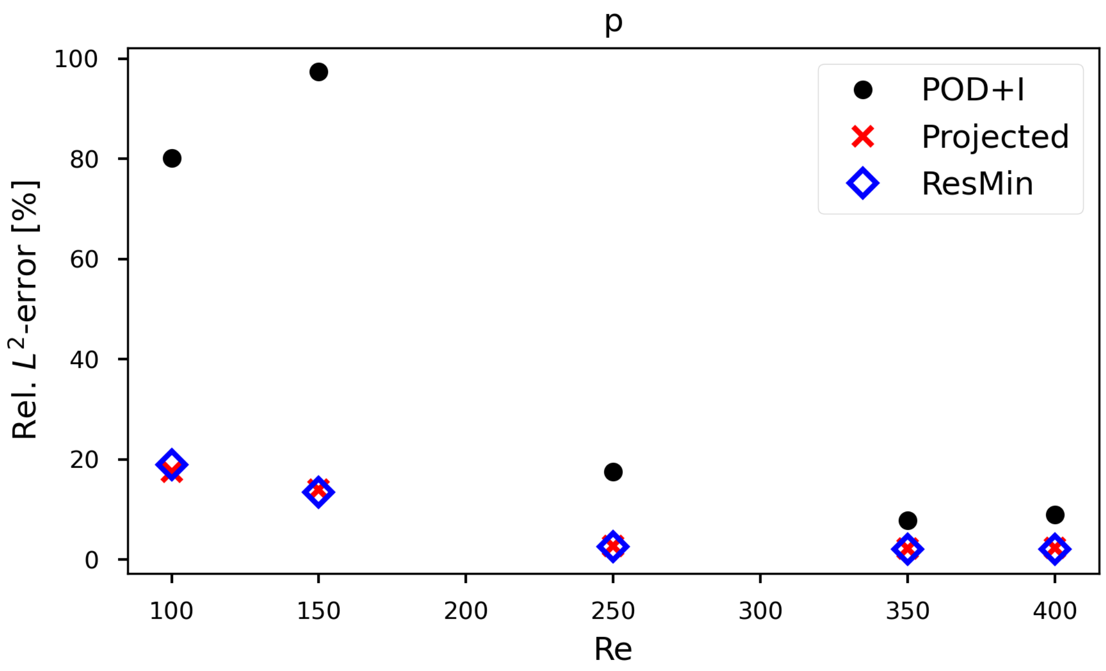


Re: 50 - 450

Error Velocity (Ux and Uy)



Error Pressure



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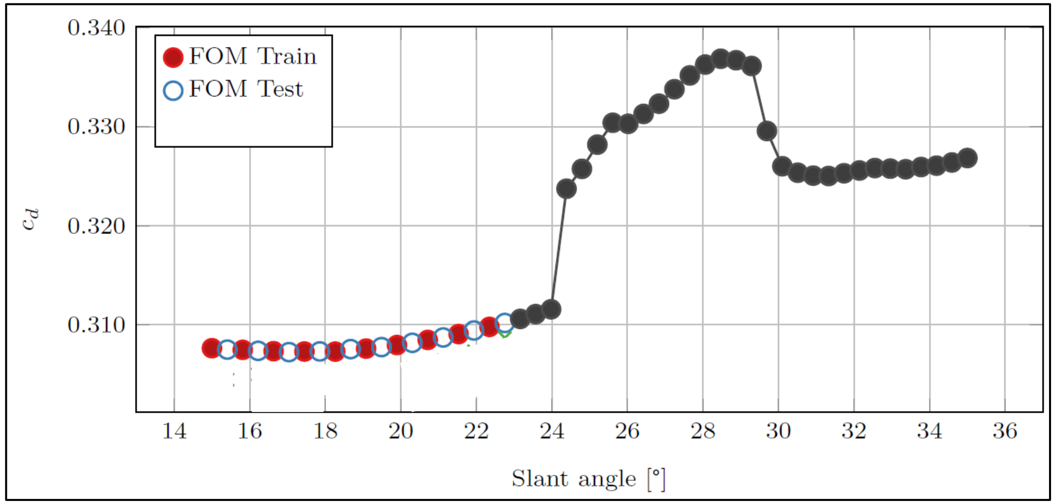
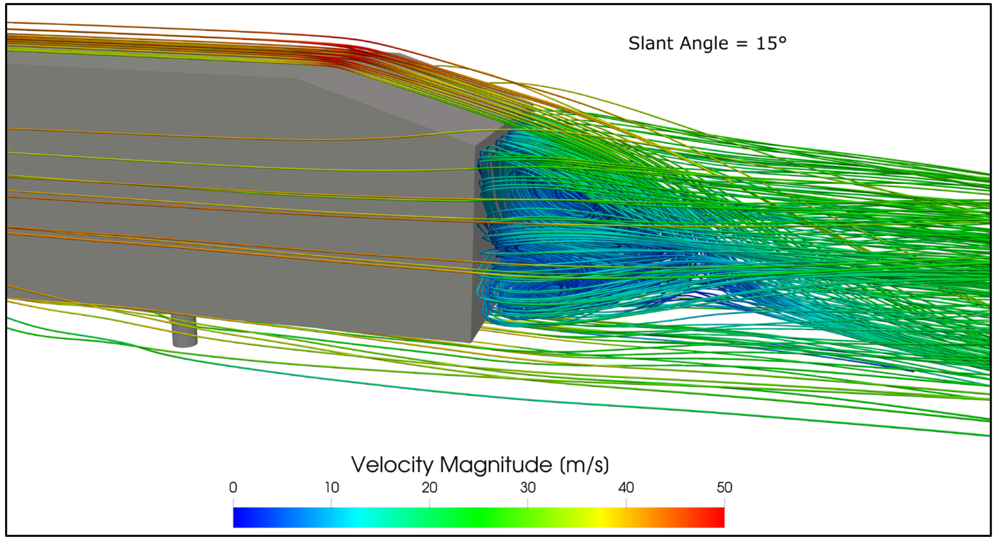
Ahmed Body

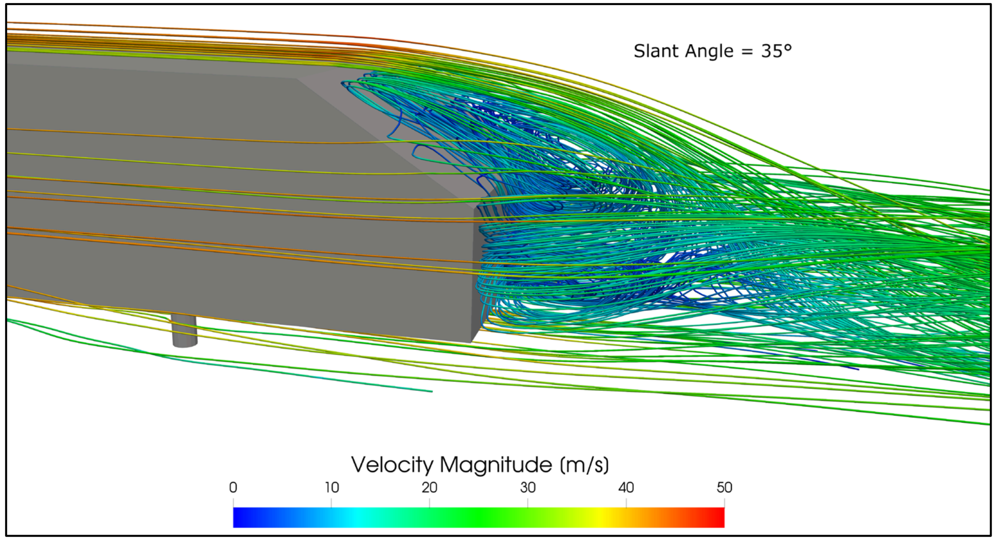
Training Data



Full order model (FOM):

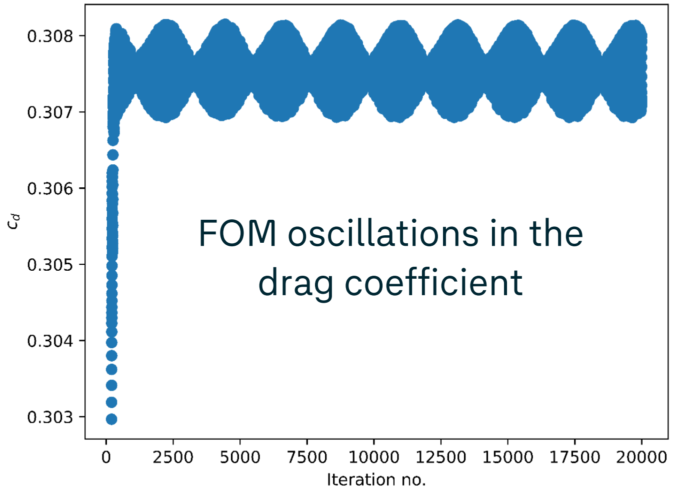
- 50 RANS simulations, evenly spaced for slant angles from 15° to 35°
- SIMPLE algorithm from OpenFOAM ®

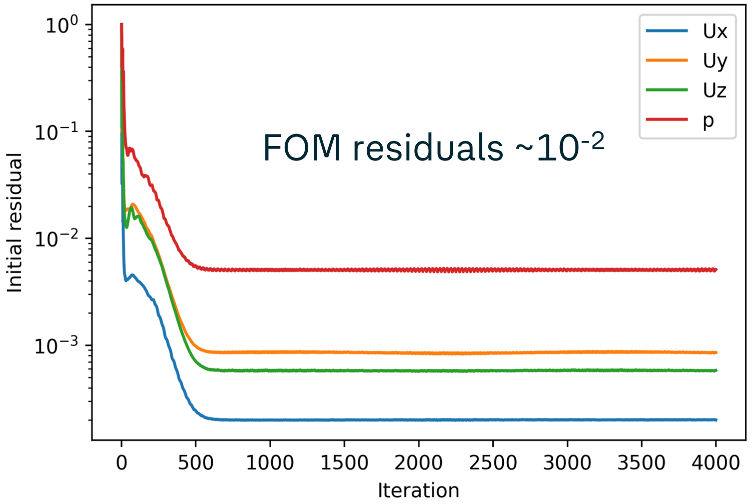


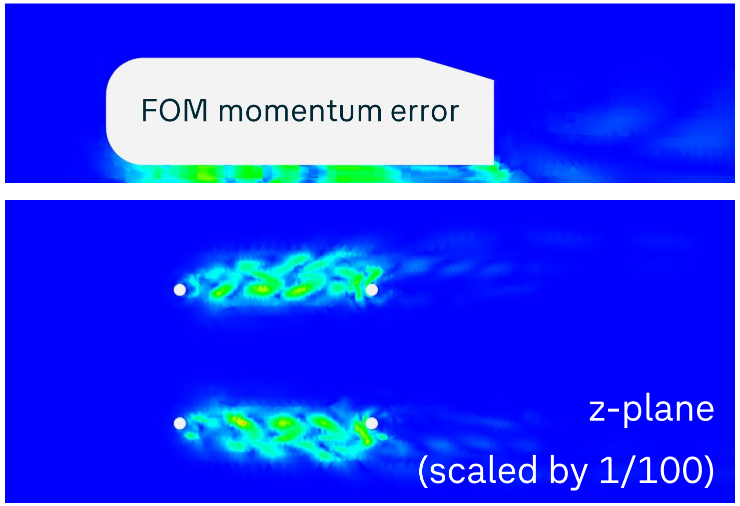


Ahmed Body

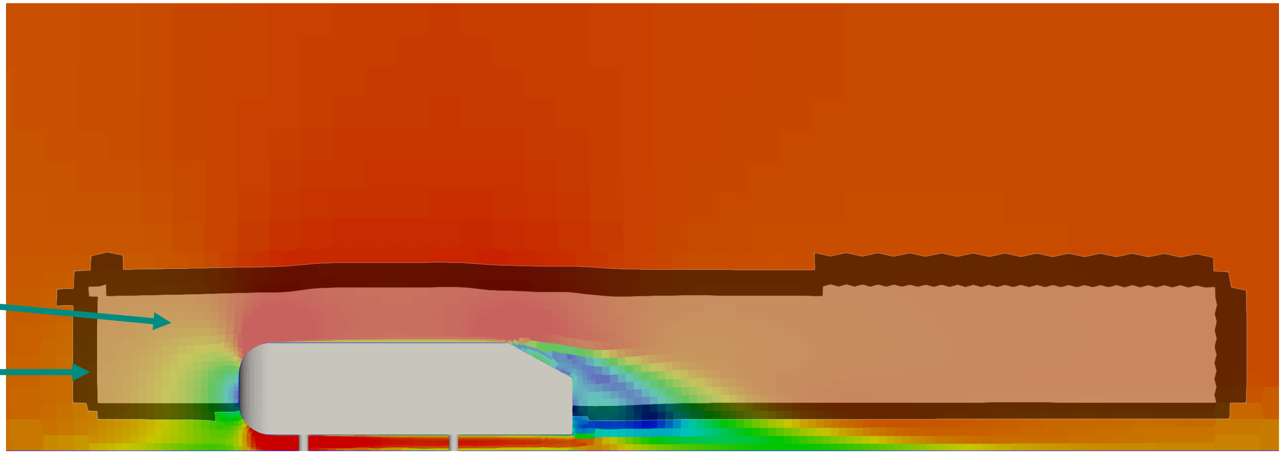
Challenge #2: Poor FOM Convergence





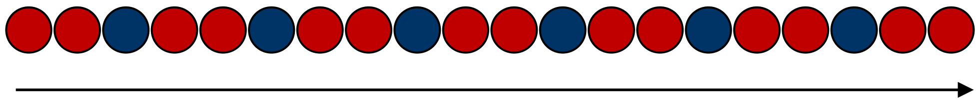


- To exclude high residuals in underbody region, ROM is restricted to subvolume shown right
- **Obj-Fn Evaluation, Error Evaluation**: HotCells
- **POD**: HotCells + HaloCells



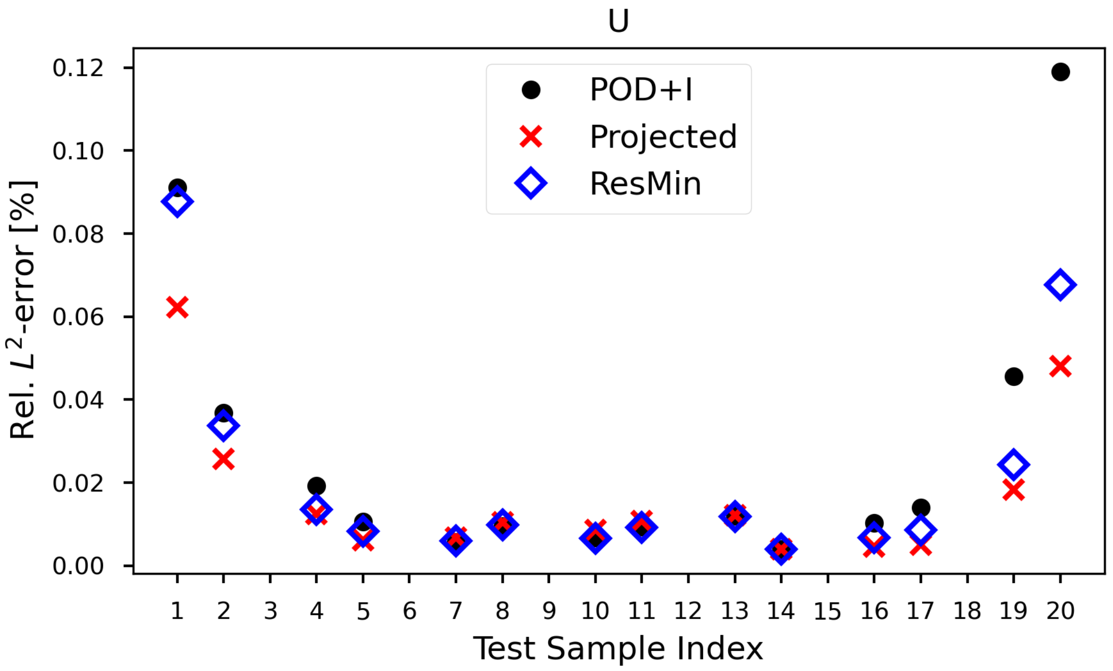
Ahmed First Flow Region

Train snapshots
Test snapshots

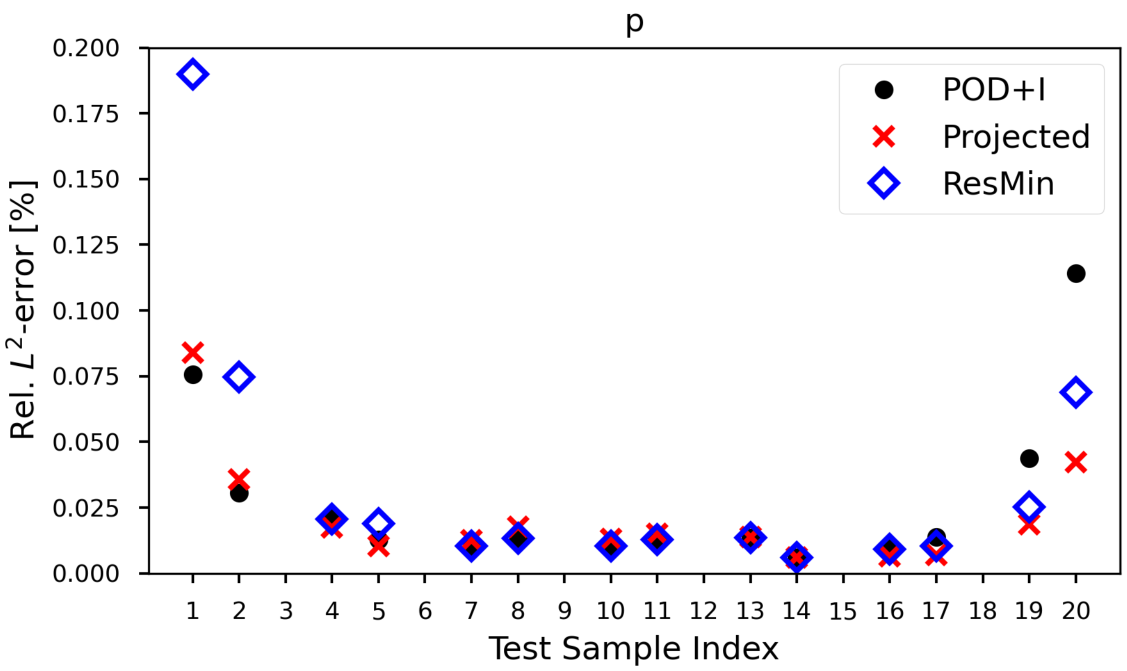


Slant angle: 15° - 22.8°

Error Velocity (Ux, Uy and Uz)



Error Pressure



Ahmed First Flow Region

$\mathbf{f}(\tilde{x})$: residual vector

$$\frac{\text{Ux,res}(\tilde{x})}{\|\text{Ux,res}(\tilde{x}^0)\|_2}$$

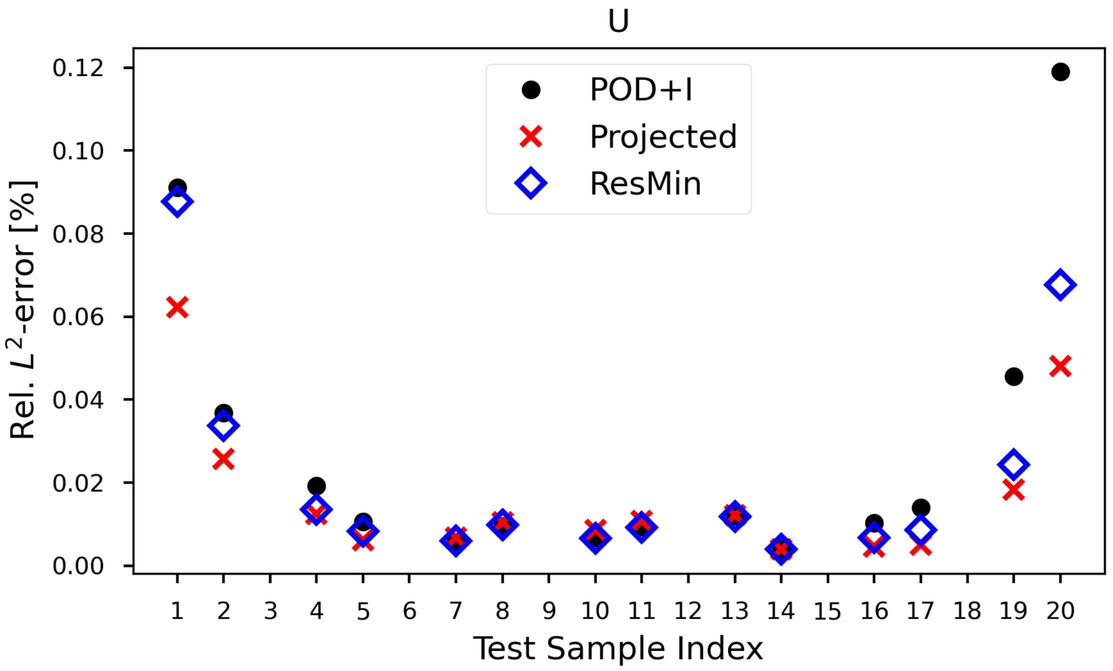
$$\frac{\text{Uy,res}(\tilde{x})}{\|\text{Uy,res}(\tilde{x}^0)\|_2}$$

$$\frac{\text{Uz,res}(\tilde{x})}{\|\text{Uz,res}(\tilde{x}^0)\|_2}$$

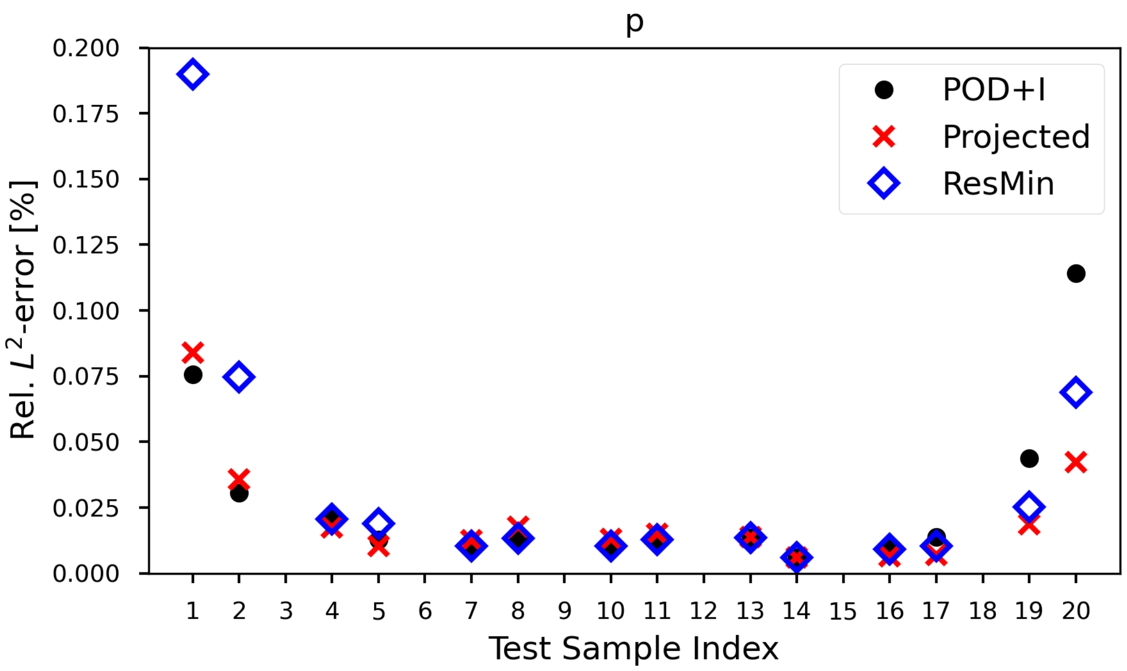
$$\frac{\text{p,res}(\tilde{x})}{\|\text{p,res}(\tilde{x}^0)\|_2}$$

$$\frac{\text{phi,res}(\tilde{x})}{\|\text{phi,res}(\tilde{x}^0)\|_2}$$

Error Velocity (Ux, Uy and Uz)

