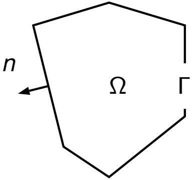
First kind boundary integral formulation for the Hodge-Helmholtz equation

X.Claeys* & R.Hiptmair†

* LJLL UPMC / INRIA Alpines,
† SAM ETH Zürich

Integral equation for low frequency Maxwell

Ω = bounded Lipschitz, $\Gamma = \partial\Omega$



Find $\boldsymbol{E} \in \mathbf{H}(\mathbf{curl}, \Omega)$ such that
$\mathbf{curl}(\mathbf{curl}\,\boldsymbol{E}) - \kappa^2\boldsymbol{E} = 0$ in Ω
$\boldsymbol{n} \times (\boldsymbol{E}|_\Gamma \times \boldsymbol{n}) = \boldsymbol{g}$ on Γ.

Boundary integral formulation ? at low frequency ?

Electric field integral equation (EFIE) : find $\boldsymbol{u} \in \mathrm{H}^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$ such that

$$\int_{\Gamma\times\Gamma} \mathscr{G}_\kappa(\boldsymbol{x}-\boldsymbol{y})\big(\ \kappa^{-2}\mathrm{div}_\Gamma\boldsymbol{v}(\boldsymbol{x})\mathrm{div}_\Gamma\boldsymbol{u}(\boldsymbol{y}) - \boldsymbol{v}(\boldsymbol{x})\cdot\boldsymbol{u}(\boldsymbol{y})\ \big)d\sigma(\boldsymbol{x},\boldsymbol{y})$$
$$= \int_\Gamma \boldsymbol{g}(\boldsymbol{x})\cdot\boldsymbol{v}(\boldsymbol{x})d\sigma(\boldsymbol{x}) \qquad \forall \boldsymbol{v} \in \mathrm{H}^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$$

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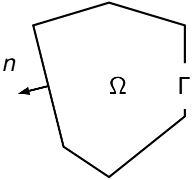

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Green kernel : $\mathscr{G}_\kappa(\boldsymbol{x}) := \exp(\imath\kappa|\boldsymbol{x}|)/(4\pi|\boldsymbol{x}|)$

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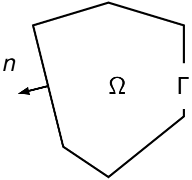

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Low frequency breakdown

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Loop-star/tree stabilisation : Wilton & Glisson (1981), Vecchi (1999), Zhao & Chew (2000), Lee & Burkholder (2003), Eibert (2004), Andriulli (2012).

Debye sources : Greengard & Epstein (2010), Greengard, Epstein & O'Neil (2013 & 2015), Vico, Ferrando, Greengard & Gimbutas (2016).

Current and charge formulation : Taskinen & Ylä-Oijala (2006) Taskinen & Vanska (2007), Taskinen (2009), Bendali & al. (2012), Vico & al. (2013), Ganesh, Hawkins & Volkov (2014).

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Current and charge formulations have been shown to stem from Maxwell's equations by relaxing the contraint associated to divergence by means of Lagrange multipliers. The later formulation of electromagnetics is known as Picard's system (introduced by Picard, 1984).

Goal of this work : clarifying the analysis of this approach (Calderón calculus ? traces ? well-posedness ? etc. . .)

Initial idea : rewrite Picard's system as a 2nd order problem and try to adapt the analysis presented in Costabel (1988).

Outline

I. Maxwell as vector Helmholtz equation

II. Hodge-Helmholtz potential theory

III. Low frequency regime

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I. **Maxwell as vector Helmholtz equation**

II. Hodge-Helmholtz potential theory

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Reformulation of Maxwell's BVP

Proposition (Hazard & Lenoir, 1996) : Let $\Omega \subset \mathbb{R}^3$ be bounded Lipschitz, and assume that $\kappa^2 \notin \mathfrak{S}(-\Delta_{\mathrm{Dir}})$, then for any $\boldsymbol{g} \in \mathbf{H}^{-1/2}(\mathbf{curl}_\Gamma, \Gamma)$, we have

