EIGENPAIRS OF THE CURL OPERATOR ON AXISYMMETRIC TORI

MONIQUE DAUGE

Let Ω be a bounded three-dimensional domain. The main subtlety of the curl eigenproblem

$$(*) \qquad \mathbf{curl}\, \boldsymbol{u} = \kappa \boldsymbol{u} \quad \text{in} \quad \Omega$$

is to find the right complementary conditions so that $(*)$ can be interpreted as the eigenequation associated with an unbounded self-adjoint operator. The gauge condition $\operatorname{div} \boldsymbol{u} = 0$ has to be added, and, roughly, the right boundary condition is $\boldsymbol{u} \cdot \boldsymbol{n} = 0$ on $\partial\Omega$, which is sufficient if Ω is simply connected, in contrast with the more interesting case when Ω has a non-trivial homotopy group. We found various elements of answer in [2, 1].

For a same domain Ω a set of distinct boundary conditions can be considered: They differ from each other by a finite dimensional space of circulation conditions on a set of mutually dual cycles. The theoretical considerations and the numerical methods presented in [1] raise a couple of intriguing questions.

(1) How does the choice of cycles influence the boundary conditions?
(2) Does there exist a continuous choice of suitable boundary conditions?
(3) How do eigenvalues depend on boundary conditions? Is there only a finite number of them that are modified?

In this talk, we address the particular situation when Ω is axisymmetric, which allows for a scalar reduction of problem $(*)$ and a diagonalization of the eigenproblems along angular frequencies.

From a work in progress with Martin COSTABEL and Yvon LAFRANCHE.

REFERENCES

[1] A. ALONSO RODRÍGUEZ, J. CAMAÑO, R. RODRÍGUEZ, A. VALLI, P. VENEGAS, *Finite element approximation of the spectrum of the curl operator in a multiply-connected domain*. UDEC preprint (2016).

[2] R. HIPTMAIR, P. R. KOTIUGA, AND S. TORDEUX, *Self-adjoint curl operators*. Ann. Mat. Pura Appl. (4), 191 (2012), pp. 431–457.