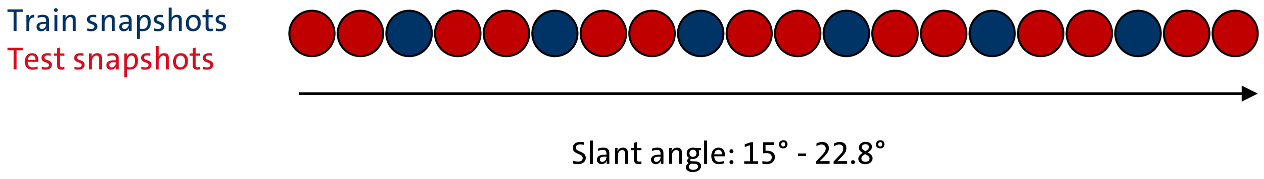


Error Pressure

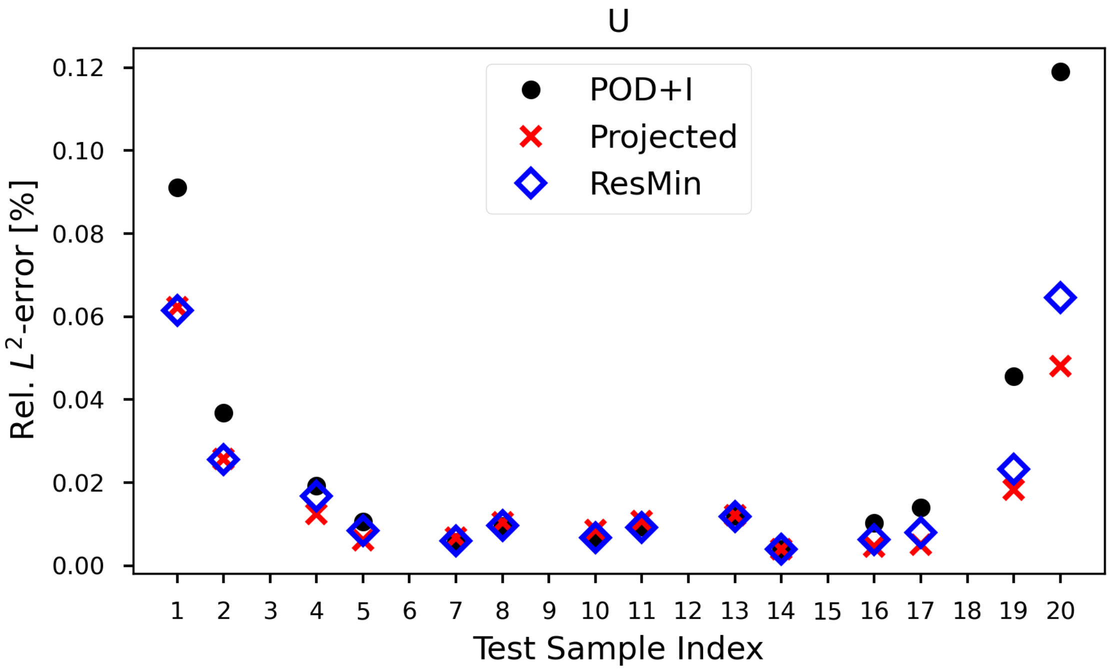


Ahmed First Flow Region

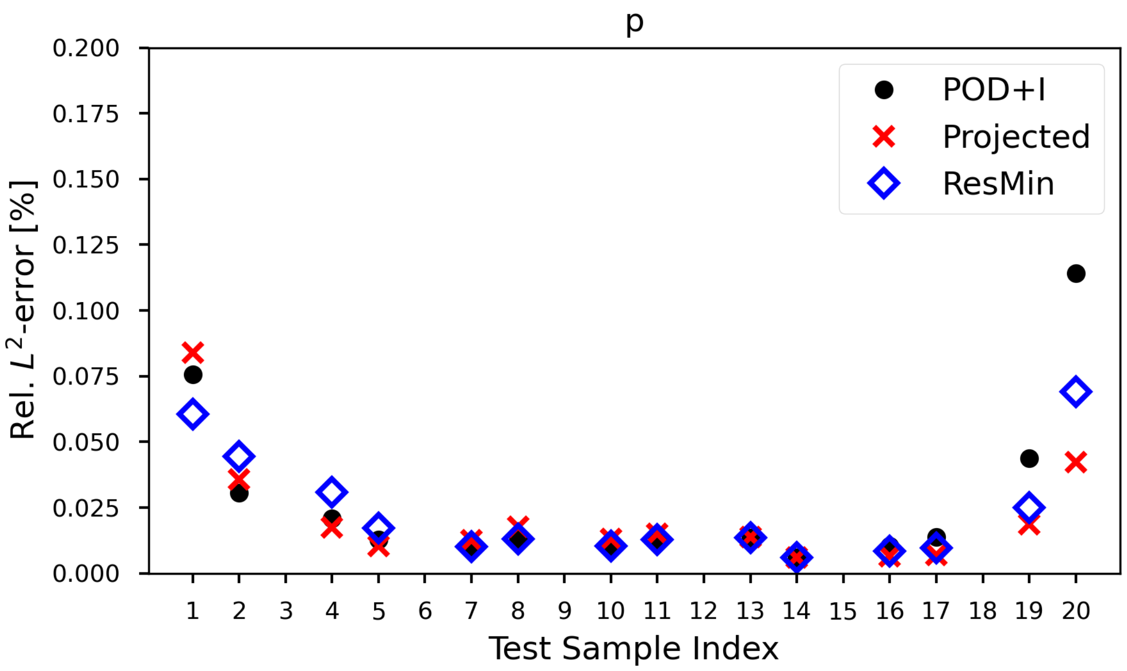
Residual Scaling Factor from Projected Solution



Error Velocity (Ux, Uy and Uz)

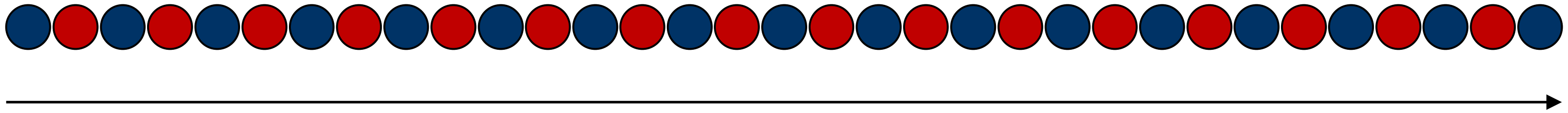


Error Pressure



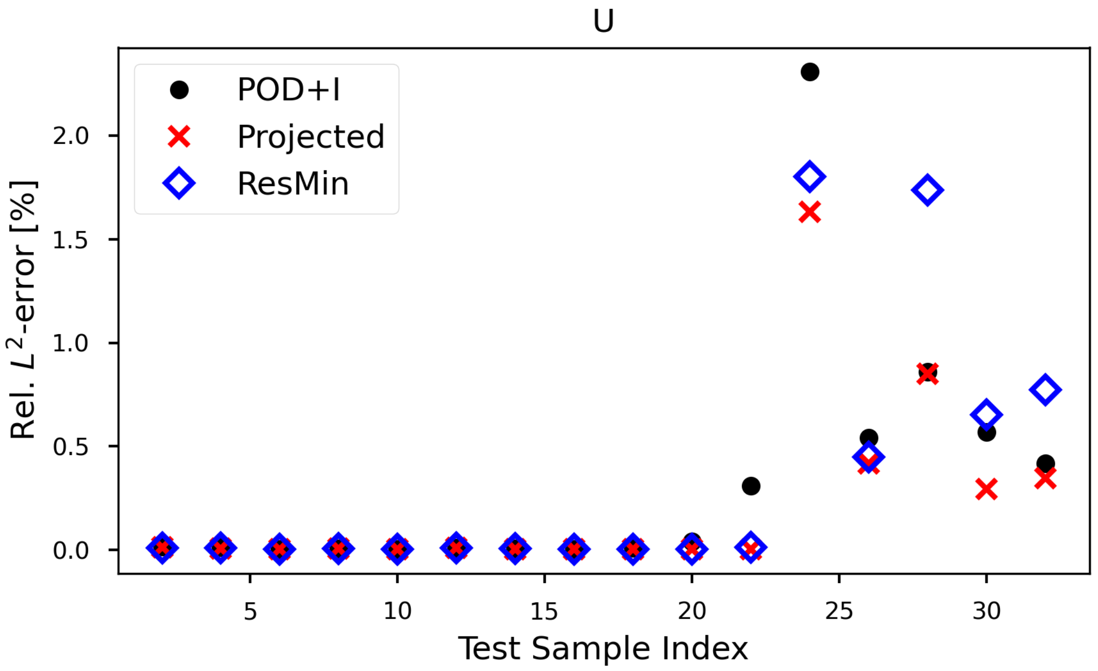
Ahmed First and Second Flow Region

Train snapshots
Test snapshots

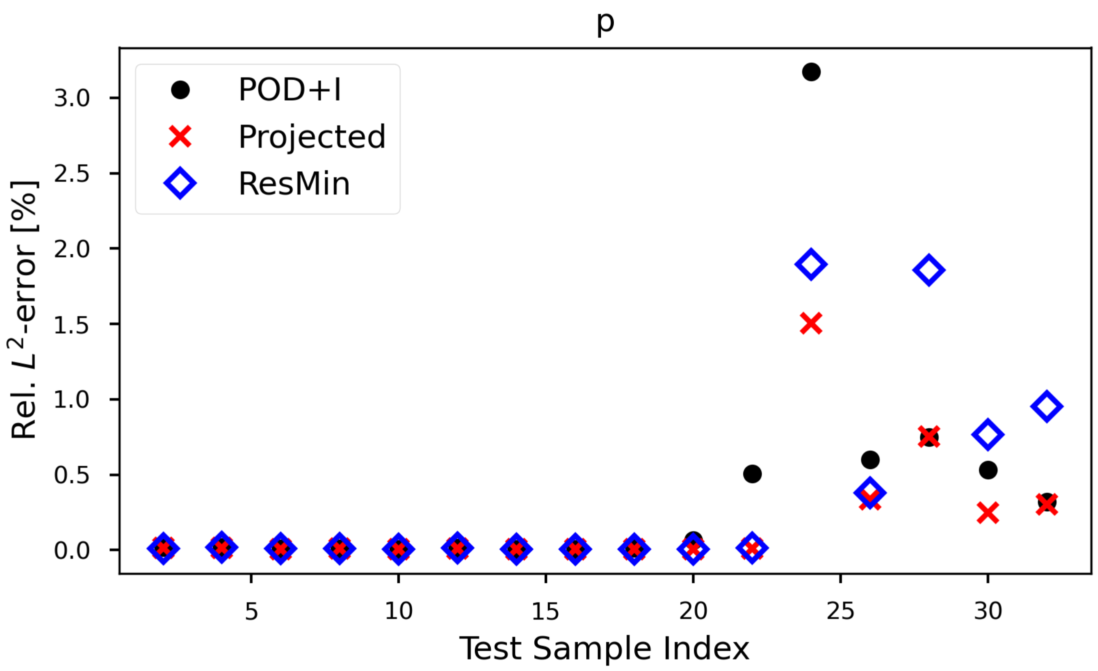


Slant angle: 15° - 28.1°

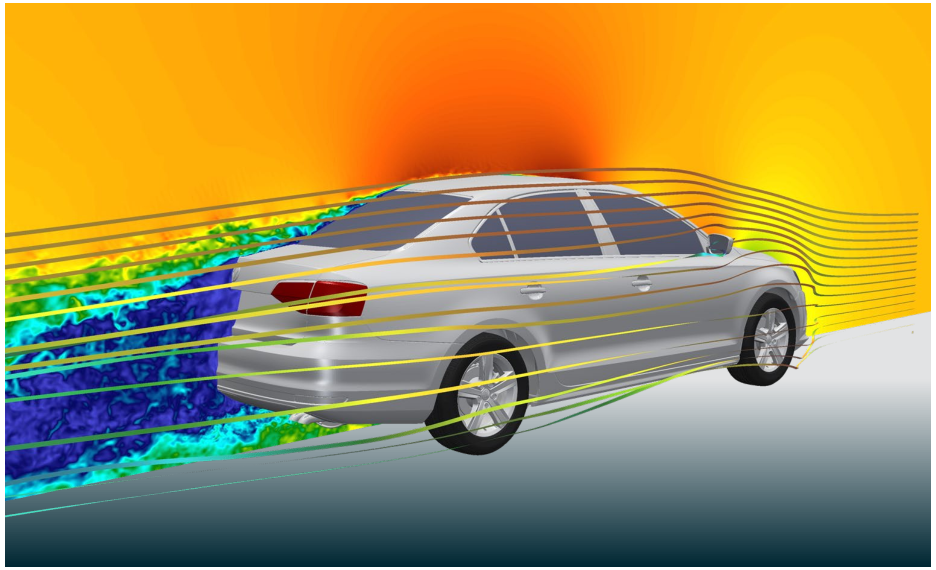
Error Velocity (Ux, Uy and Uz)



Error Pressure



Challenge #3: Prediction of Time-averaged Quantities of Unsteady Simulations

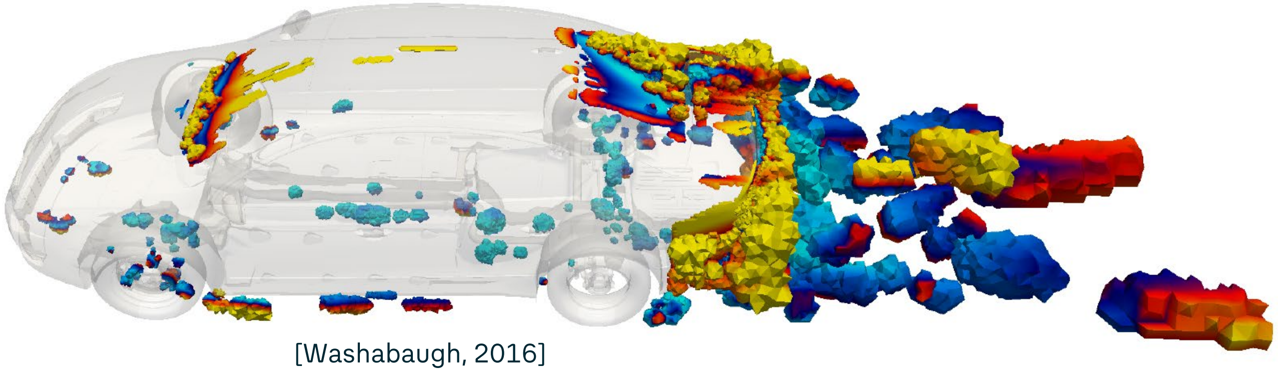


Full order model in car aerodynamics is unsteady:

- DDES or Lattice-Boltzmann, typically 4 sec of physical time, 100-200 million cells
- Quantities of interests are time-averaged (last 1.5 sec): fields and coefficients

- Residual Equations?

Challenge #4: Hyper-Reduction



[Washabaugh, 2016]

Choice of the submesh:

- Several approaches in literature
- Avoid regions of high FOM residuals?
- Problems for industrial geometries and flows to be expected

As noted before: hyper-reduction not only to accelerate the online computation, but likely essential for accuracy

Summary

Challenges of ROMs in Car Aerodynamics:

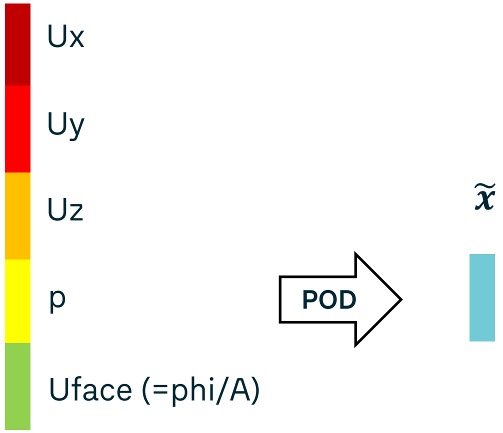
1. "Shift problem": Minima of objective function and state error do not coincide
2. Poor FOM convergence
3. Prediction of time-averaged quantities of unsteady simulations
4. Hyperreduction

Backup

Steady-State Least Squares Petrov Galerkin Reduced Order Model

Overview

x : state vector (snapshot vector)



$\tilde{x}$ **:** reduced state (POD coefficients)

f($\tilde{x}$) : residual vector

$$\frac{\text{Ux,res}\,(U(\tilde{x}),\, p(\tilde{x}),\, phi(\tilde{x}))}{\|\text{Ux,res}\,(U(\tilde{x}^0),\, p(\tilde{x}^0),\, phi(\tilde{x}^0)\|_2}$$

$$\frac{\text{Uy,res}\,(U(\tilde{x}),\, p(\tilde{x}),\, phi(\tilde{x}))}{\|\text{Uy,res}\,(U(\tilde{x}^0),\, p(\tilde{x}^0),\, phi(\tilde{x}^0)\|_2}$$

$$\frac{\text{Uz,res}\,(U(\tilde{x}),\, p(\tilde{x}),\, phi(\tilde{x}))}{\|\text{Uz,res}\,(U(\tilde{x}^0),\, p(\tilde{x}^0),\, phi(\tilde{x}^0)\|_2}$$

$$\frac{\text{p,res}\,(U(\tilde{x}),\, p(\tilde{x}),\, phi(\tilde{x}))}{\|\text{p,res}\,(U(\tilde{x}^0),\, p(\tilde{x}^0),\, phi(\tilde{x}^0)\|_2}$$

$$\frac{\text{phi,res}\,(U(\tilde{x}),\, p(\tilde{x}),\, phi(\tilde{x}))}{\|\text{phi,res}\,(U(\tilde{x}^0),\, p(\tilde{x}^0),\, phi(\tilde{x}^0)\|_2}$$

Objective Function = $\|A\, f(\tilde{x})\|_2$

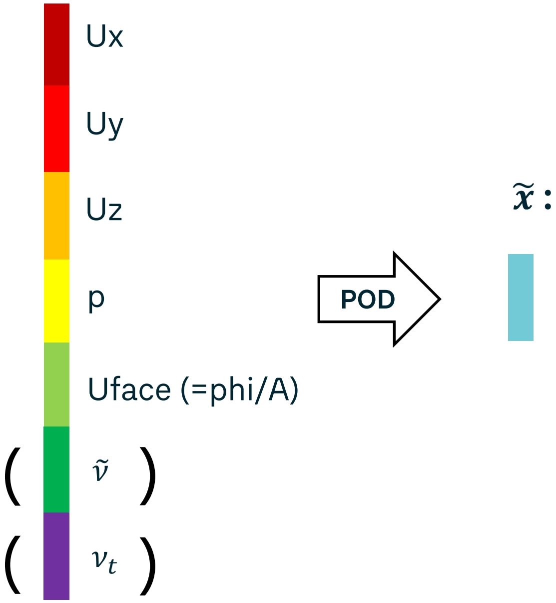
- Discrete **residual scaling** via norm of residuals of initial solution (POD+I)
- **Nonlinear least-squares** solver from Python package scipy

Steady-State Least Squares Petrov Galerkin Reduced Order Model

Overview

x : state vector (snapshot vector)

$\tilde{x}$ **:** reduced state (POD coefficients)



f($\tilde{x}$) : residual vector

$$\left(\begin{array}{c} \frac{Ux,res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|Ux,res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \\ \frac{Uy,res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|Uy,res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \\ \frac{Uz,res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|Uz,res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \\ \frac{p,res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|p,res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \\ \frac{phi,res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|phi,res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \\ \frac{\tilde{\nu},res\ (U(\tilde{x}),\ p(\tilde{x}),\ phi(\tilde{x}))}{\|\tilde{\nu},res\ (U(\tilde{x}^0),\ p(\tilde{x}^0),\ phi(\tilde{x}^0))\|_2} \end{array} \right)$$

Objective Function = $\|A\ f(\tilde{x})\|_2$

- Discrete **residual scaling** via norm of residuals of initial solution (POD+I)
- **Nonlinear least-squares** solver from Python package scipy

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Steady-State Least Squares Petrov Galerkin Reduced Order Model

Dimensionally-Consistent Inner Products for ROM-Projection and POD

ROM-Projection

- Dimensionally-consistent inner product **via discrete residual scaling**

$f(\tilde{x})$: residual vector

$$\frac{Ux,res\,(U(\tilde{x}),\,p(\tilde{x}),\,phi(\tilde{x}))}{\|Ux,res\,(U(\tilde{x}^0),\,p(\tilde{x}^0),\,phi(\tilde{x}^0))\|_2}$$

$$\frac{Uy,res\,(U(\tilde{x}),\,p(\tilde{x}),\,phi(\tilde{x}))}{\|Uy,res\,(U(\tilde{x}^0),\,p(\tilde{x}^0),\,phi(\tilde{x}^0))\|_2}$$

$$\frac{Uz,res\,(U(\tilde{x}),\,p(\tilde{x}),\,phi(\tilde{x}))}{\|Uz,res\,(U(\tilde{x}^0),\,p(\tilde{x}^0),\,phi(\tilde{x}^0))\|_2}$$

$$\frac{p,res\,(U(\tilde{x}),\,p(\tilde{x}),\,phi(\tilde{x}))}{\|p,res\,(U(\tilde{x}^0),\,p(\tilde{x}^0),\,phi(\tilde{x}^0))\|_2}$$

$$\frac{phi,res\,(U(\tilde{x}),\,p(\tilde{x}),\,phi(\tilde{x}))}{\|phi,res\,(U(\tilde{x}^0),\,p(\tilde{x}^0),\,phi(\tilde{x}^0))\|_2}$$