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Green's formula

$$\int_\Omega \mathbf{curl}^2(\boldsymbol{u}) \cdot \boldsymbol{v} - \boldsymbol{u} \cdot \mathbf{curl}^2(\boldsymbol{v})\, d\boldsymbol{x} = \int_\Gamma \boldsymbol{v} \cdot (\boldsymbol{n} \times \mathbf{curl}\, \boldsymbol{u}) - \boldsymbol{u} \cdot (\boldsymbol{n} \times \mathbf{curl}\, \boldsymbol{v})\, d\sigma$$

$$\int_\Omega \nabla(\mathrm{div} \boldsymbol{v}) \cdot \boldsymbol{u} - \boldsymbol{v} \cdot \nabla(\mathrm{div} \boldsymbol{u})\, d\boldsymbol{x} = \int_\Gamma \mathrm{div}(\boldsymbol{v})\, \boldsymbol{n} \cdot \boldsymbol{u} - \mathrm{div}(\boldsymbol{u})\, \boldsymbol{n} \cdot \boldsymbol{v}\, d\sigma$$

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"Neumann trace" "Dirichlet trace"

$\mathcal{T}_{\mathrm{N}}(\boldsymbol{u}) := (\boldsymbol{n} \times \mathbf{curl}(\boldsymbol{u}), \boldsymbol{n} \cdot \boldsymbol{u})$ $\mathcal{T}_{\mathrm{D}}(\boldsymbol{u}) := (\boldsymbol{n} \times \boldsymbol{u} \times \boldsymbol{n}, \mathrm{div}(\boldsymbol{u}))$

Green's formula for the Hodge Laplacian

$$\int_\Omega \boldsymbol{u}\cdot\Delta\boldsymbol{v} - \boldsymbol{v}\cdot\Delta\boldsymbol{u}\, d\boldsymbol{x} = \int_\Gamma \mathcal{T}_{\mathsf{D}}(\boldsymbol{v})\cdot\mathcal{T}_{\mathsf{N}}(\boldsymbol{u}) - \mathcal{T}_{\mathsf{D}}(\boldsymbol{u})\cdot\mathcal{T}_{\mathsf{N}}(\boldsymbol{v})\, d\sigma$$

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Proposition

Denote $\mathbf{X}(\Delta,\Omega) := \{\boldsymbol{u}\in\mathbf{X}(\Omega),\ \mathbf{curl}^2(\boldsymbol{u})\in \mathrm{L}^2(\Omega)^3,\ \nabla(\mathrm{div}\,\boldsymbol{u})\in\mathrm{L}^2(\Omega)^3\}$. Then the following trace operators are continuous, surjective, and admit a continuous right inverse,

$$\mathcal{T}_{\mathsf{D}} : \mathbf{X}(\Delta,\Omega)\to\mathcal{H}_{\mathsf{D}}(\Gamma) := \mathrm{H}^{-\frac{1}{2}}(\mathrm{div}_\Gamma,\Gamma)\times\mathrm{H}^{+\frac{1}{2}}(\Gamma)$$

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This choice for for Dirichlet and Neumann traces is apparently the only one that garantees good continuity/surjectivity properties. It was also considered in [Mitrea& al, 2016] and [Schwarz, 1995].

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Important :

$$\int_\Gamma \mathcal{T}_{\mathrm{D}}(\boldsymbol{v})\cdot\mathcal{T}_{\mathrm{N}}(\boldsymbol{u}) \neq \int_\Omega \mathbf{curl}(\boldsymbol{u})\cdot\mathbf{curl}(\boldsymbol{v}) + \mathrm{div}(\boldsymbol{u})\mathrm{div}(\boldsymbol{v})d\boldsymbol{x} \pm \int_\Omega \boldsymbol{v}\cdot\Delta\boldsymbol{u}d\boldsymbol{x}$$

which makes using Costabel's variational analysis (apparently) impossible.

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Repesentation formula

Green's formula may be rewritten as $\Delta 1_\Omega - 1_\Omega \Delta = \mathcal{T}_{\mathsf{D}}' \mathcal{T}_{\mathsf{N}} - \mathcal{T}_{\mathsf{N}}' \mathcal{T}_{\mathsf{D}}$ in the sense of distributions. Hence for any $\boldsymbol{u} \in \mathbf{X}(\Omega)$ satisfying $\Delta \boldsymbol{u} + \kappa^2 \boldsymbol{u} = 0$ in Ω, we have

$$-(\Delta + \kappa^2) 1_\Omega \boldsymbol{u} = \mathcal{T}_{\mathsf{N}}' \mathcal{T}_{\mathsf{D}}(\boldsymbol{u}) - \mathcal{T}_{\mathsf{D}}' \mathcal{T}_{\mathsf{N}}(\boldsymbol{u})$$

