Recent progress on genuinely multidimensional and steady state preserving discretizations for hyperbolic balance laws

Abstract. In this talk I will review some recent work on the design of discretizations of hyperbolic balance laws. We aim at embedding in the schemes a genuinely multidimensional nature, and possibly a stationary preserving (SP) property. The first aspect requires going beyond the classical definition of numerical fluxes, and allow the use of fluxes which depend on three or more states. The second property is more delicate and requires on one hand the numerical dissipation to have the appropriate structure, and on the other to share with the consistent part of the scheme to share a discrete kernel which is non-trivial. We review these conditions, and propose a framework to design SP methods based on the introduction of pseudo-potentials given by primitive functions of the fluxes and of the source terms. We apply the method to the construction of an SP variant of SUPG which is shown to outperform the classical method on a variety of linear and non-linear problems involving both multidimensional stationary states, and low Mach perturbations.