ResMin Objective Function = $\|A\, f(\tilde{x})\|_2$

POD

- Dimensionally-consistent inner product via **non-dimensionalizing state vectors before POD**

x : state vector (snapshot vector)

$$\frac{Ux}{\|Ux_{\text{all train}}\|_2}$$

$$\frac{Uy}{\|Uy_{\text{all train}}\|_2}$$

$$\frac{Uz}{\|Uz_{\text{all train}}\|_2}$$

$$\frac{p}{\|p_{\text{all train}}\|_2}$$

$$\frac{Uface}{\|Uface_{\text{all train}}\|_2}$$

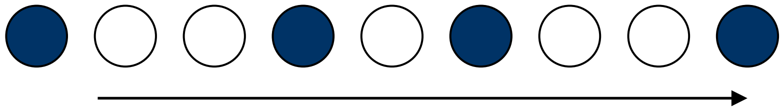
POD

$\tilde{x}$: reduced state (POD coefficients)

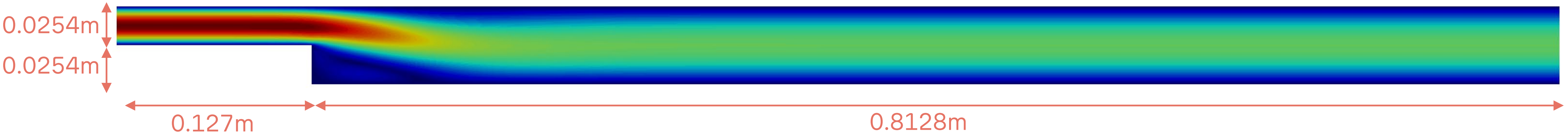
Backward Facing Step - Varying Re

Test Case Description

Train snapshots
Test snapshots



Re: 50 - 450



$Re = \frac{U * L}{\nu}$, with L=0.0254m and U=1 m/s

Re	Kinematic viscosity ν [m^2/s]
50	5.080E-04
200	1.270E-04
300	8.467E-05
450	5.644E-05

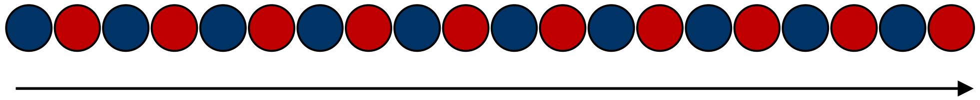
Velocity Profile at Inlet

```
    name            parabolicFixedValue;
    code            #{
        Foam::vectorField values=this->patch().Cf();
        Foam::scalarField ycoords=this->patch().Cf().component(vector::Y);
        forAll(values, id)
        {
            values[id].component(vector::Y)= 0;
            values[id].component(vector::X)=-6/(0.0254*0.0254)*pow(ycoords[id],2)+6/0.0254*ycoords[id];
            values[id].component(vector::Z)= 0;
        }

        operator==(values);
     #};
```

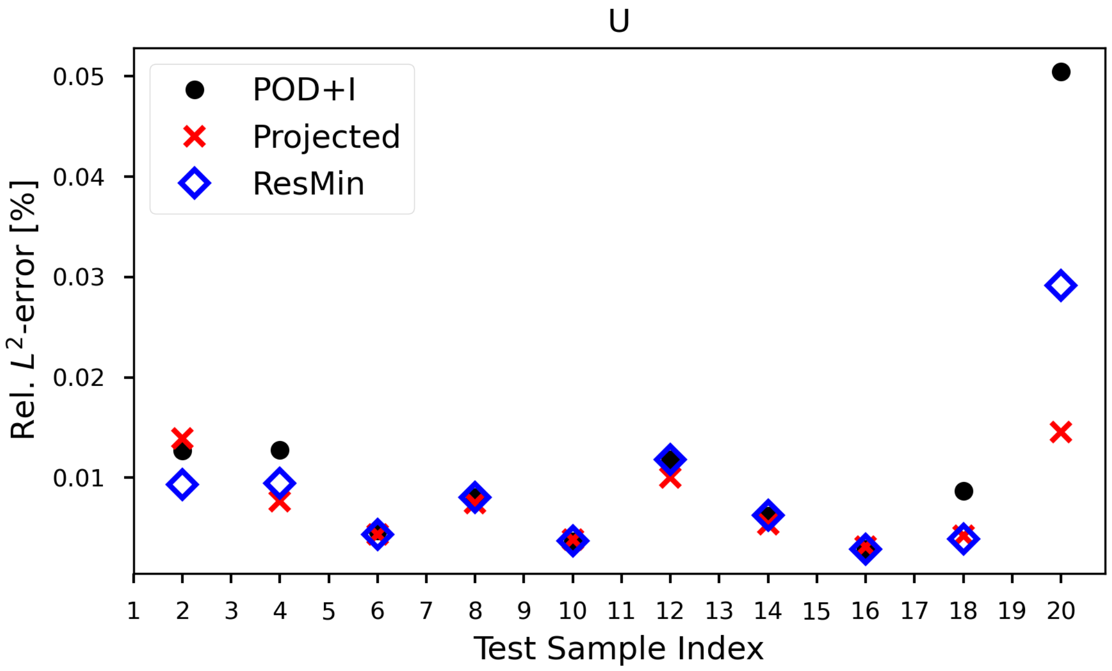
Ahmed First Flow Region

Train snapshots
Test snapshots

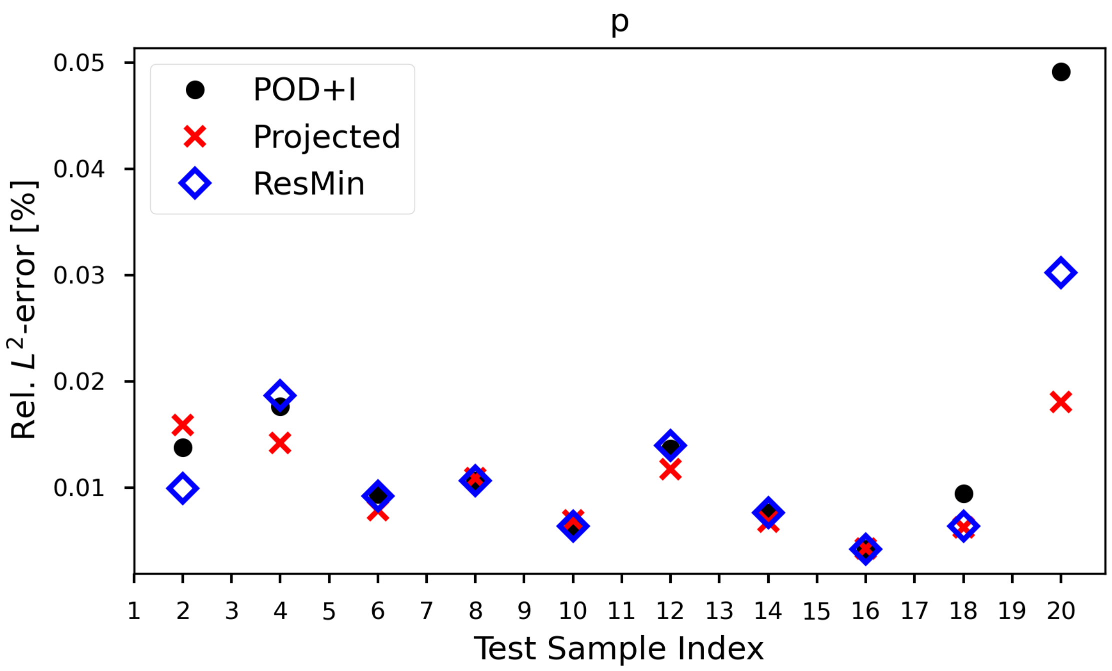


Slant angle: 15° - 22.8°

Error Velocity (Ux, Uy and Uz)



Error Pressure

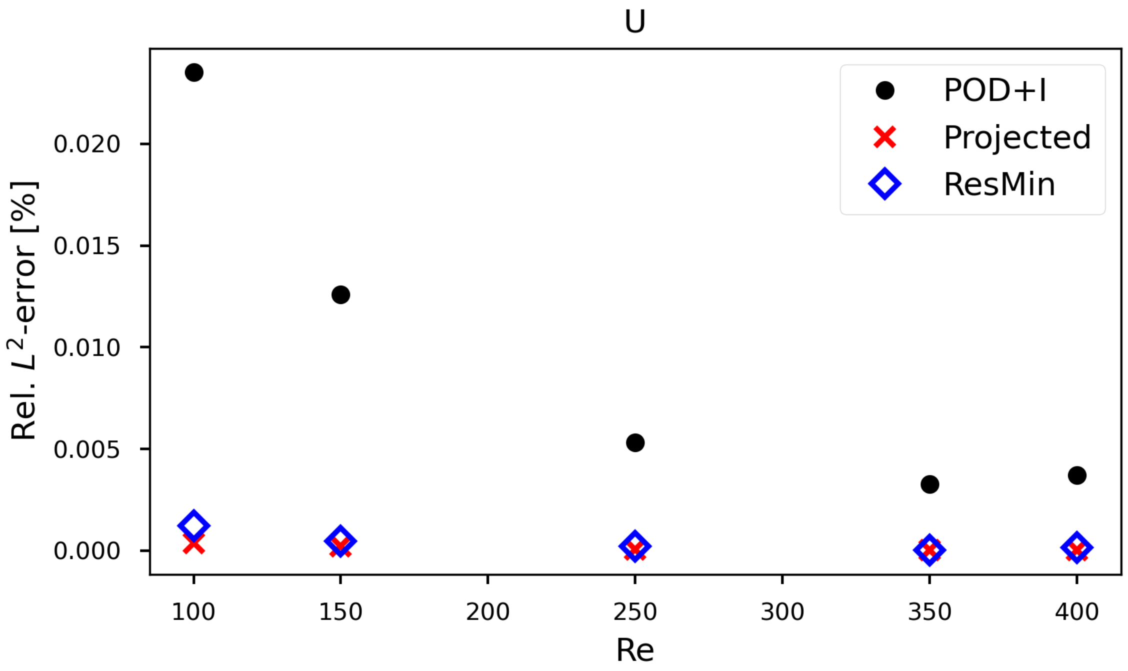


Backward Facing Step - Varying Re

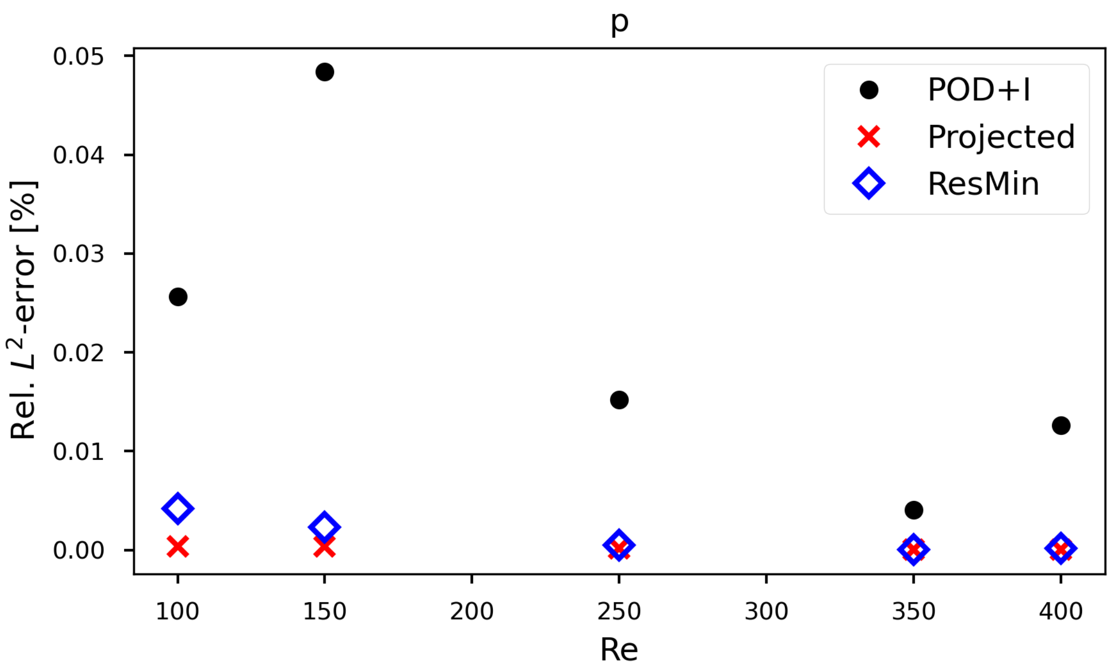
More Training Snaps

Train snapshots: 50, 60, 70, …, 450 (excluding test snaps, 36 in total)
Test snapshots: 100, 150, 250, 350, 400 (same as before)

Error Velocity (Ux and Uy)



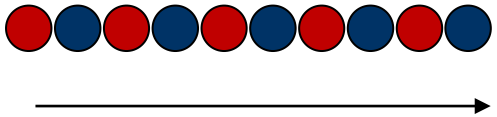
Error Pressure



Ahmed Second Flow Region

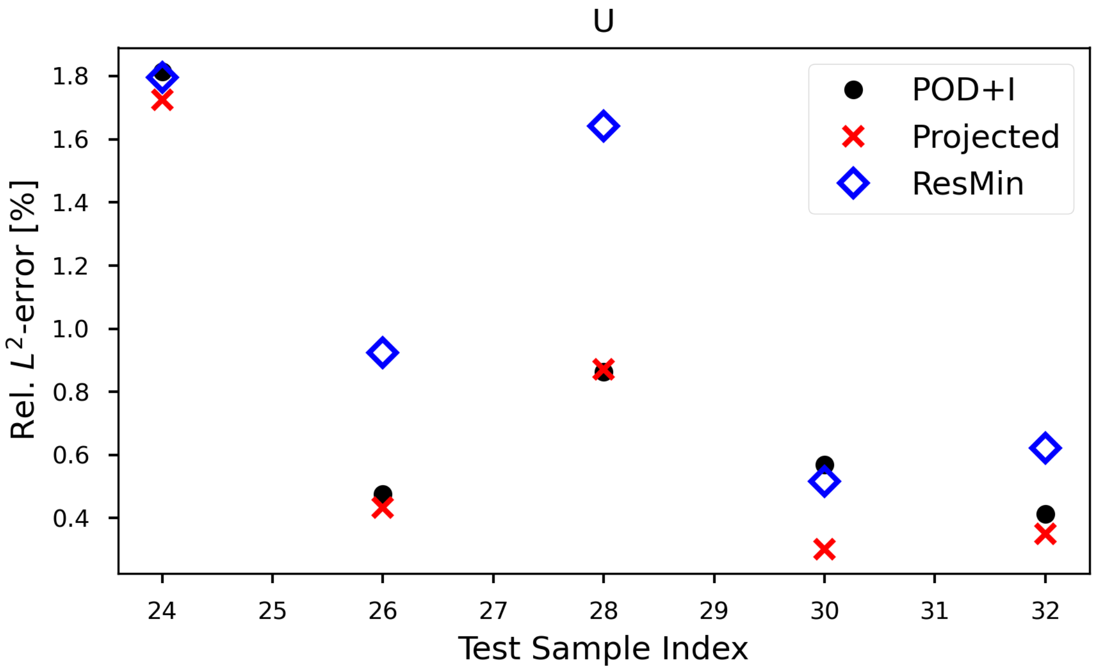
Start from POD+I

Train snapshots
Test snapshots

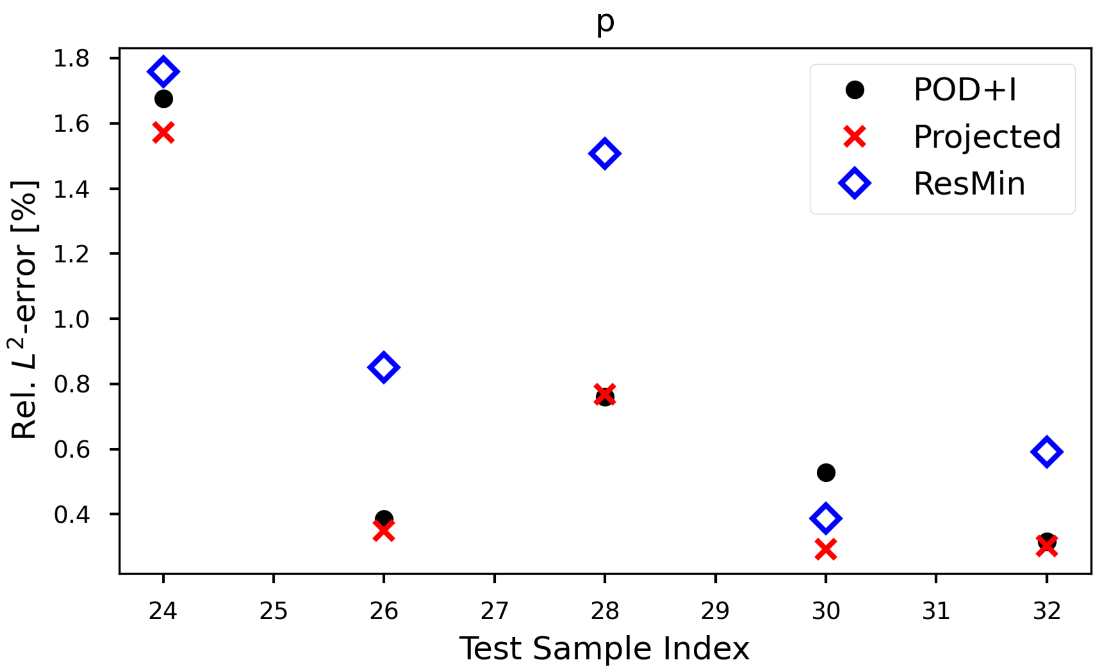


Slant angle: 24.4° - 28.1°

Error Velocity (Ux, Uy and Uz)

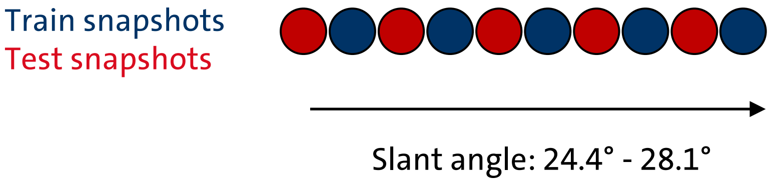


Error Pressure

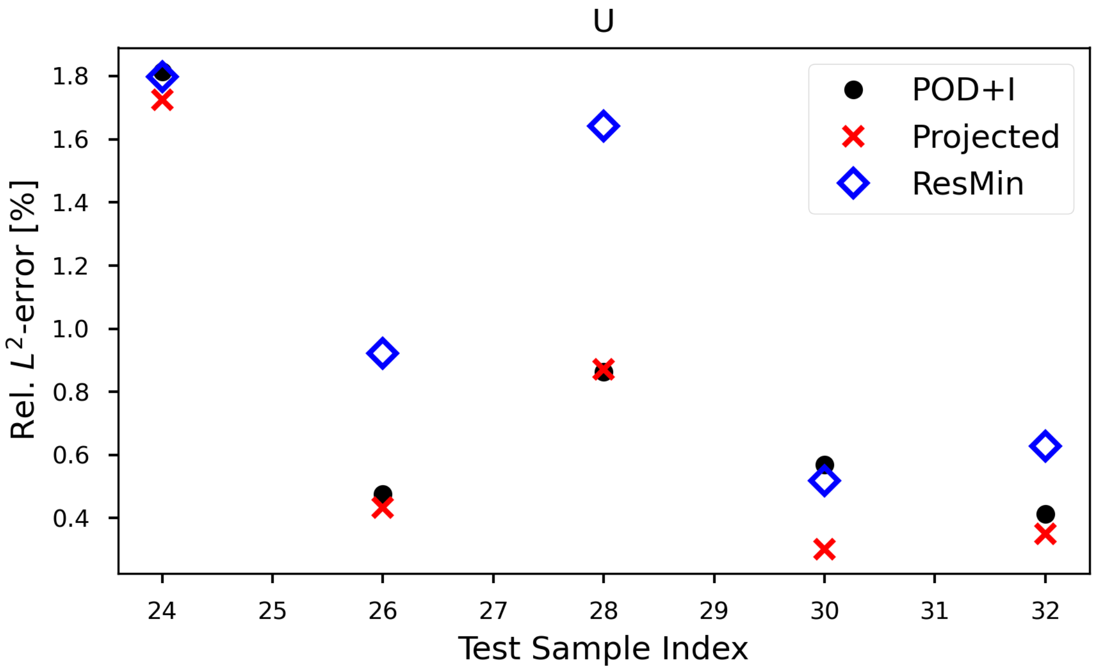


Ahmed Second Flow Region

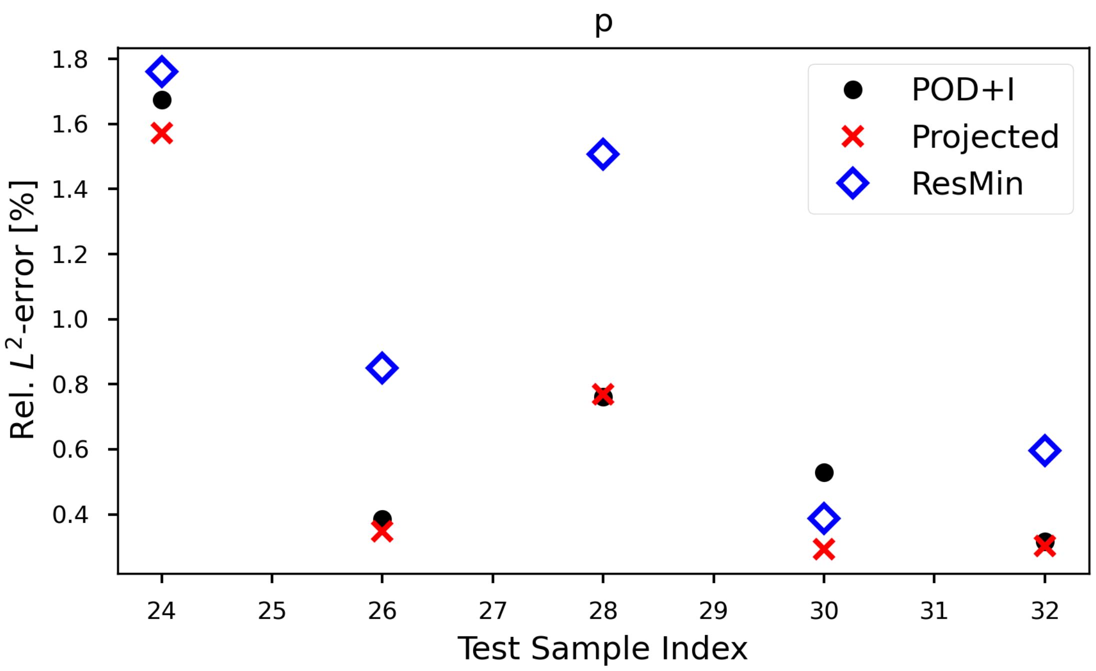
Start from Projected Solution



Error Velocity (Ux, Uy and Uz)