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$$\mathscr{G}_\kappa * \big(\quad - (\Delta + \kappa^2) 1_\Omega \boldsymbol{u} = \mathcal{T}'_{\mathrm{N}}\mathcal{T}_{\mathrm{D}}(\boldsymbol{u}) - \mathcal{T}'_{\mathrm{D}}\mathcal{T}_{\mathrm{N}}(\boldsymbol{u}) \quad \big)$$

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$$\mathcal{DL}_\kappa(\mathcal{T}_\mathsf{D}(\boldsymbol{u}))(\boldsymbol{x}) + \mathcal{SL}_\kappa(\mathcal{T}_\mathsf{N}(\boldsymbol{u}))(\boldsymbol{x}) = 1_\Omega(\boldsymbol{x})\boldsymbol{u}(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in \mathbb{R}^3$$

$$\text{where} \quad \begin{aligned} \mathcal{DL}_\kappa &:= +\mathscr{G}_\kappa * \mathcal{T}_\mathsf{N}' : \mathcal{H}_\mathsf{D}(\Gamma) \to \mathbf{X}(\Delta, \Omega) \\ \mathcal{SL}_\kappa &:= -\mathscr{G}_\kappa * \mathcal{T}_\mathsf{D}' : \mathcal{H}_\mathsf{N}(\Gamma) \to \mathbf{X}(\Delta, \Omega) \end{aligned}$$

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Explicit expression

$$\mathcal{DL}_\kappa(\boldsymbol{q}, \beta)(\boldsymbol{x}) := \int_\Gamma (\nabla \mathscr{G}_\kappa)(\boldsymbol{x} - \boldsymbol{y}) \times (\boldsymbol{q}(\boldsymbol{y}) \times \boldsymbol{n}(\boldsymbol{y})) + \mathscr{G}_\kappa(\boldsymbol{x} - \boldsymbol{y})\boldsymbol{n}(\boldsymbol{y})\beta(\boldsymbol{y})\, d\sigma(\boldsymbol{y})$$

$$\mathcal{SL}_\kappa(\boldsymbol{p}, \alpha)(\boldsymbol{x}) := \int_\Gamma -\mathscr{G}_\kappa(\boldsymbol{x} - \boldsymbol{y})\boldsymbol{p}(\boldsymbol{y}) + (\nabla \mathscr{G})(\boldsymbol{x} - \boldsymbol{y})\alpha(\boldsymbol{y})\, d\sigma(\boldsymbol{y})$$

No blow up at low frequency as the kernel $\mathscr{G}_\kappa(\boldsymbol{x}) \to \mathscr{G}_0(\boldsymbol{x})$ remains bounded for $\kappa \to 0$.

Jump formula and Calderón's operator

Proposition : for $\star = \mathsf{D}, \mathsf{N}$, denoting $\mathcal{T}_{\star,c}$ = exterior traces, and $[\mathcal{T}_\star] := \mathcal{T}_\star - \mathcal{T}_{\star,c}$, we have

$$[\mathcal{T}_{\mathsf{D}}] \cdot \mathcal{DL}_\kappa = \mathrm{Id}, \quad [\mathcal{T}_{\mathsf{D}}] \cdot \mathcal{SL}_\kappa = 0,$$
$$[\mathcal{T}_{\mathsf{N}}] \cdot \mathcal{DL}_\kappa = 0, \quad [\mathcal{T}_{\mathsf{N}}] \cdot \mathcal{SL}_\kappa = \mathrm{Id}.$$

Proposition : the matrix of continuous boundary integral operators

$$\mathcal{C} := \begin{bmatrix} \mathcal{T}_{\mathsf{D}} \cdot \mathcal{DL}_\kappa & \mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_\kappa \\ \mathcal{T}_{\mathsf{N}} \cdot \mathcal{DL}_\kappa & \mathcal{T}_{\mathsf{N}} \cdot \mathcal{SL}_\kappa \end{bmatrix}$$

is a projector $\mathcal{C}^2 = \mathcal{C}$ mapping $\mathcal{H}_{\mathsf{D}}(\Gamma) \times \mathcal{H}_{\mathsf{N}}(\Gamma) \to \mathcal{H}_{\mathsf{D}}(\Gamma) \times \mathcal{H}_{\mathsf{N}}(\Gamma)$. For $\boldsymbol{u} \in \mathbf{X}(\Delta, \Omega)$ we have $\Delta \boldsymbol{u} + \kappa^2 \boldsymbol{u} = 0$ in $\Omega \iff (\mathcal{T}_{\mathsf{D}}(\boldsymbol{u}), \mathcal{T}_{\mathsf{N}}(\boldsymbol{u})) \in \mathrm{Range}(\mathcal{C})$.

