Approximate Deconvolution Leray Reduced Order Model

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Galerkin reduced order models (G-ROMs) are advanced techniques that efficiently reduce the complexity of full order models (FOMs) derived from traditional numerical methods like the finite element method (FEM). G-ROMs have successfully reduced computational costs in simulating laminar fluid flows governed by the Navier-Stokes equations (NSE). However, in cases where the number of ROM degrees of freedom is too low to capture complex flow dynamics, known as under-resolved regimes, G-ROMs can produce inaccuracies, often seen as numerical oscillations.

To stabilize G-ROMs for turbulent flows in under-resolved regime, regularized ROMs (Reg-ROMs) use filtering techniques to smooth out terms in the NSE. A classical example, the Leray ROM (L-ROM), replaces the nonlinear term with a filtered velocity term, enhancing stability and accuracy. However, the L-ROM can become over-diffusive when the filter radius is too large, resulting in excessive dissipation and sensitivity to small variations in the filter radius.

To address all these limitations, we propose a new type of Reg-ROM, the approximate deconvolution Leray ROM (ADL-ROM) [2]. This new approach incorporates the technique of approximate deconvolution, a strategy widely used in image processing and inverse problems, to enhance the accuracy and reduce the sensitivity of the L-ROM. Specifically, in the ADL-ROM, the filtered velocity in the L-ROM is replaced with an approximately deconvoluted velocity in the nonlinear term, thereby improving the model's performance.

This talk aims to present and analyze this novel Reg-ROM, the ADL-ROM. In the first part of the talk, I will introduce the well-known L-ROM and elaborate on the development and mechanics of our new ADL-ROM. This includes a detailed discussion of how the approximate deconvolution technique is applied to the L-ROM framework and the theoretical foundations that support its use. I will also provide a detailed commentary and comparison of these Reg-ROMs and the standard G-ROM. This will include presenting numerical results obtained from the ADL-ROM, highlighting its superior performance in various test cases. Additionally, I will thoroughly analyze the associated errors, examining how the ADL-ROM mitigates issues like over-diffusivity and sensitivity to filter radius perturbations.

The second part of the talk will offer some insights into our theoretical work [1], where we are providing rigorous numerical analysis results, such as stability and convergence, for the ADL-ROM. This theoretical analysis includes error bounds for both the AD operators and the ADL-ROM itself, along with numerical experiments demonstrating the accuracy and stability improvements of ADL-ROM over traditional methods. Results indicate that ADL-ROM provides superior flow control with lower computational costs, validated through various test cases. The presentation concludes with discussions on future directions, emphasizing ADL-ROM's potential for broader applications in complex fluid dynamics modelling.

References

[1] I. Moore, A. Sanfilippo, F. Ballarin, and T. Iliescu. Numerical Analysis of the Approximate Deconvolution Leray Reduced Order Model. *arXiv preprint, https://arxiv.org/abs/2410.02673*, 2024.

[2] A. Sanfilippo, I. Moore, F. Ballarin, and T. Iliescu. Approximate deconvolution Leray reduced order model. *Finite Elem. Anal. Des.*, 226, 2023.