Error Pressure



1

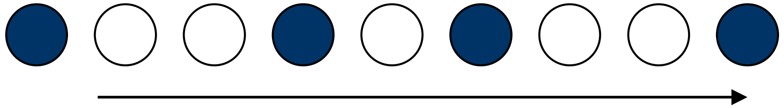
Sanity Checks:
Predict Training Snapshots

VOLKSWAGEN GROUP

Backward Facing Step - Varying Re

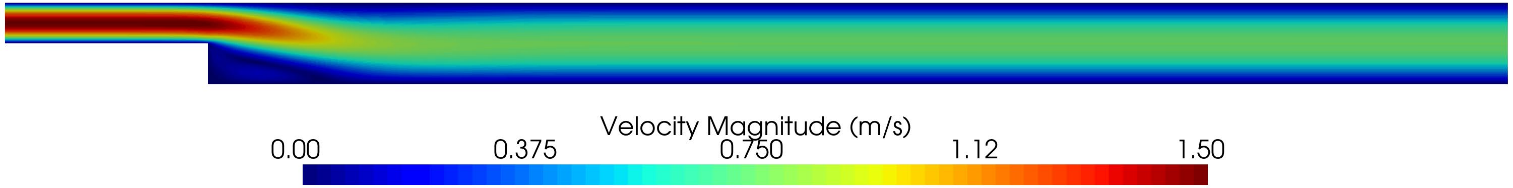
FOM Velocity

Train snapshots
Test snapshots

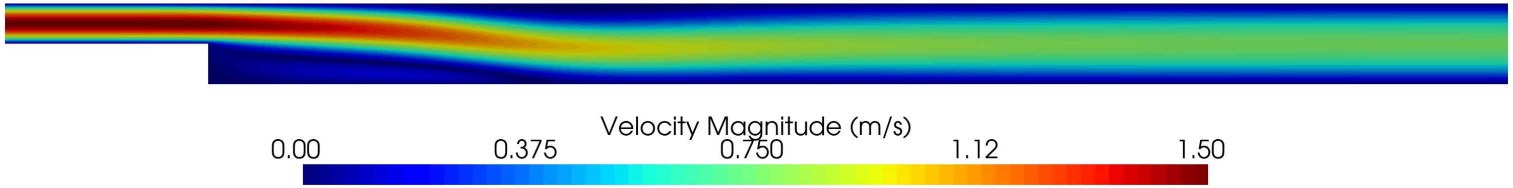


Re: 50 - 450

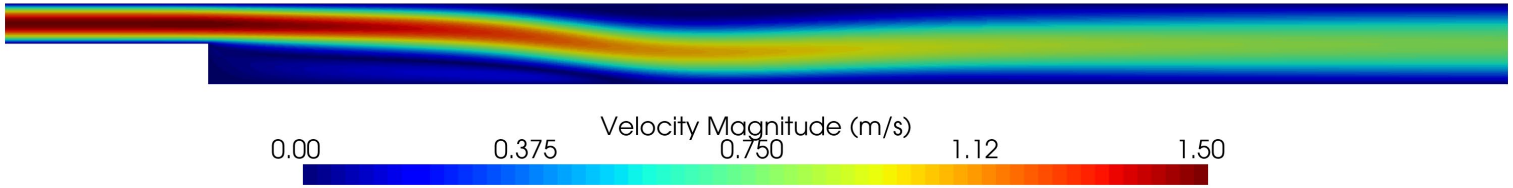
Re 50



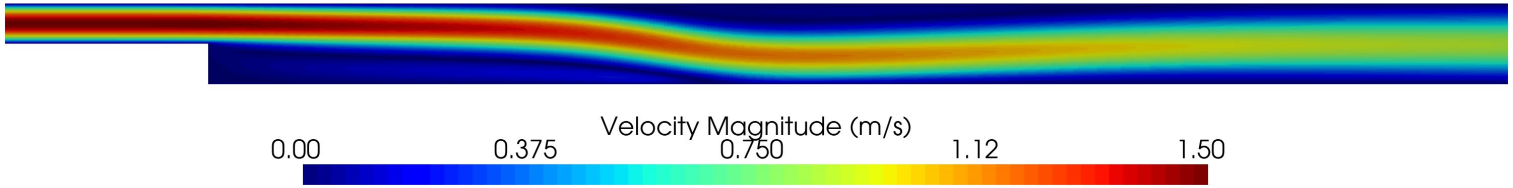
Re 200



Re 300

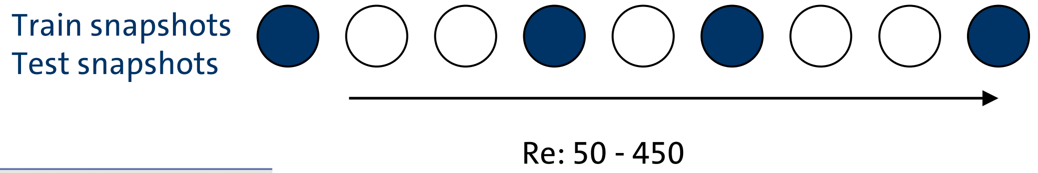


Re 450

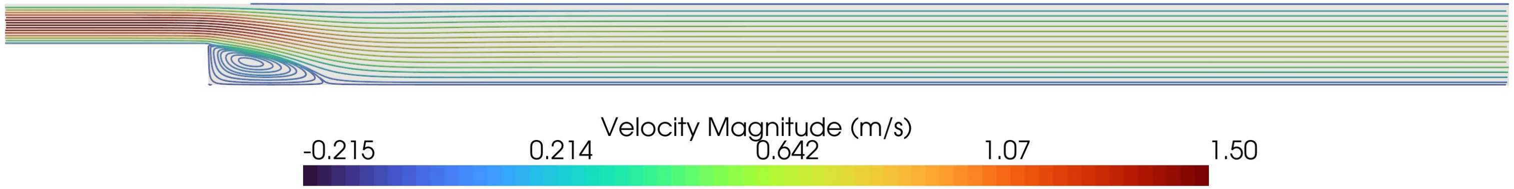


Backward Facing Step - Varying Re

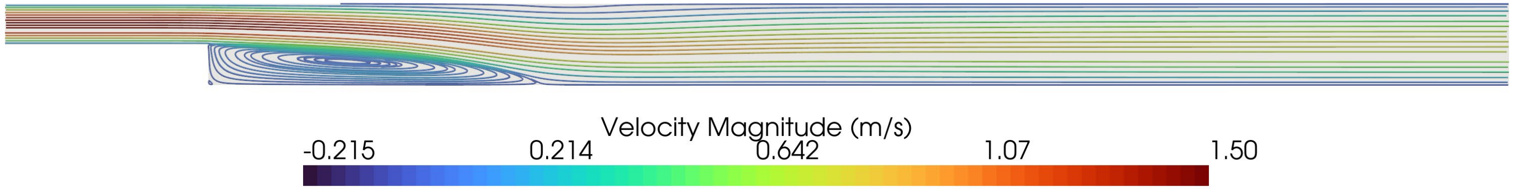
FOM Velocity



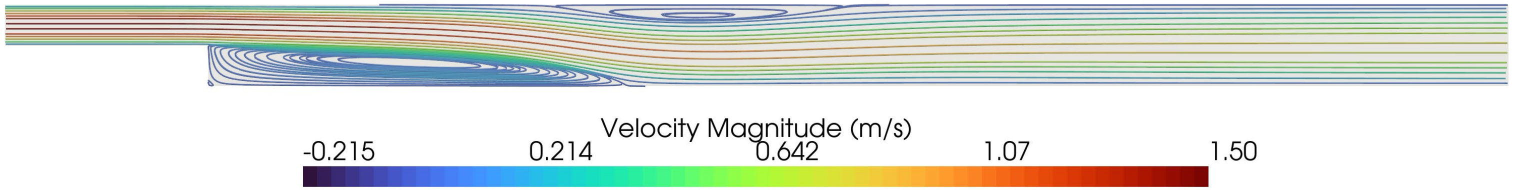
Re 50



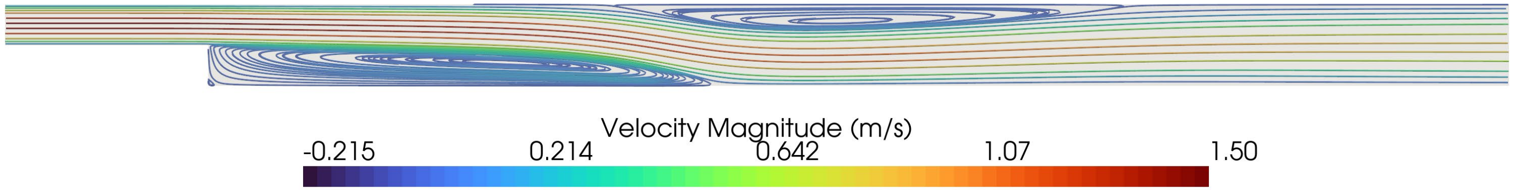
Re 200



Re 300

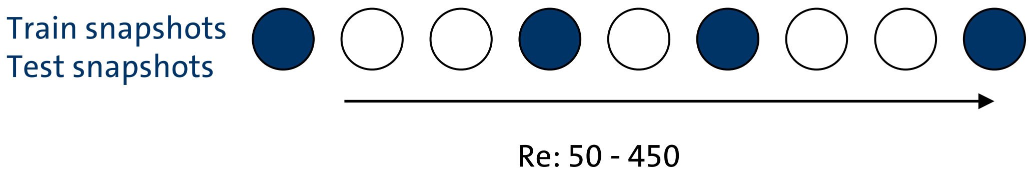


Re 450



Backward Facing Step - Varying Re

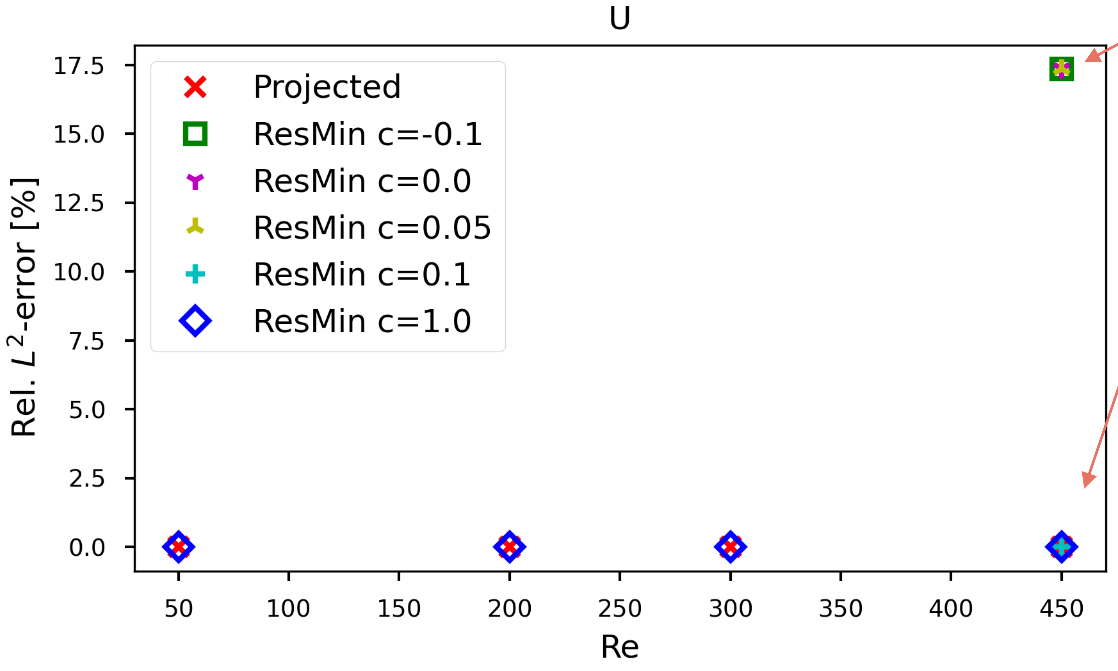
ROM with Varying Initial Coefficients



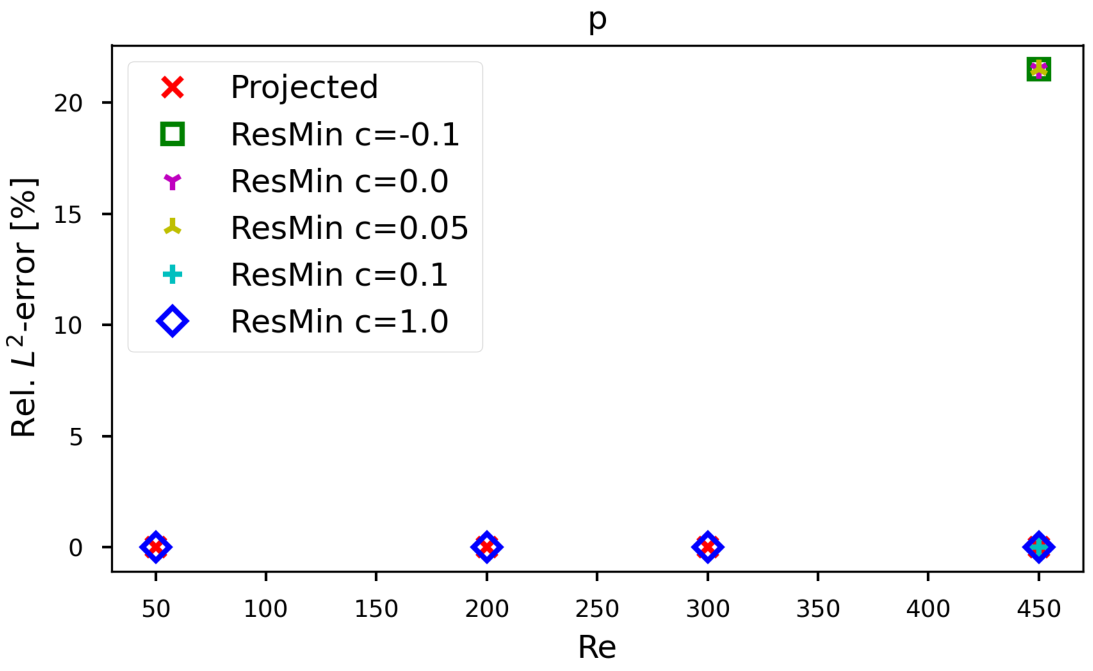
- $\boldsymbol{x}^{\mathbf{0}} = c\boldsymbol{x}^{proj}$, c is varied in plots
- Discrete scaling factors computed with residuals corresponding to c=0

Solutions subsequently referred to as :
- „Resmin“ (top)
- „Projected“ (bottom)

Error Velocity (Ux and Uy)