Proposition : Each of the four entries of the Calderón projector $\mathcal{C}$ is an invertible operator, unless κ^2 is an eigenvalue of Δ in Ω.

First kind boundary integral operators

In [Mitrea & al, 2016], focus was on the BIOs of the second kind $\mathcal{T}_{\mathsf{D}} \cdot \mathcal{DL}_{\kappa}$ and $\mathcal{T}_{\mathsf{N}} \cdot \mathcal{SL}_{\kappa}$. Here, we focus on integral operators of the first kind. They admit the following variational form.

$$\left\langle \mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_{\kappa} \begin{pmatrix} \boldsymbol{p} \\ \alpha \end{pmatrix}, \begin{pmatrix} \boldsymbol{q} \\ \beta \end{pmatrix} \right\rangle = -\int_{\Gamma\times\Gamma} \mathscr{G}_{\kappa}(\boldsymbol{x}-\boldsymbol{y})[\, \boldsymbol{p}(\boldsymbol{y}) \cdot \boldsymbol{q}(\boldsymbol{x}) + \alpha(\boldsymbol{y}) \mathrm{div}_{\Gamma} \boldsymbol{q}(\boldsymbol{x}) \,] d\sigma$$
$$- \int_{\Gamma\times\Gamma} \mathscr{G}_{\kappa}(\boldsymbol{x}-\boldsymbol{y})[\, \mathrm{div}_{\Gamma} \boldsymbol{q}(\boldsymbol{y}) \beta(\boldsymbol{x}) + \kappa^2 \alpha(\boldsymbol{y}) \beta(\boldsymbol{x}) \,] d\sigma$$

and denoting $\boldsymbol{p}_{\times}(\boldsymbol{y}) = \boldsymbol{n}(\boldsymbol{y}) \times \boldsymbol{p}(\boldsymbol{y})$,

$$\left\langle \mathcal{T}_{\mathsf{N}} \cdot \mathcal{DL}_{\kappa} \begin{pmatrix} \boldsymbol{p} \\ \alpha \end{pmatrix}, \begin{pmatrix} \boldsymbol{q} \\ \beta \end{pmatrix} \right\rangle =$$
$$- \int_{\Gamma\times\Gamma} \mathscr{G}_{\kappa}(\boldsymbol{x}-\boldsymbol{y})[\, \mathrm{div}_{\Gamma} \boldsymbol{q}_{\times}(\boldsymbol{x}) \mathrm{div}_{\Gamma} \boldsymbol{p}_{\times}(\boldsymbol{y}) - \kappa^2 \boldsymbol{q}_{\times}(\boldsymbol{x}) \cdot \boldsymbol{p}_{\times}(\boldsymbol{y}) \,] d\sigma$$
$$+ \int_{\Gamma\times\Gamma} \mathscr{G}_{\kappa}(\boldsymbol{x}-\boldsymbol{y})[\, \boldsymbol{q}_{\times}(\boldsymbol{x}) \cdot (\boldsymbol{n}(\boldsymbol{y}) \times \nabla_{\Gamma} \alpha(\boldsymbol{y})) + \boldsymbol{p}_{\times}(\boldsymbol{y}) \cdot (\boldsymbol{n}(\boldsymbol{x}) \times \nabla_{\Gamma} \beta(\boldsymbol{x})) \,] d\sigma$$
$$+ \int_{\Gamma\times\Gamma} \mathscr{G}_{\kappa}(\boldsymbol{x}-\boldsymbol{y}) \alpha(\boldsymbol{y}) \beta(\boldsymbol{x}) \boldsymbol{n}(\boldsymbol{y}) \cdot \boldsymbol{n}(\boldsymbol{x}) \, d\sigma$$

Garding's inequality

A classical tool for proving Garding's inequality for Maxwell related operators (see e.g. [Buffa & Hiptmair, 2002]) is the existence of a projector $\mathrm{Q} : \mathbf{H}^{-1/2}(\mathrm{div}_\Gamma, \Gamma) \to \mathbf{H}^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$ such that

- Q maps continuously into $\mathbf{H}^{1/2}_{\mathrm{R}}(\Gamma) := \{\boldsymbol{n} \times \boldsymbol{u}|_\Gamma,\ \boldsymbol{u} \in \mathrm{H}^1(\Omega)^3\}$
- $\ker(\mathrm{Q}) = \{\boldsymbol{u} \in \mathbf{H}^{-1/2}(\mathrm{div}_\Gamma, \Gamma),\ \mathrm{div}_\Gamma(\boldsymbol{u}) = 0\}$.