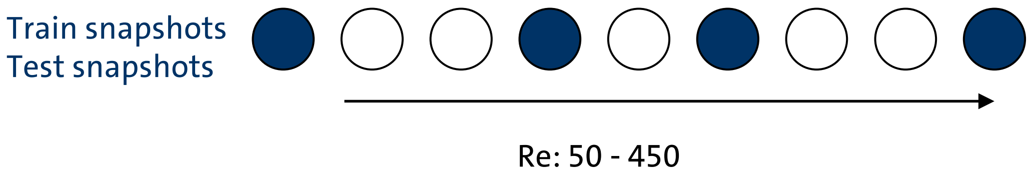


Error Pressure

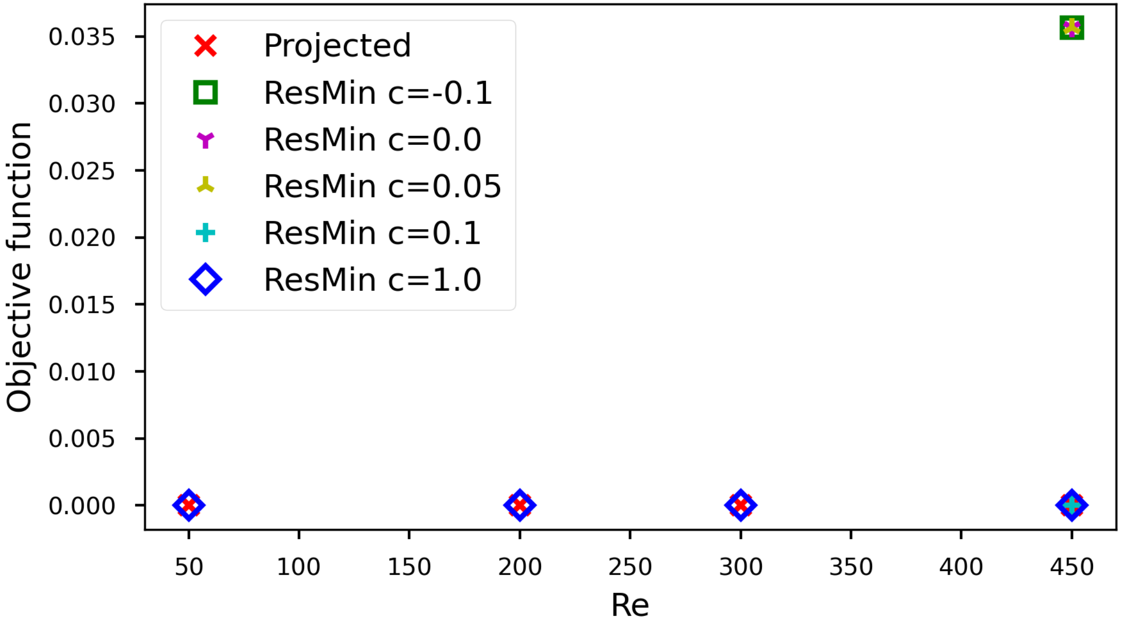


Backward Facing Step - Varying Re

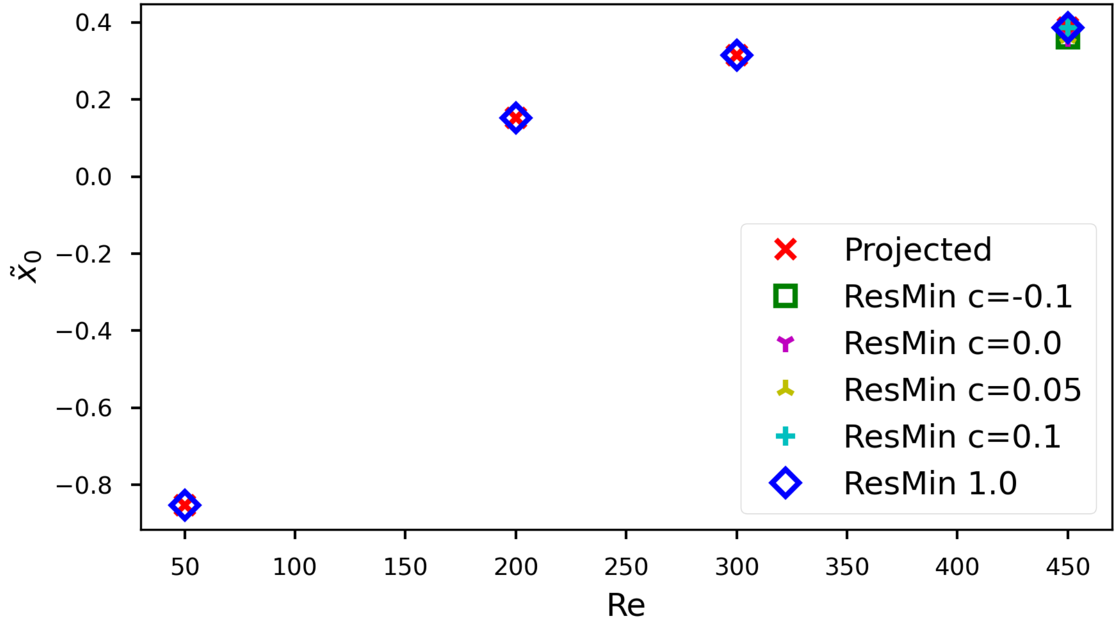
ROM with Varying Initial Coefficients



Objective-function



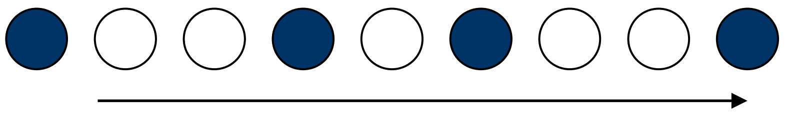
POD coefficient 1



Backward Facing Step - Varying Re

ROM with Varying Initial Coefficients

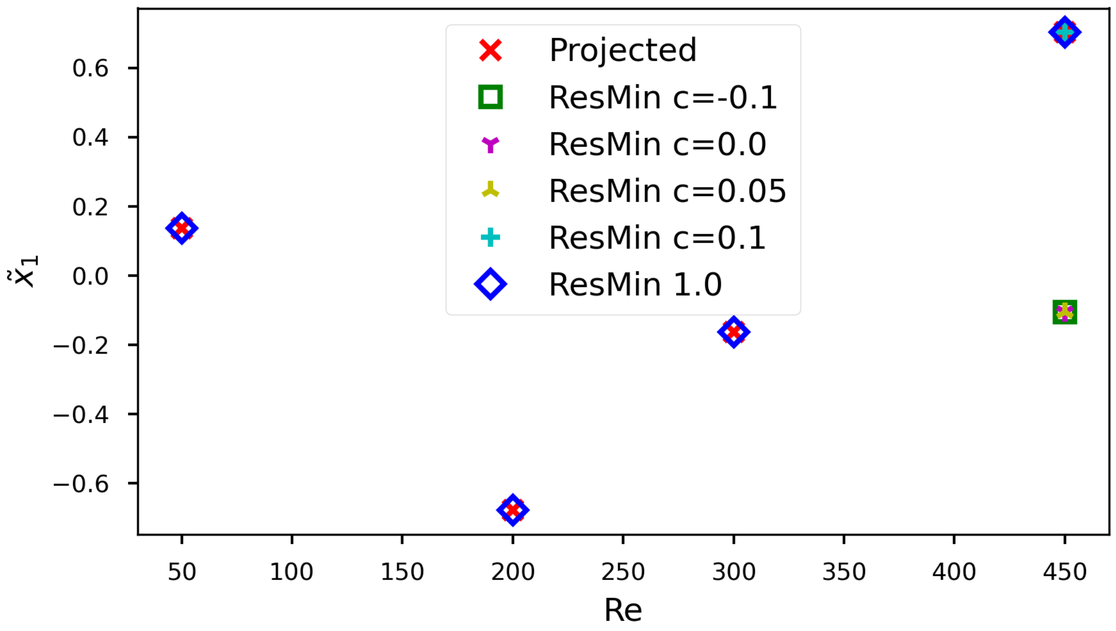
Train snapshots
Test snapshots



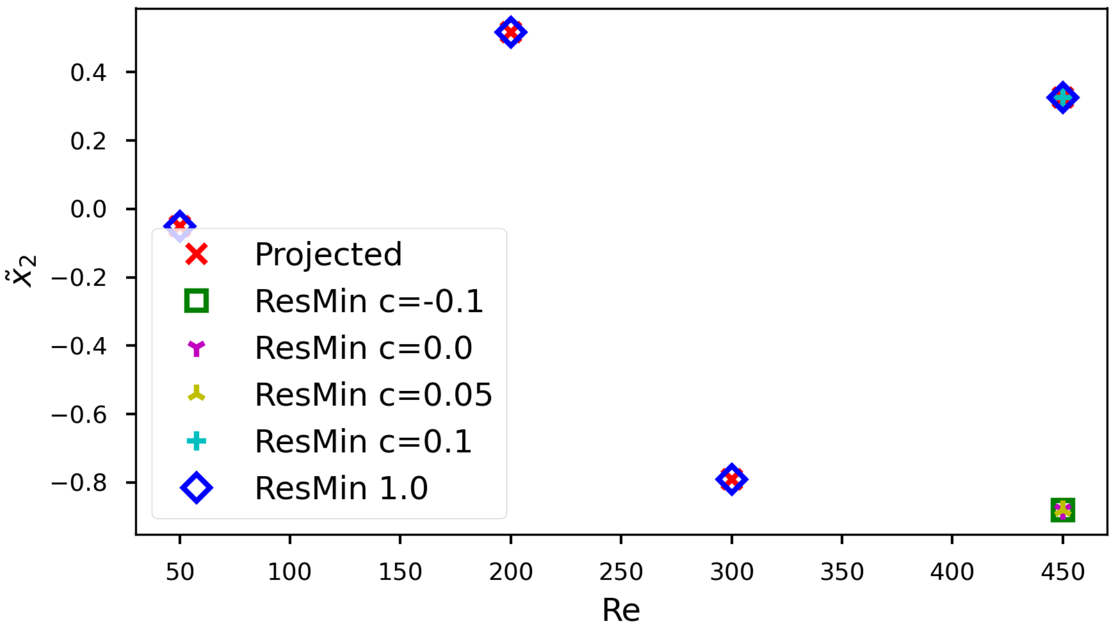
Re: 50 - 450

- **pod.A =** [[-0.85348906 0.13748433 -0.05152169]
  [0.15252261 -0.67777536 0.51706616]
  [0.31431486 -0.16330657 -0.79027662] #Re 300
  [0.38665159 0.7035976   0.32473215]]   #Re 450
- **For local minimum: a_resmin =** [0.36214574 -0.10471274 -0.88161874]

POD coefficient 2



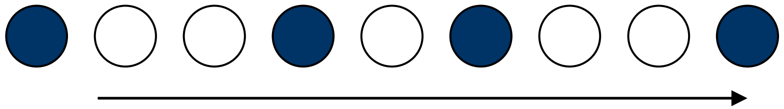
POD coefficient 3



Backward Facing Step - Varying Re

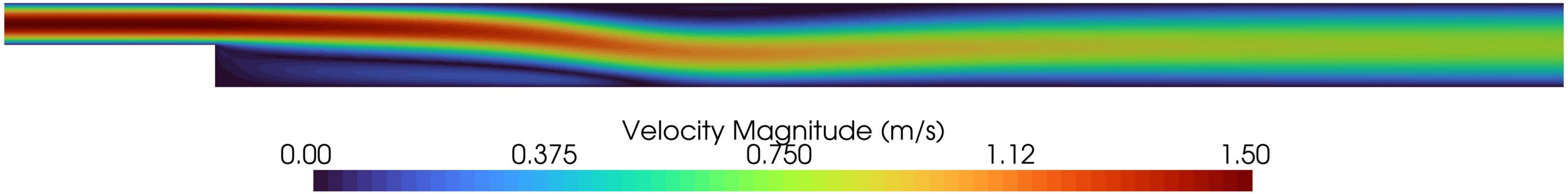
ResMin @ Re450

Train snapshots
Test snapshots

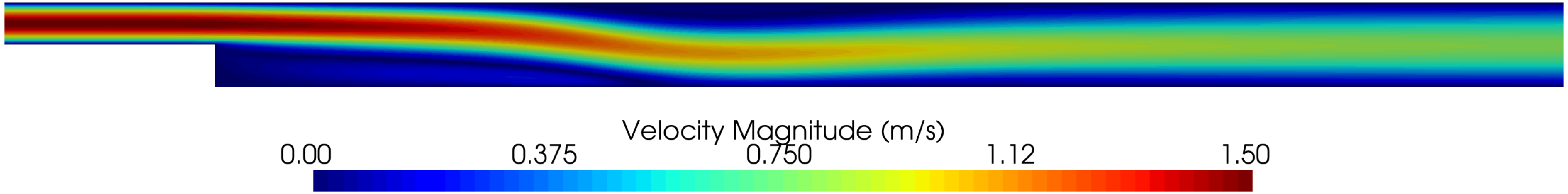


Re: 50 - 450

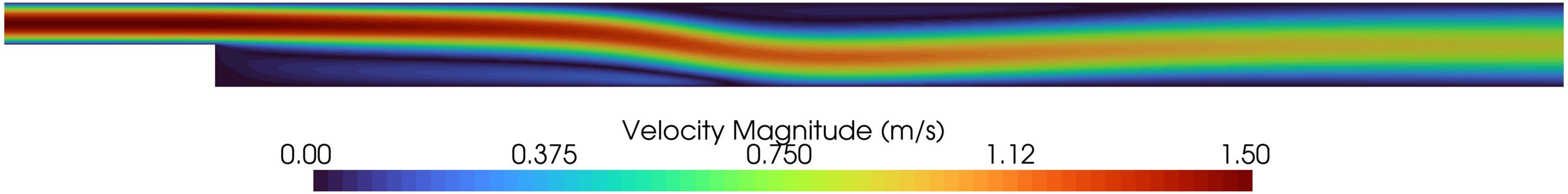
FOM Re = 300



Resmin Re = 450

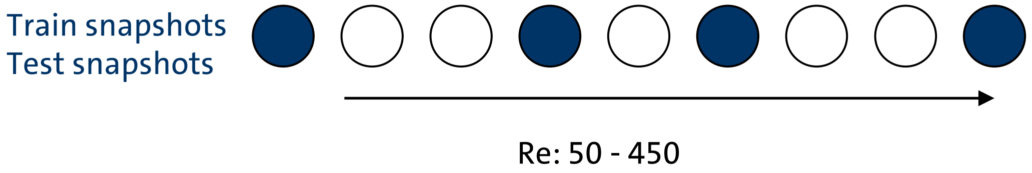


FOM Re = 450

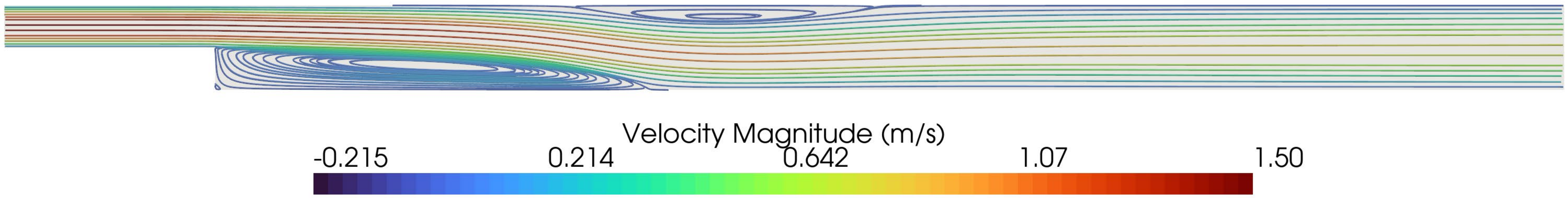


Backward Facing Step - Varying Re

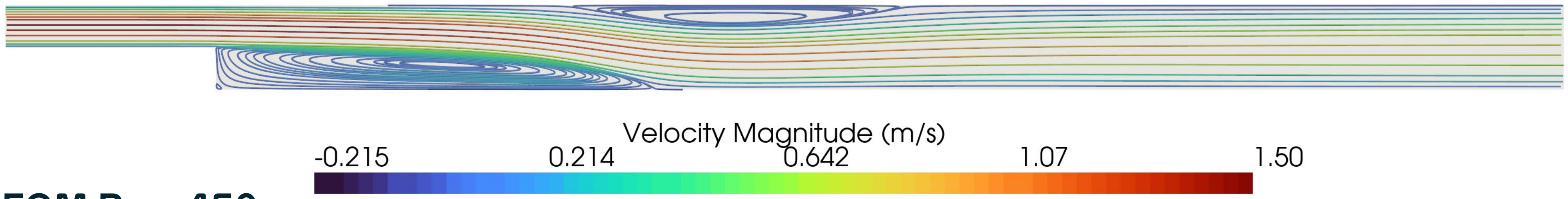
ResMin @ Re450



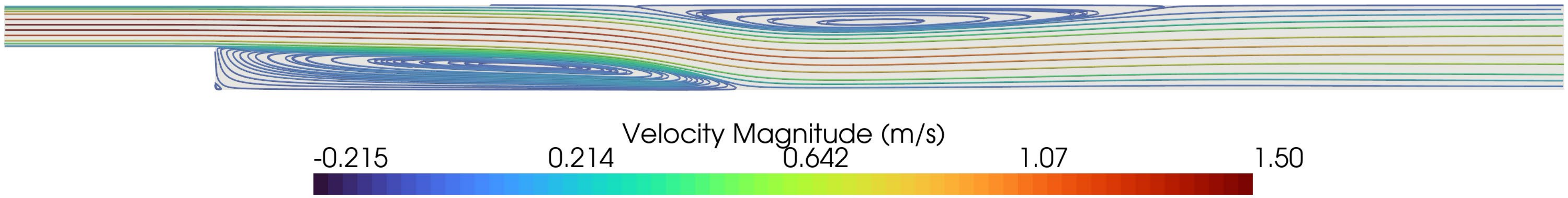
FOM Re = 300



Resmin Re = 450

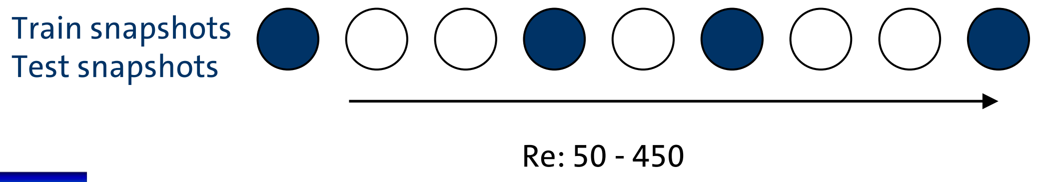


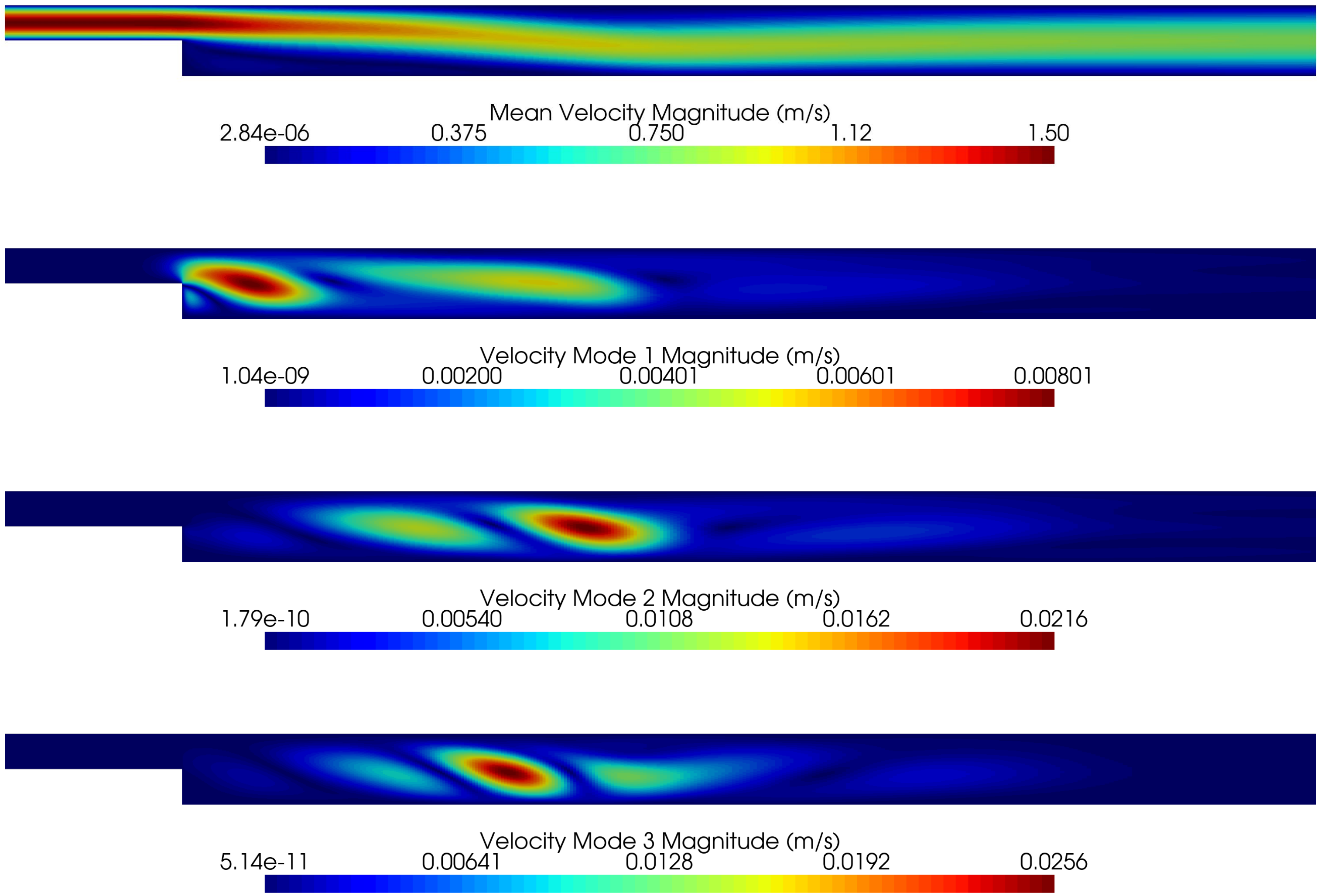
FOM Re = 450



Backward Facing Step - Varying Re

POD Modes Velocity





Singular Values

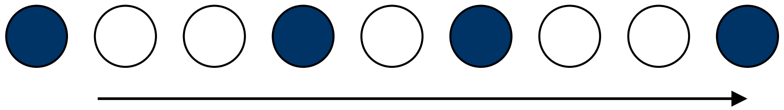
- **Mode 1:** 1.14
- **Mode 2:** 0.40
- **Mode 3:** 0.17

- Same for pressure (Global POD)

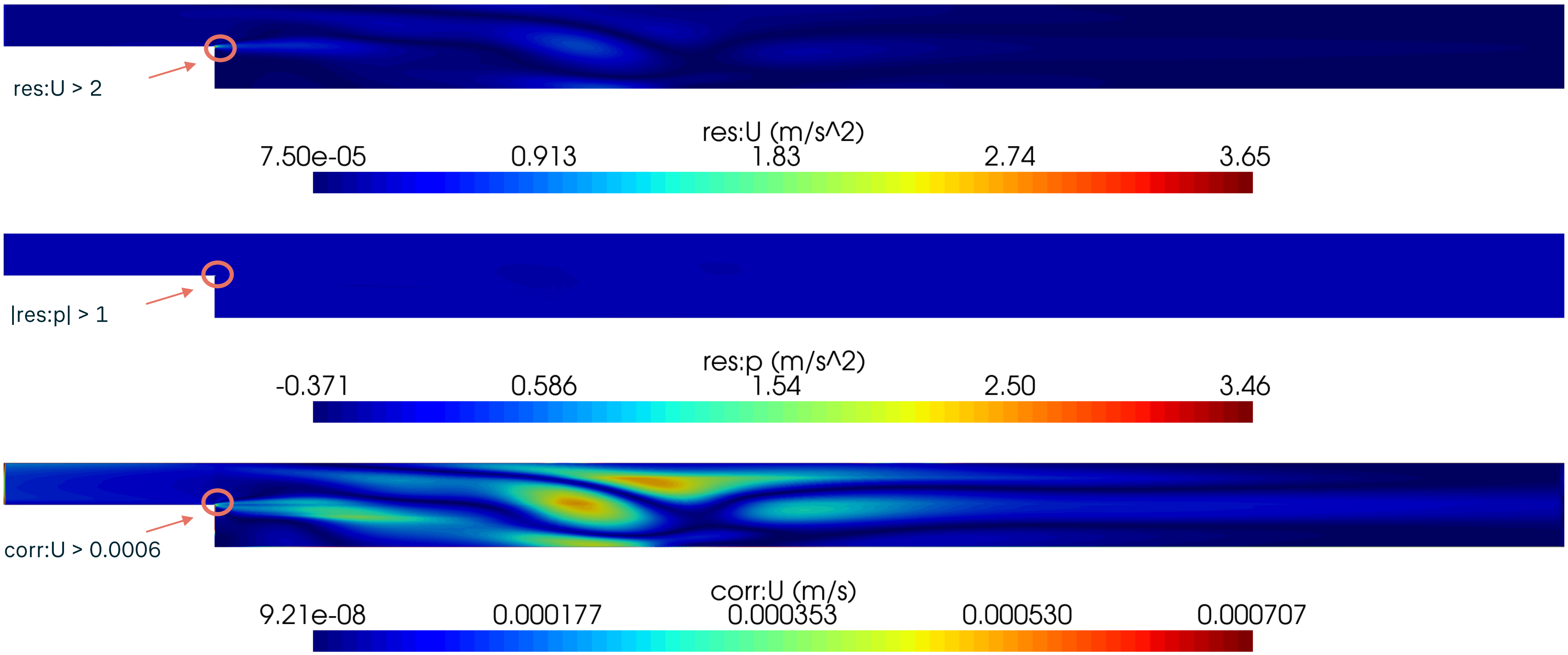
Backward Facing Step - Varying Re

ResMin @ Re450 - Residuals

Train snapshots
Test snapshots

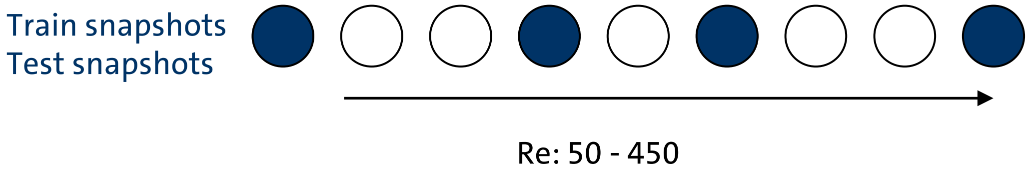


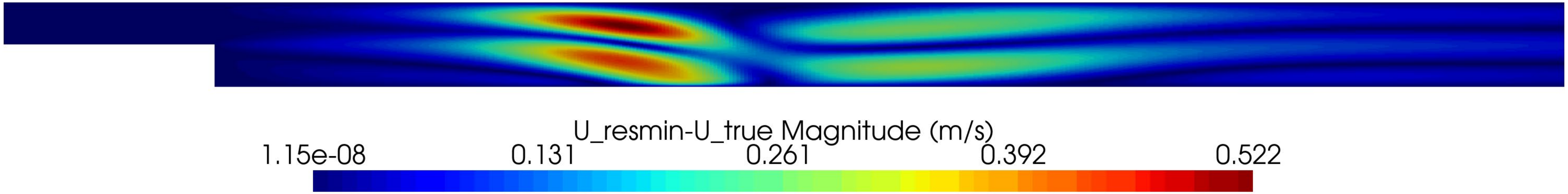
Re: 50 - 450

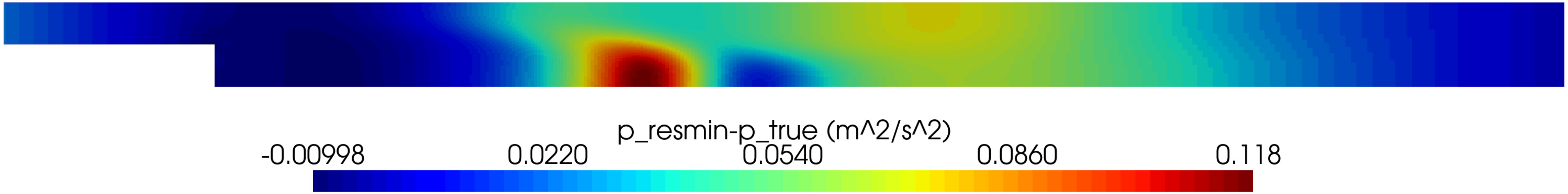


Backward Facing Step - Varying Re

ResMin @ Re450 – Error w.r.t. FOM



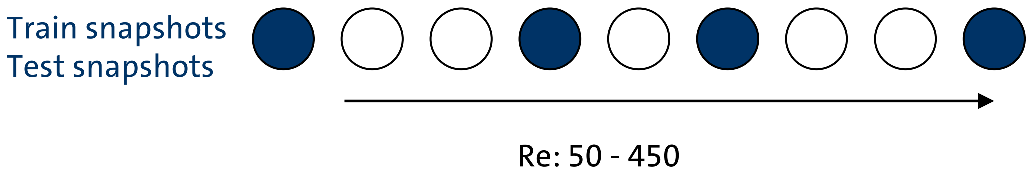


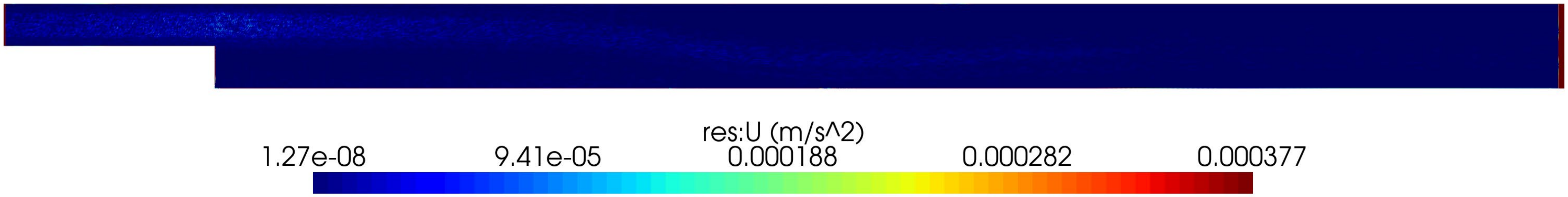


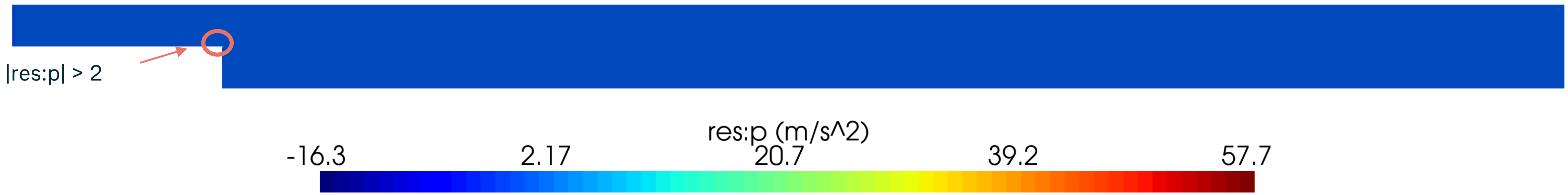
VOLKSWAGEN GROUP

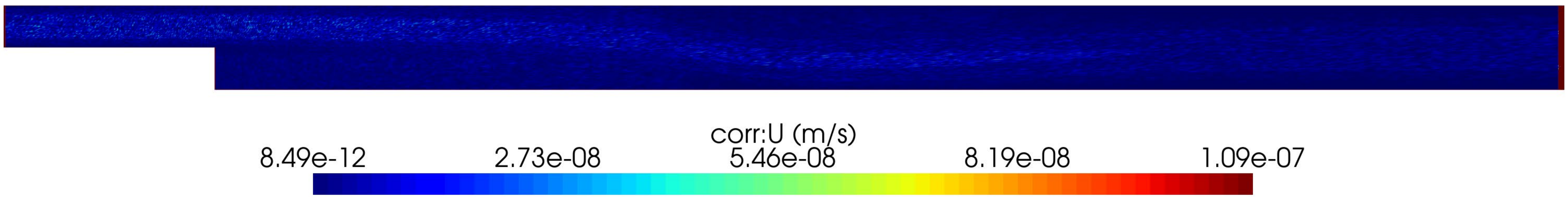
Backward Facing Step - Varying Re

Projected @ Re450 - Residuals





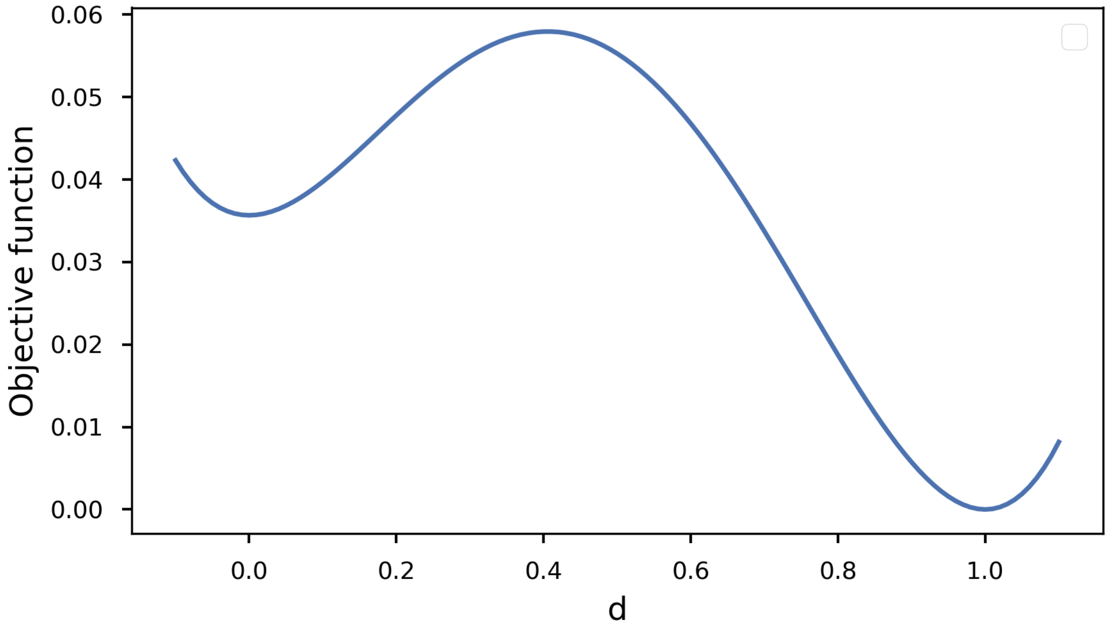




Backward Facing Step - Varying Re

Objective-function on path between Resmin and Projected Solution

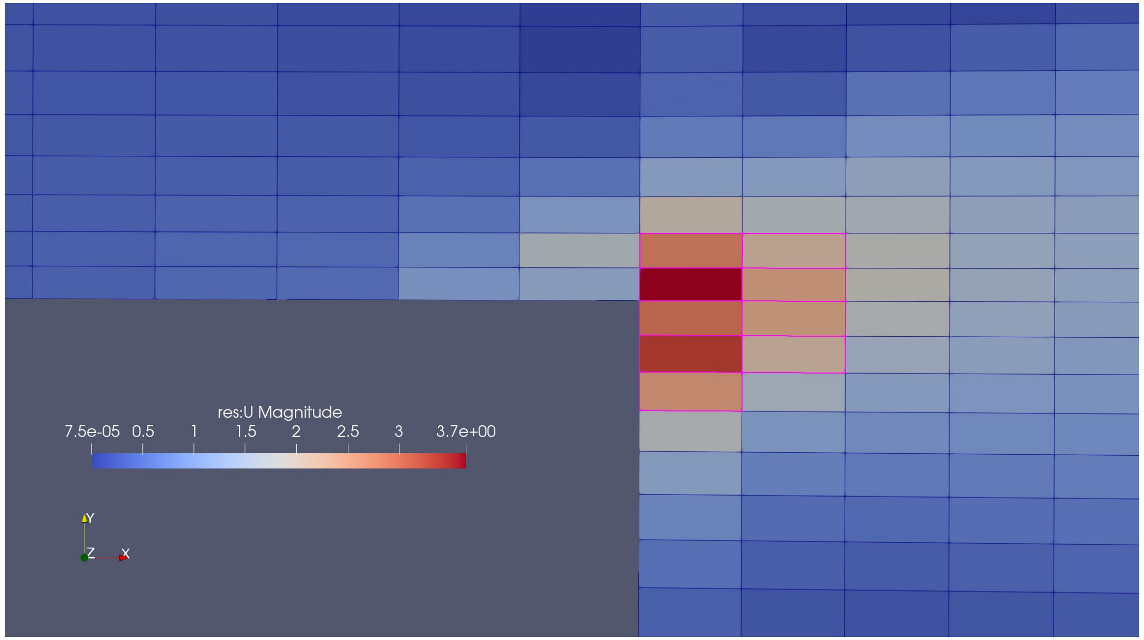
- $\boldsymbol{x}^{\mathbf{0}} = d\left(\boldsymbol{x}^{proj} - \boldsymbol{x}^{resmin}\right) + \boldsymbol{x}^{resmin}$

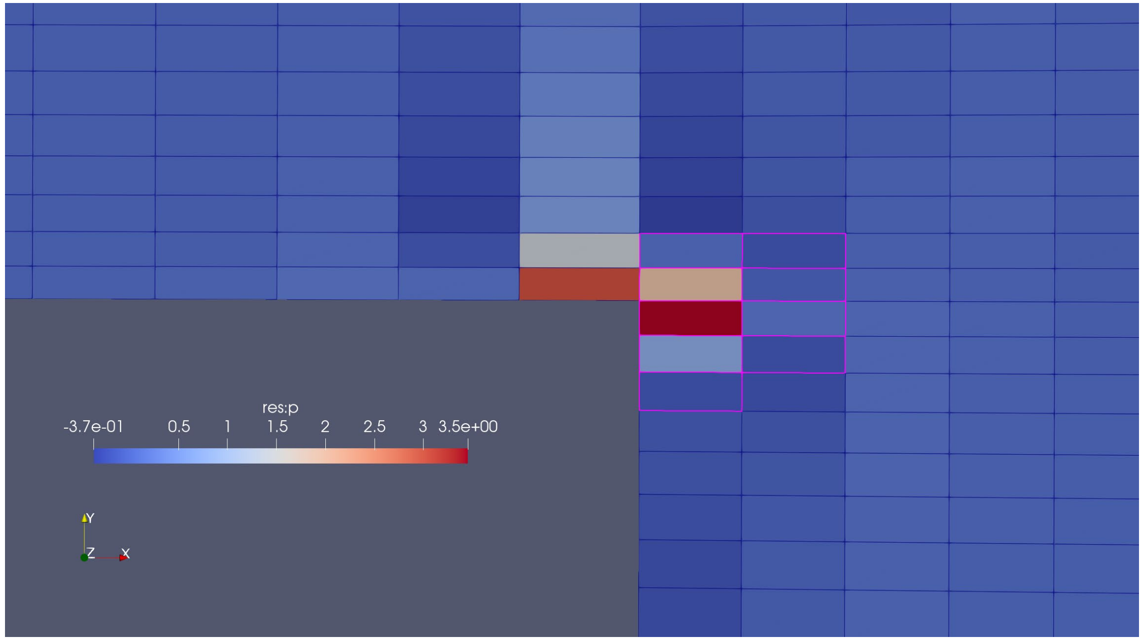


→ Resmin solution is a local minimum!!

Backward Facing Step - Varying Re

Mask for High Residuals near Boundary



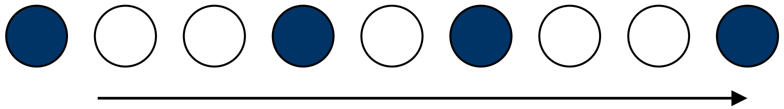


- ResMin @ Re450: magnitude(res:U) >= 2 used as a criterion to find 9 cells close to boundary

Backward Facing Step - Varying Re

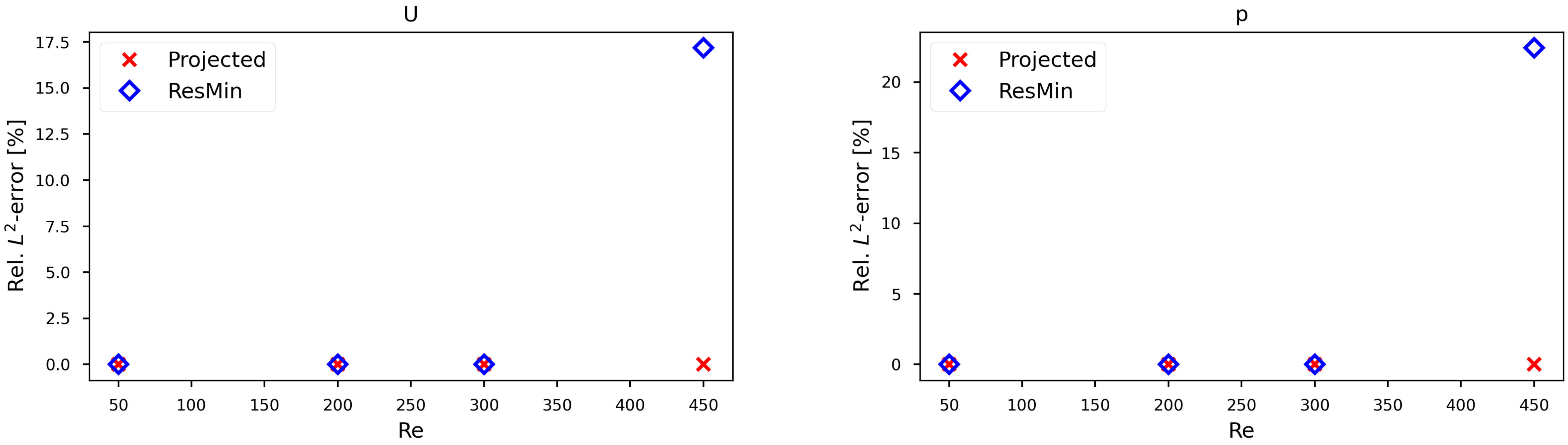
Mask for High Residuals near Boundary

Train snapshots
Test snapshots



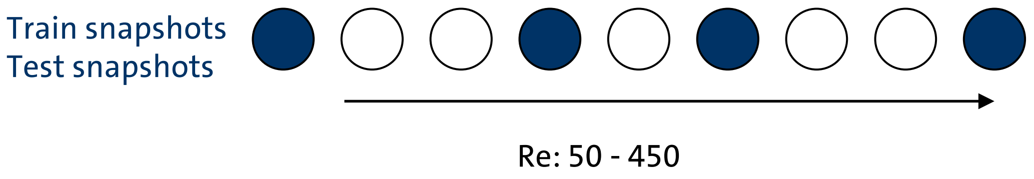
Re: 50 - 450

Only residuals in bulk region

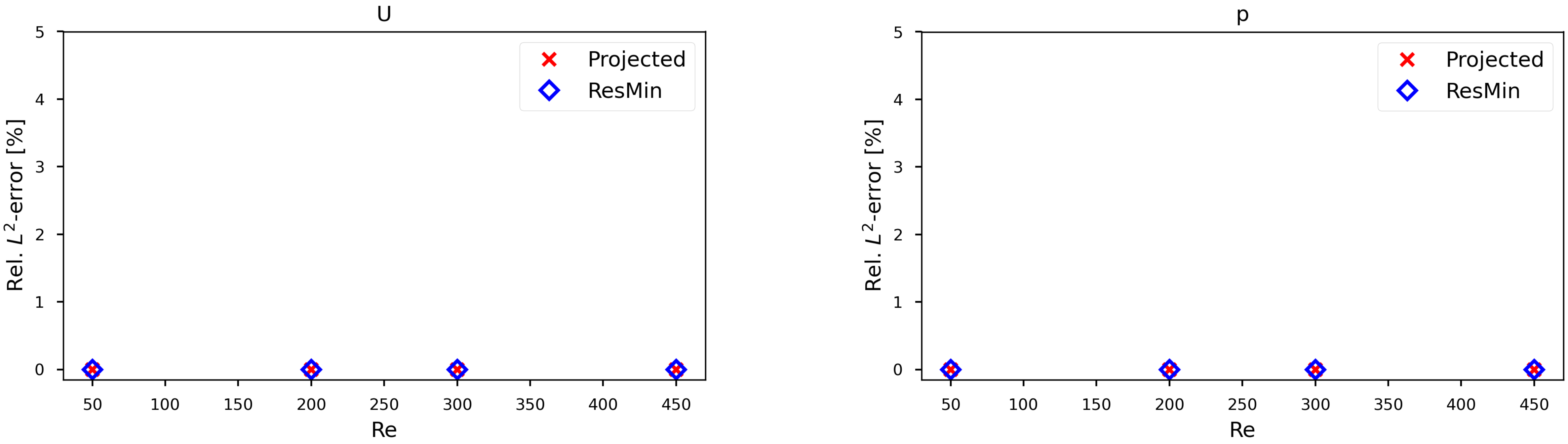


Backward Facing Step - Varying Re

Mask for High Residuals near Boundary



Only residuals in boundary region

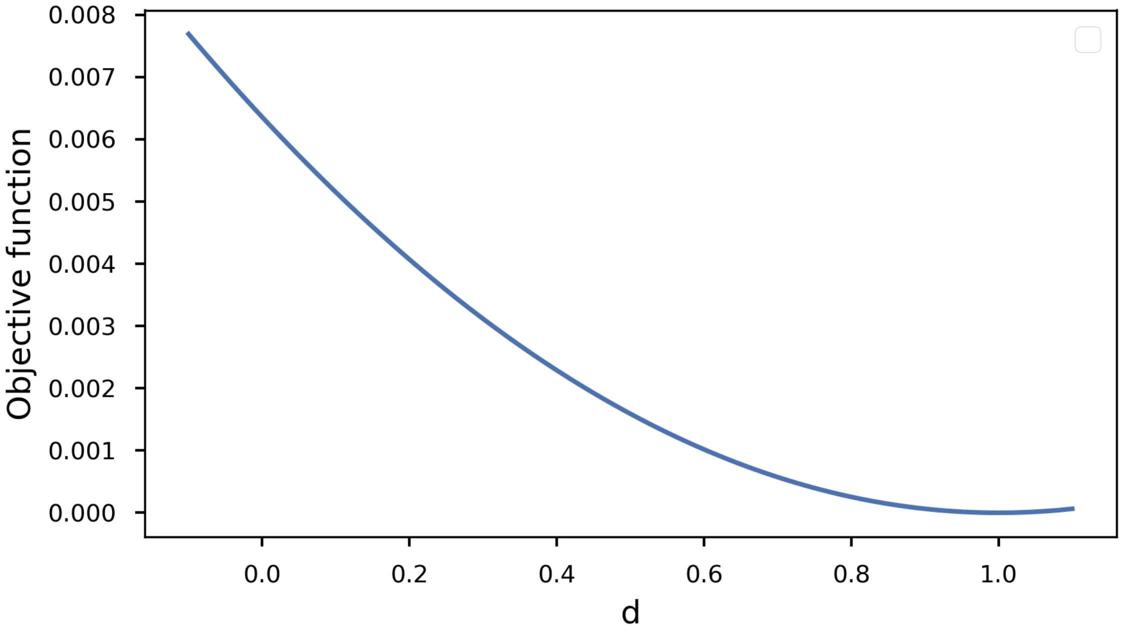


Backward Facing Step - Varying Re

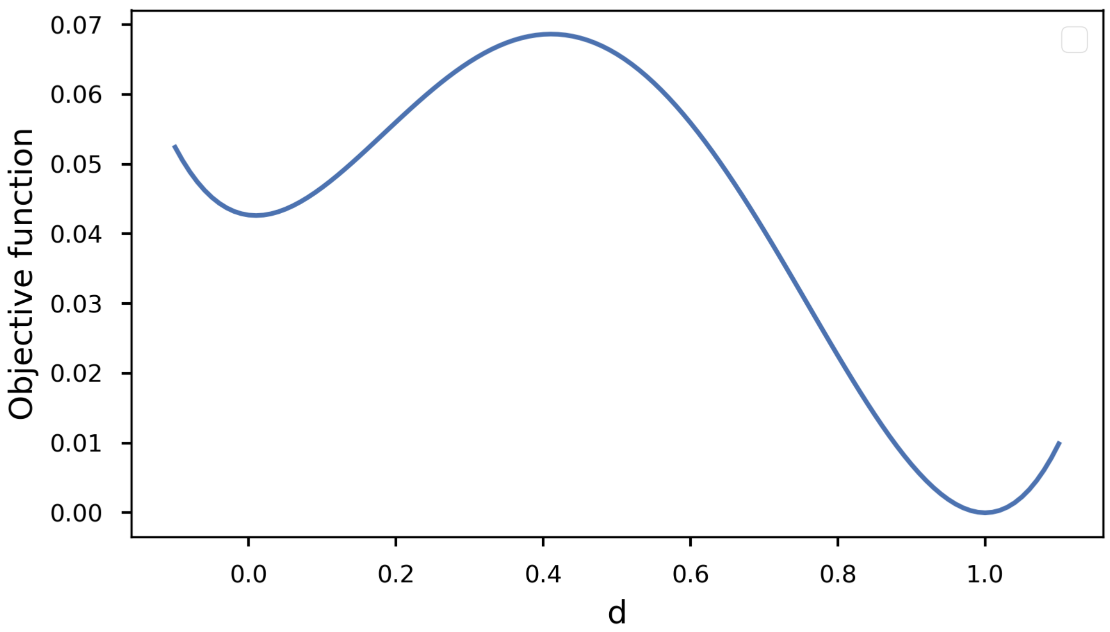
Mask for High Residuals near Boundary

- $\boldsymbol{x} = d\left(\boldsymbol{x}^{proj} - \boldsymbol{x}^{resmin}\right) + \boldsymbol{x}^{resmin}$

Only residuals in boundary region



Only residuals in bulk region



→ local minimum vanished!