Define an involution $\Theta^2 = \mathrm{Id}$ by

$$\Theta(\begin{bmatrix} \boldsymbol{p} \\ \alpha \end{bmatrix}) := \begin{bmatrix} \boldsymbol{p} \\ \alpha \end{bmatrix} - 2 \begin{bmatrix} \mathrm{Q}(\boldsymbol{p}) \\ \kappa^{-2}\mathrm{div}_\Gamma(\boldsymbol{p}) \end{bmatrix}.$$

Theorem : For any $\kappa \in \mathbb{R}_+$, there exists a constant $c(\kappa) > 0$ and a compact operator $\mathrm{K} : \mathcal{H}_{\mathrm{N}}(\Gamma) \to \mathcal{H}_{\mathrm{N}}(\Gamma)$ such that

$$\Re e\{\langle(\mathcal{T}_{\mathrm{D}} \cdot \mathcal{SL}_\kappa + \mathrm{K}_\kappa)\mathrm{u}, \Theta(\overline{\mathrm{u}})\rangle\} \geq c(\kappa)\|\mathrm{u}\|^2_{\mathcal{H}_{\mathrm{N}}(\Gamma)} \quad \forall \mathrm{u} \in \mathcal{H}_{\mathrm{N}}(\Gamma).$$

Remarks :

- Analogous result holds for $\mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_\kappa$
- The coercivity constant depends on κ

Outline

I. Maxwell as vector Helmholtz equation

II. Hodge-Helmholtz potential theory

III. Low frequency regime

Explicit expression of low frequency operators

We are particularly interested in studying the operators at vanishing frequency.

$$\left\langle \mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_0 \begin{pmatrix} \boldsymbol{p} \\ \alpha \end{pmatrix}, \begin{pmatrix} \boldsymbol{q} \\ \beta \end{pmatrix} \right\rangle =$$
$$-\int_{\Gamma\times\Gamma} \mathscr{G}_0(\boldsymbol{x}-\boldsymbol{y})[\,\boldsymbol{p}(\boldsymbol{y})\cdot\boldsymbol{q}(\boldsymbol{x}) + \alpha(\boldsymbol{y})\mathrm{div}_\Gamma\boldsymbol{q}(\boldsymbol{x}) + \beta(\boldsymbol{x})\mathrm{div}_\Gamma\boldsymbol{q}(\boldsymbol{y})\,]d\sigma$$

Unfortunately this operator admits a finite dimensional but systematically non trivial kernel. Indeed we have.

Proposition : We have $\mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_0(\boldsymbol{p}, \alpha) = 0$ if and only if $\boldsymbol{p} = 0$ and $\nabla_\Gamma(\,\int_\Gamma \alpha(\boldsymbol{y})d\sigma(\boldsymbol{y})/|\boldsymbol{x}-\boldsymbol{y}|\,) = 0$, so that

$$\begin{aligned}\dim\ker(\mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_0) &= \#\{\,\text{connected components of } \Gamma\,\}\\ &= \text{1st Betti number of } \Gamma.\end{aligned}$$

Proposition : Any element $(\boldsymbol{p}, \alpha) = (0, \alpha) \in \ker(\mathcal{T}_{\mathsf{D}} \cdot \mathcal{SL}_0)$ satisfying $\int_\Gamma \alpha\beta d\sigma = 0$ for all $\beta \in \ker(\nabla_\Gamma) = \{$ locally constants $\}$ vanishes $\alpha = 0$.

Explicit expression of low frequency operators

Proposition : We have $\mathcal{T}_{\mathrm{D}} \cdot \mathcal{SL}_0(\boldsymbol{p}, \alpha) = 0$ if and only if $\boldsymbol{p} = 0$ and $\nabla_\Gamma\big(\int_\Gamma \alpha(\boldsymbol{y}) d\sigma(\boldsymbol{y})/|\boldsymbol{x} - \boldsymbol{y}| \big) = 0$, so that

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Proposition : Any element $(\boldsymbol{p}, \alpha) = (0, \alpha) \in \ker(\mathcal{T}_{\mathrm{D}} \cdot \mathcal{SL}_0)$ satisfying $\int_\Gamma \alpha\beta d\sigma = 0$ for all $\beta \in \ker(\nabla_\Gamma) = \{$ locally constants $\}$ vanishes $\alpha = 0$.