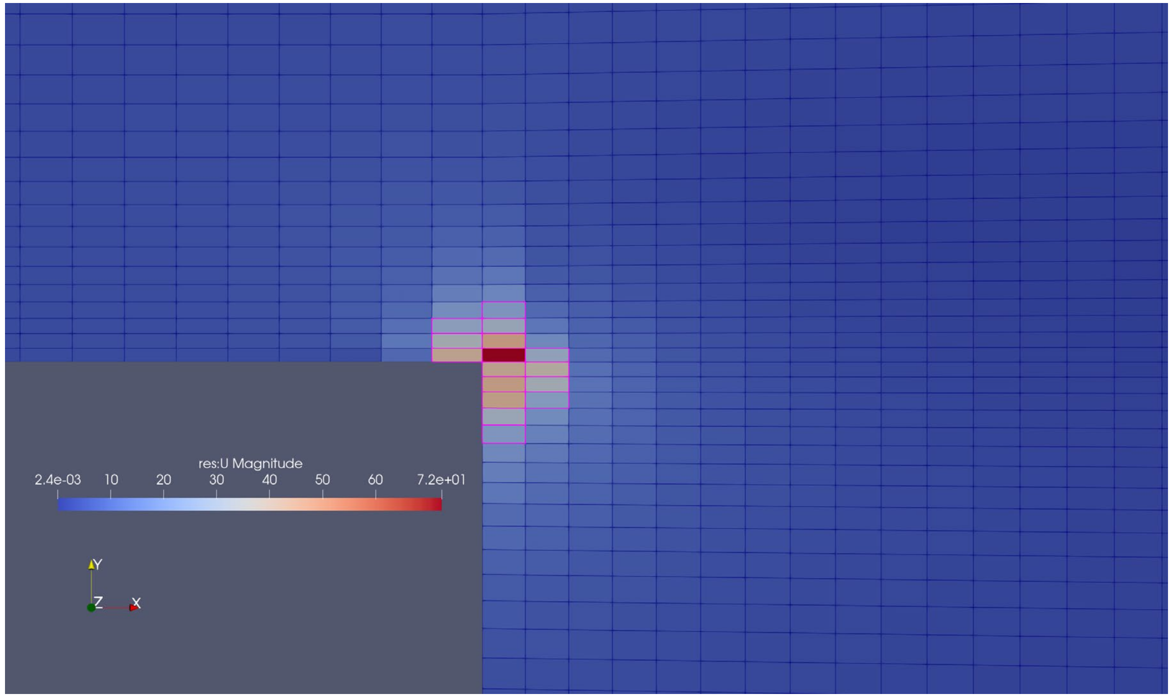
2

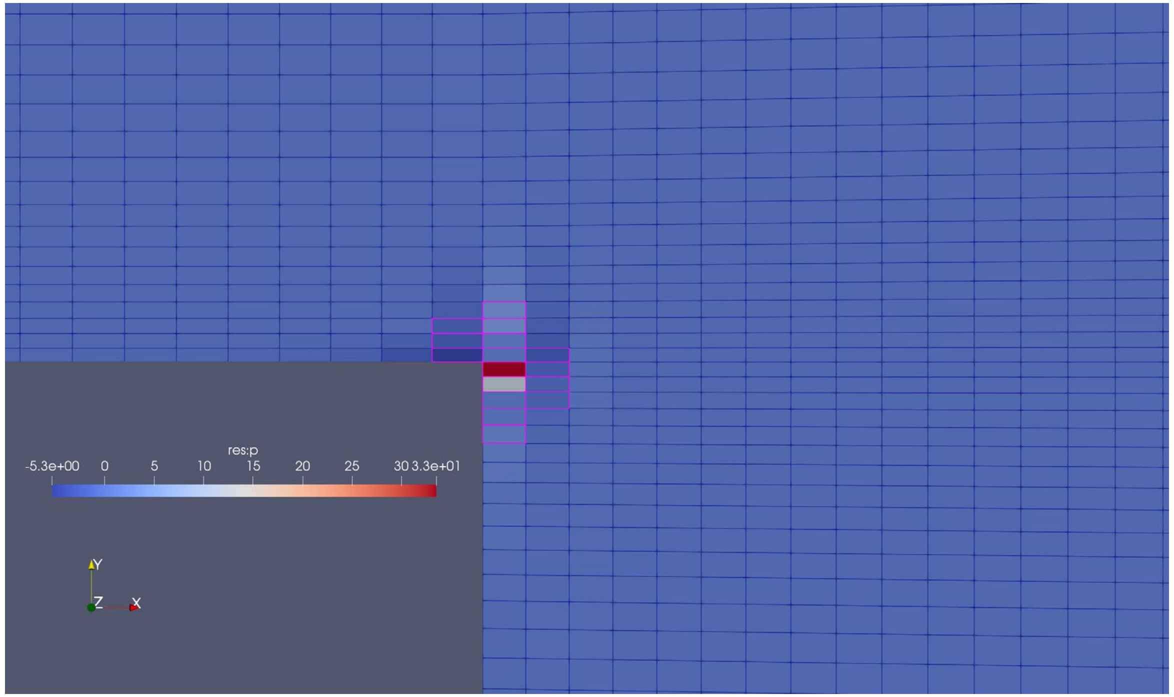
Predict Test Snapshots

Backward Facing Step - Varying Re

Mask for High Residuals near Boundary

- Mask for POD+I solution @ Re=100 (test snapshot)

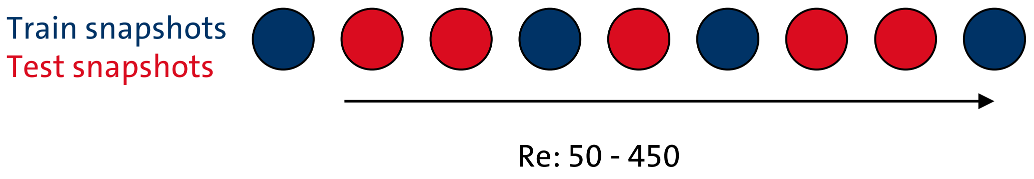




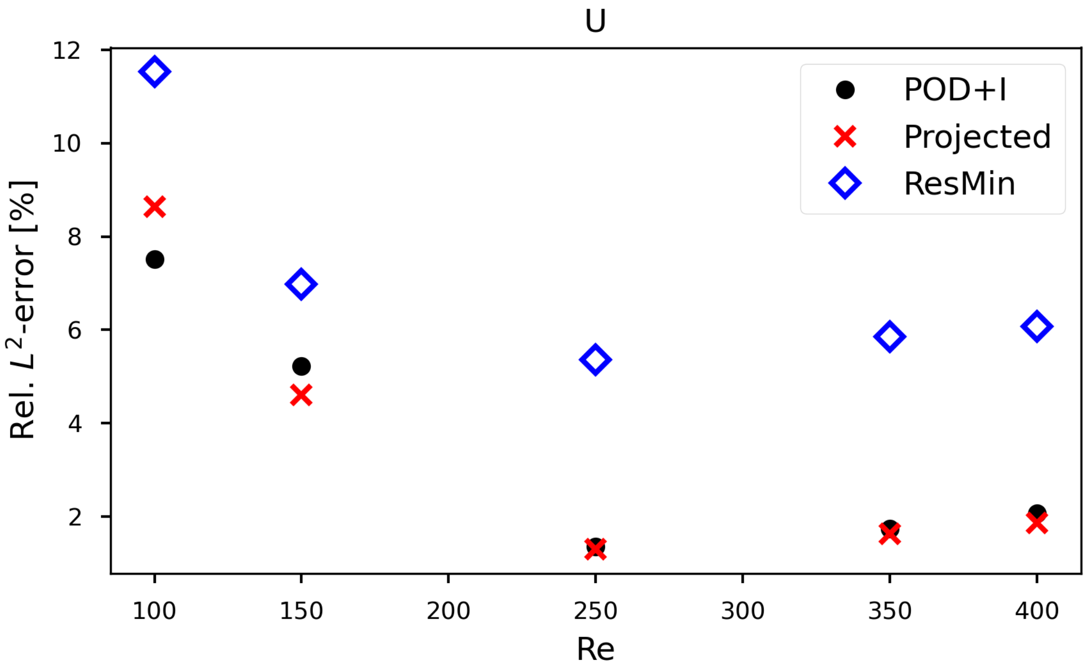
- Magnitude(res:U) >= 20 used as a criterion to find 16 cells
- Mask used for all test snapshots for now for simplicity (very similar for all test snapshots)

Backward Facing Step - Varying Re

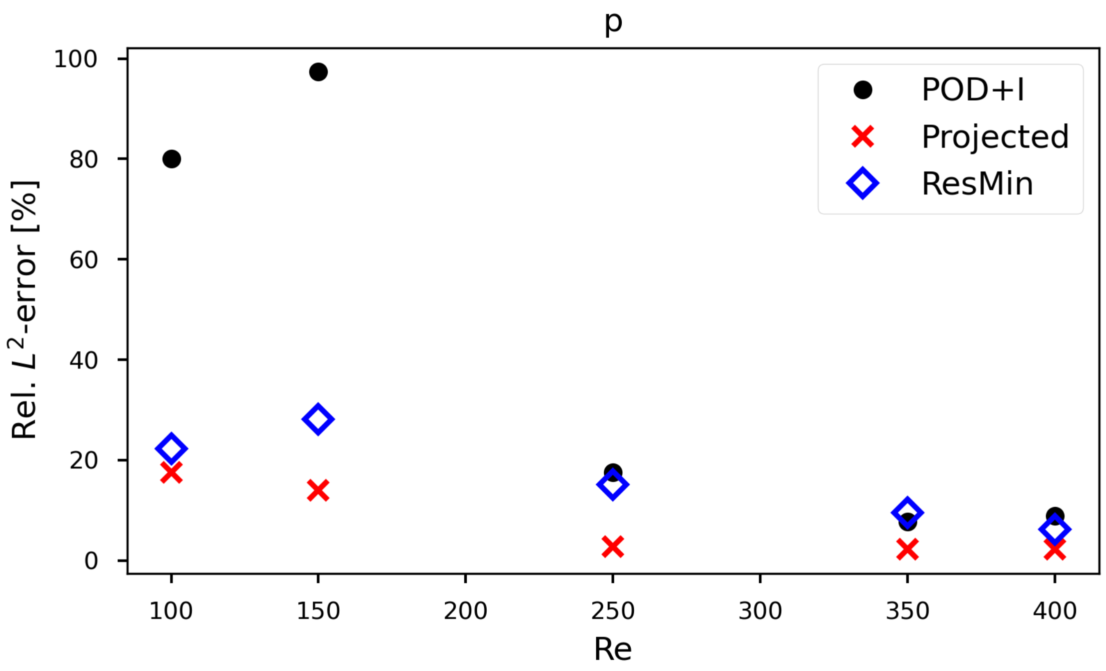
Without Mask (Bulk + Boundary Residuals)



Error Velocity (Ux and Uy)

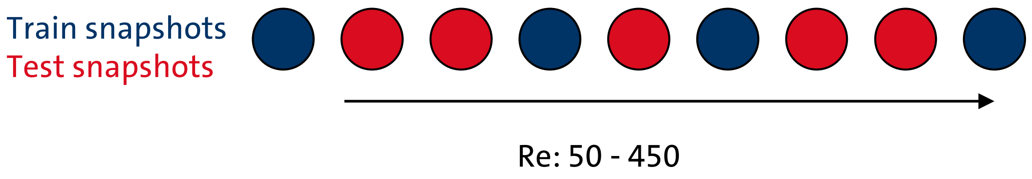


Error Pressure

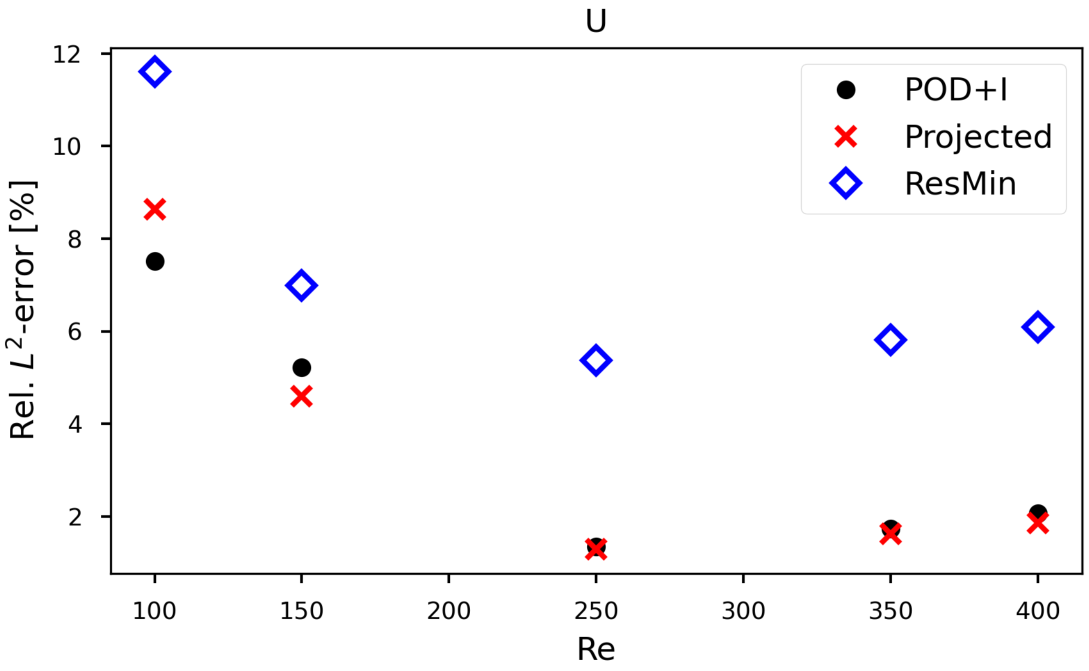


Backward Facing Step - Varying Re

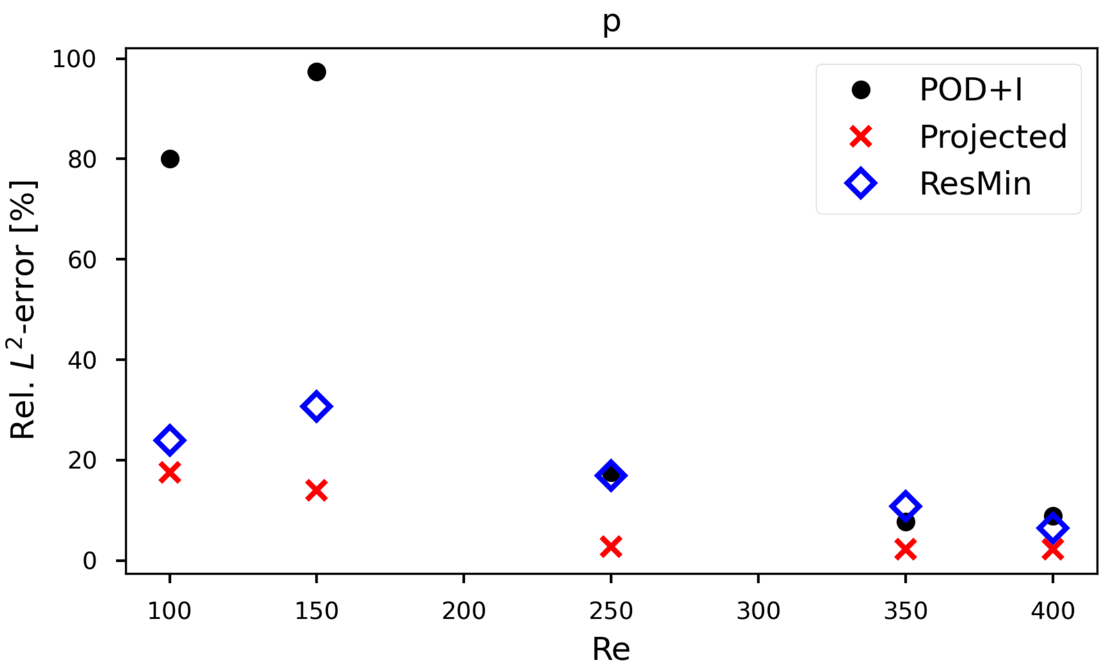
Only Bulk Residuals



Error Velocity (Ux and Uy)

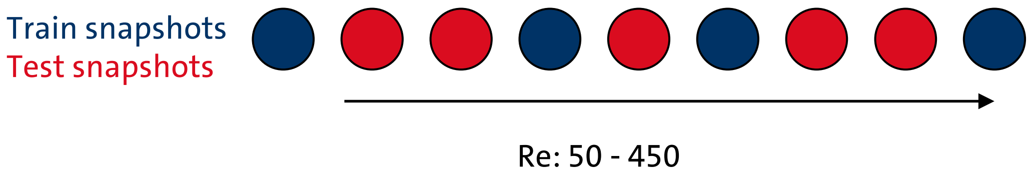


Error Pressure

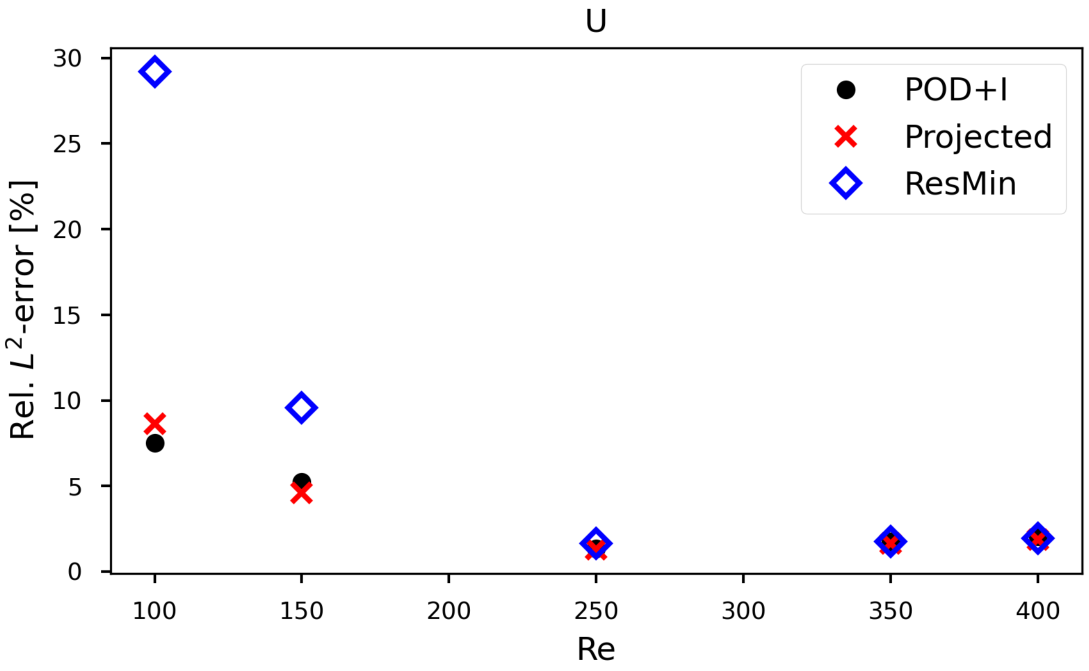


Backward Facing Step - Varying Re

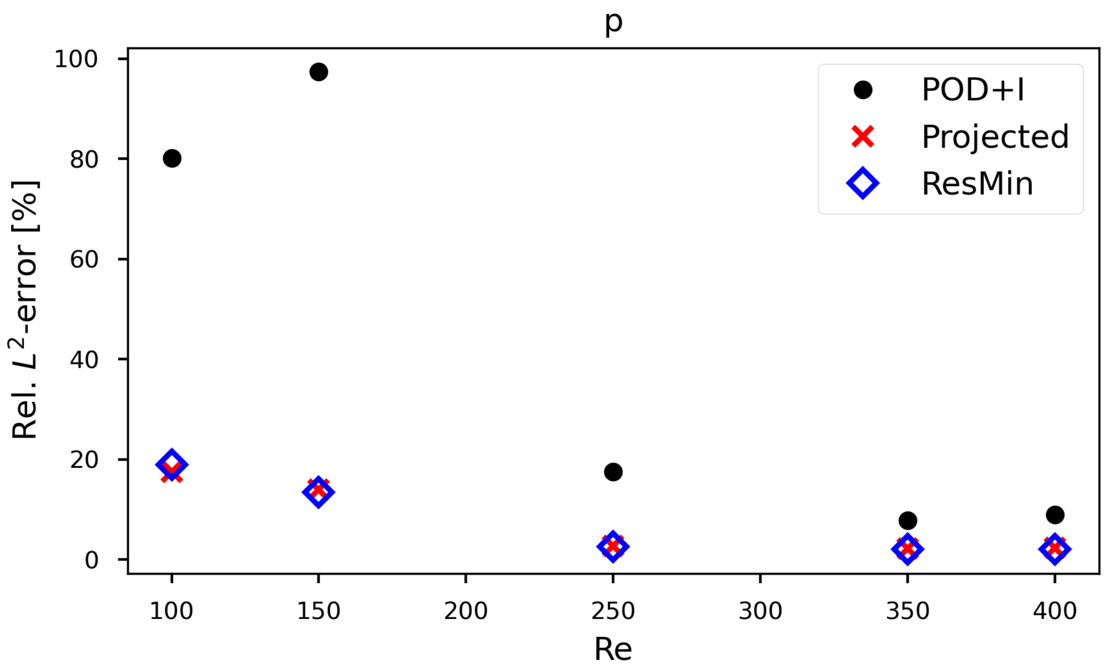
Only Boundary Residuals



Error Velocity (Ux and Uy)

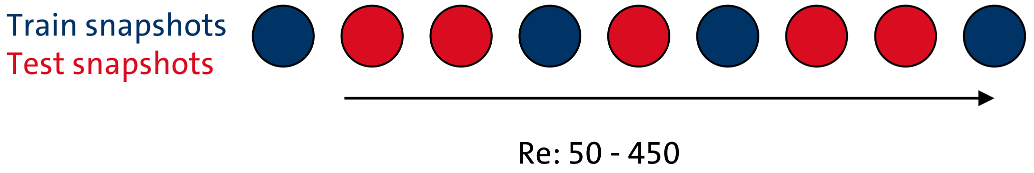


Error Pressure

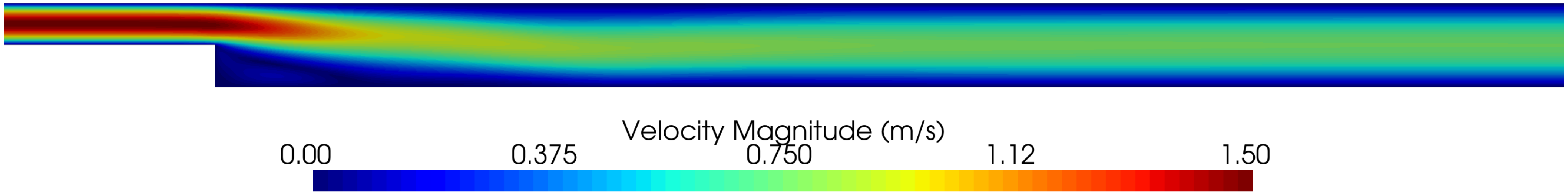


Backward Facing Step - Varying Re

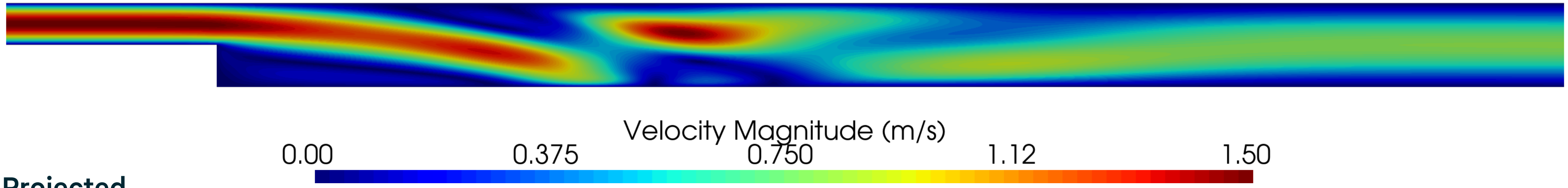
Only Boundary Residuals - Re=100



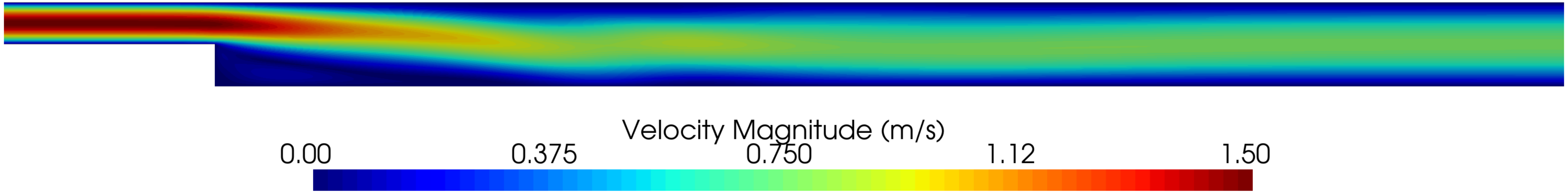
POD+I



Resmin



Projected

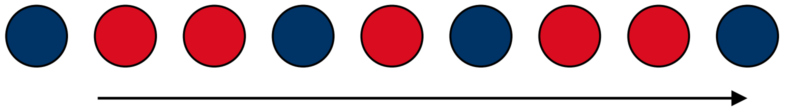


VOLKSWAGEN GROUP

Backward Facing Step - Varying Re

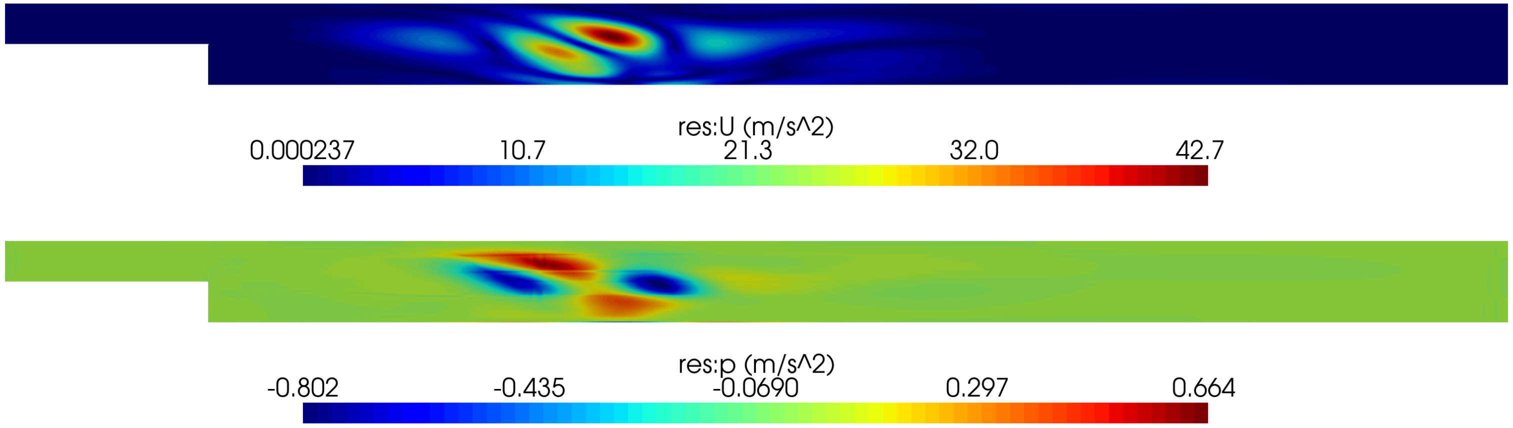
Only Boundary Residuals - Re=100

Train snapshots
Test snapshots

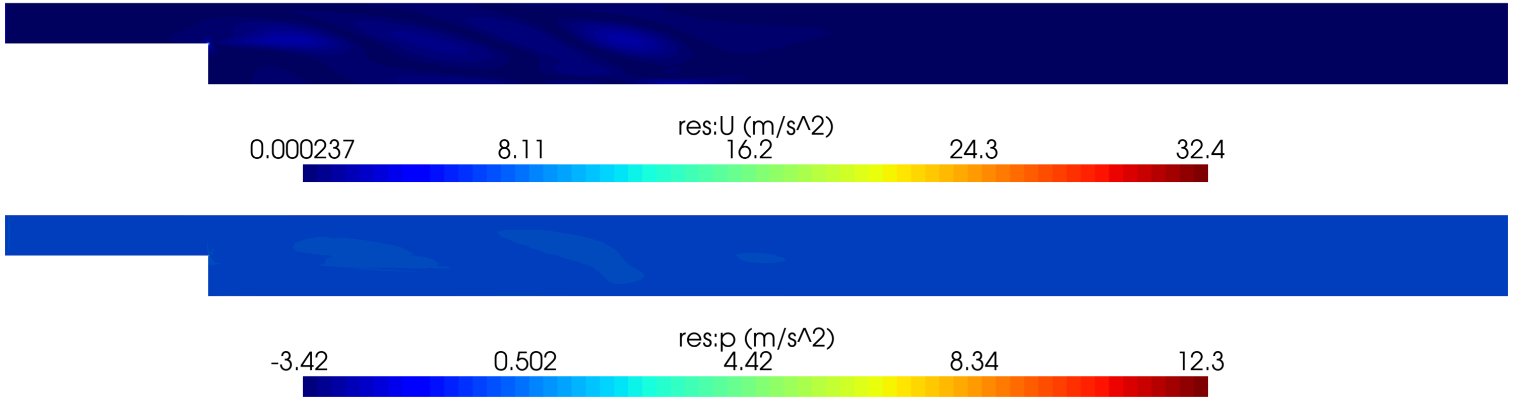


Re: 50 - 450

Resmin

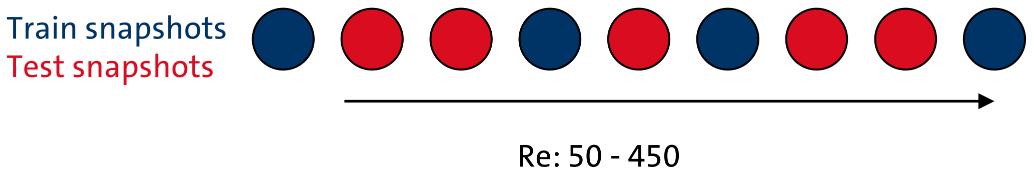


Projected



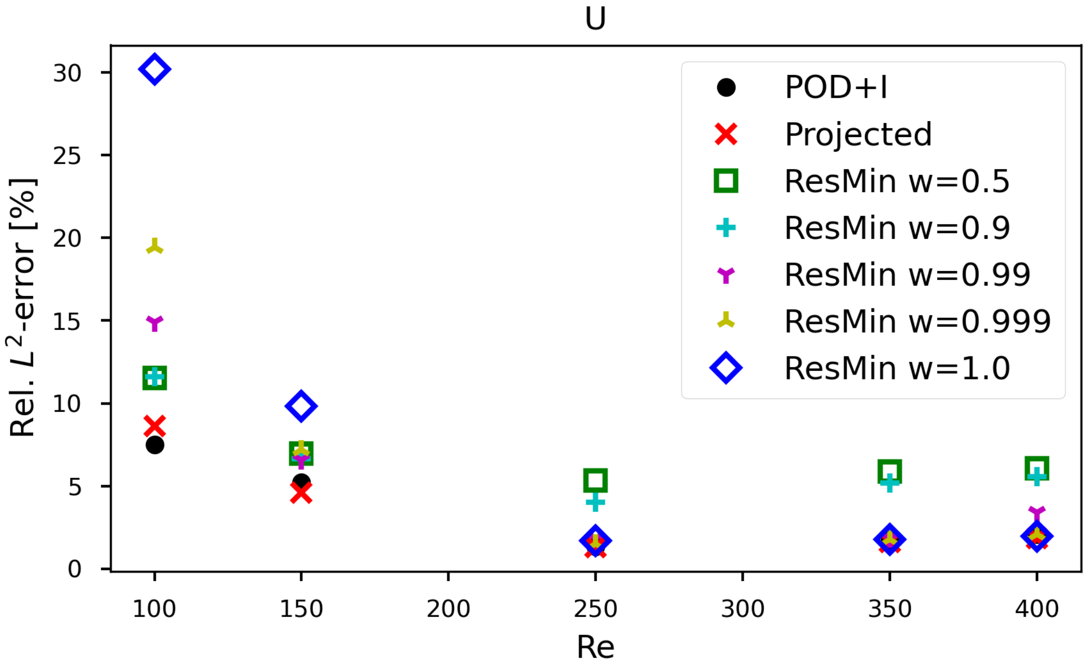
Backward Facing Step - Varying Re

Weighing of Boundary and Bulk Residuals

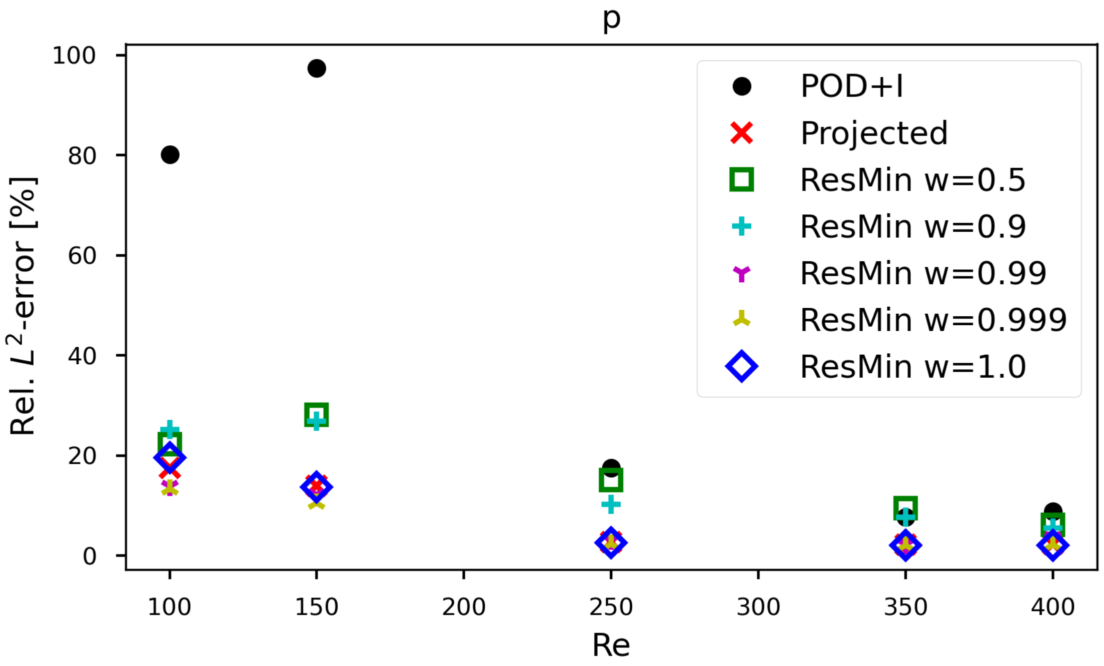


$$\mathbf{f}(\tilde{x}) = w\, \mathbf{f}(\tilde{x})_{boundary} + (1-w)\, \mathbf{f}(\tilde{x})_{bulk}$$

Error Velocity (Ux and Uy)



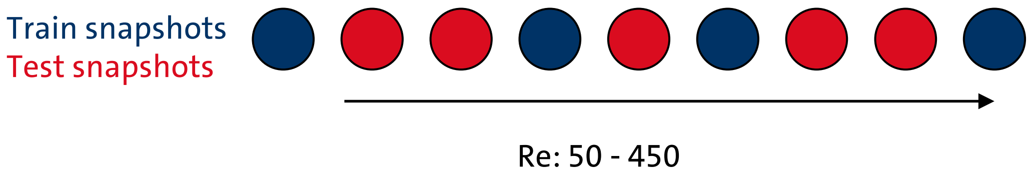
Error Pressure



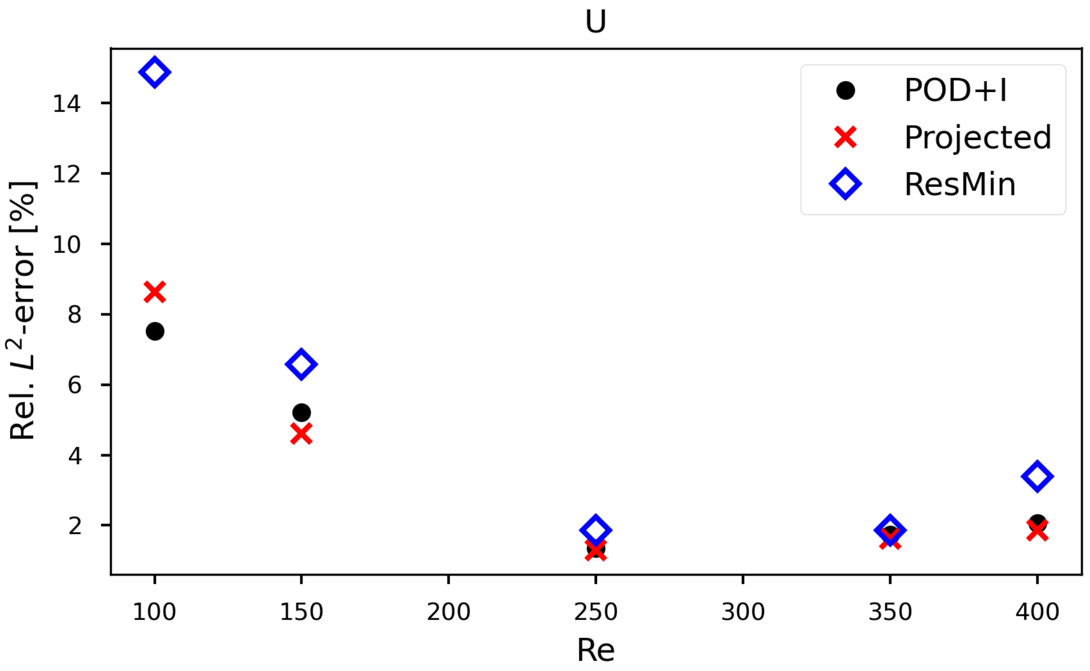
VOLKSWAGEN GROUP

Backward Facing Step - Varying Re

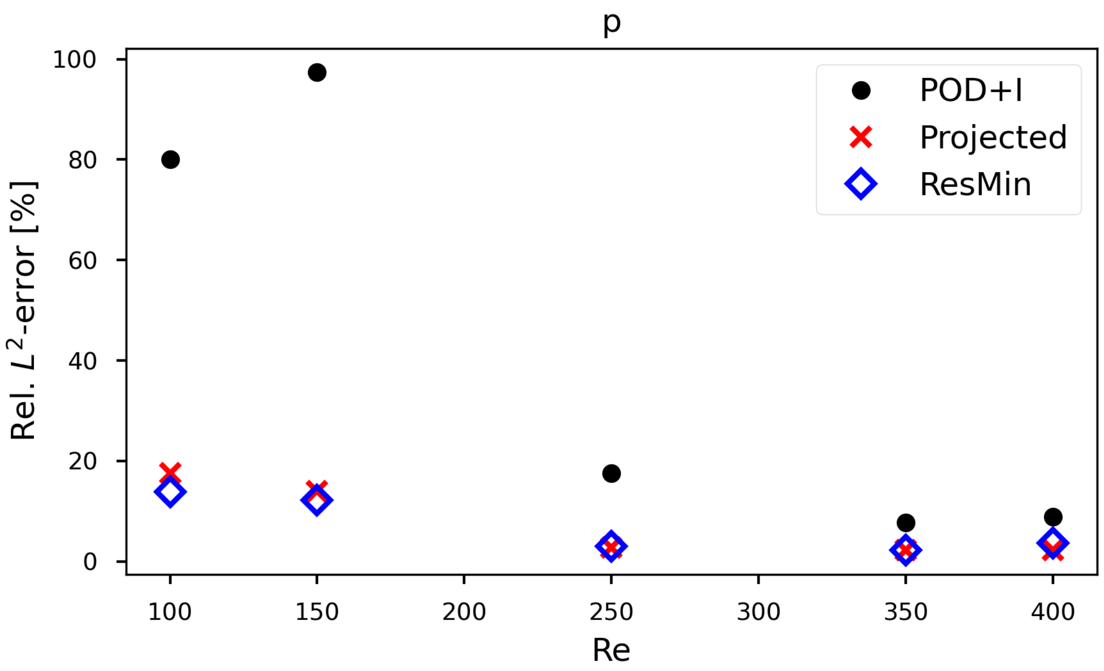
Weighing of Boundary and Bulk Residuals - w=0.99



Error Velocity (Ux and Uy)

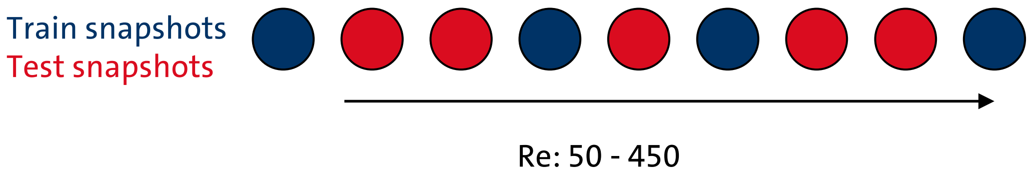


Error Pressure

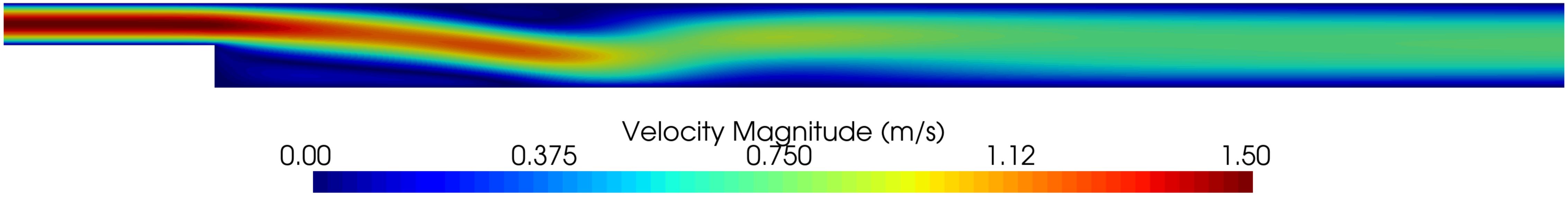


Backward Facing Step - Varying Re

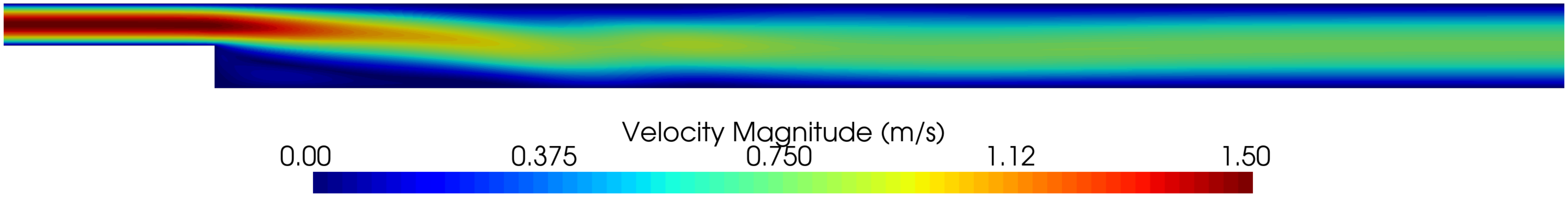
w=0.99, Re=100



Resmin

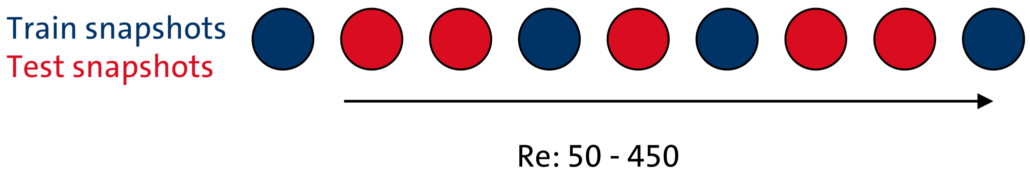


Projected

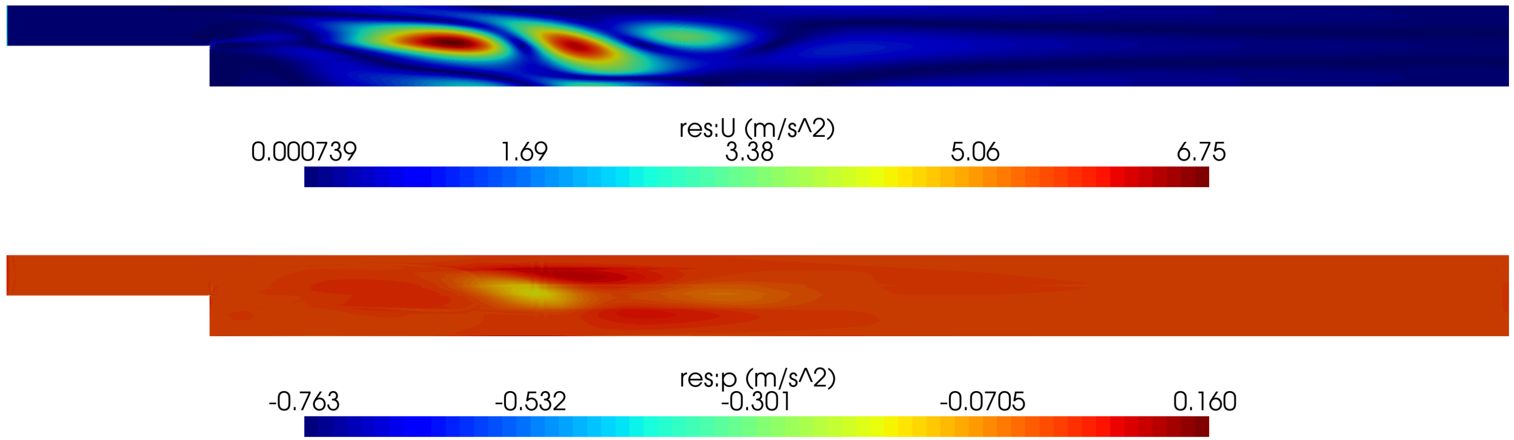


Backward Facing Step - Varying Re

w=0.99, Re=100



Resmin



Projected