Regularised formulation : elements of the kernel can be filtered out imposing constraints/Lagrange parameters, which leads to the following saddle point problem.

Find $\mathfrak{u} = (\boldsymbol{p}, \alpha) \in \mathcal{H}_{\mathrm{N}}(\Gamma)$ and $\mu \in \ker(\nabla_\Gamma)$ such that
$\langle \mathcal{T}_{\mathrm{D}} \cdot \mathcal{SL}_0(\mathfrak{u}), \mathfrak{v} \rangle + \int_\Gamma \mu\beta \, d\sigma = \langle \mathfrak{f}, \mathfrak{v} \rangle \quad \forall \mathfrak{v} = (\boldsymbol{q}, \beta) \in \mathcal{H}_{\mathrm{N}}(\Gamma)$
$\int_\Gamma \lambda\alpha \, d\sigma = 0 \quad \forall \lambda \in \ker(\nabla_\Gamma)$

Explicit expression of low frequency operators

Things are somehow more involved for $\mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_0$. Denote $\boldsymbol{p}_\times := \boldsymbol{n} \times \boldsymbol{p}$. The variationnal form of this operator is

$$\left\langle \mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_0 \begin{pmatrix} \boldsymbol{p} \\ \alpha \end{pmatrix}, \begin{pmatrix} \boldsymbol{q} \\ \beta \end{pmatrix} \right\rangle =$$

$$\int_{\Gamma\times\Gamma} \mathscr{G}_0(\boldsymbol{x}-\boldsymbol{y})[\, \alpha(\boldsymbol{y})\beta(\boldsymbol{x})\boldsymbol{n}(\boldsymbol{y}) \cdot \boldsymbol{n}(\boldsymbol{x}) - \mathrm{div}_\Gamma \boldsymbol{p}_\times(\boldsymbol{y}) \mathrm{div}_\Gamma \boldsymbol{q}_\times(\boldsymbol{x}) \,] d\sigma(\boldsymbol{x},\boldsymbol{y})$$

$$- \int_{\Gamma\times\Gamma} \mathscr{G}_0(\boldsymbol{x}-\boldsymbol{y})[\, \boldsymbol{q}_\times(\boldsymbol{x}) \cdot \mathbf{curl}_\Gamma \alpha(\boldsymbol{y}) + \boldsymbol{p}_\times(\boldsymbol{y}) \cdot \mathbf{curl}_\Gamma \beta(\boldsymbol{x}) \,] d\sigma(\boldsymbol{x},\boldsymbol{y})$$

Proposition : We have $\mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_0(\boldsymbol{p}, \alpha)$ if and only if $\alpha = 0$ and $\mathbf{curl}_\Gamma \boldsymbol{p} = 0$ and $\mathbf{curl}_\Gamma\big(\int_\Gamma \boldsymbol{n}(\boldsymbol{y}) \times \boldsymbol{p}(\boldsymbol{y}) d\sigma(\boldsymbol{y})/|\boldsymbol{x}-\boldsymbol{y}| \,\big) = 0$, and we have

$$\begin{aligned} \dim \ker(\mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_\kappa) &= \#\{\text{ non-bounding cycles on } \Gamma \,\} \\ &= \text{2nd Betti number of } \Gamma. \end{aligned}$$

Proposition : Any element $(\boldsymbol{p}, \alpha) = (\boldsymbol{p}, 0) \in \ker(\mathcal{T}_{\mathrm{N}} \cdot \mathcal{DL}_0)$ satisfying $\int_\Gamma \boldsymbol{p} \cdot \boldsymbol{q} d\sigma = 0$ for all $\boldsymbol{q} \in \ker(\mathbf{curl}_\Gamma) \cap \ker(\mathrm{div}_\Gamma)$ vanishes $\boldsymbol{p} = 0$.

Conclusion

Summary
We have devised continuity, well-posedness and coercivity analysis for layer potentials associated to the Hodge-Helmholtz in Lipschitz domains. These can be used to reformulate Maxwell's equations and they remain bounded at low frequency.

Submitted : X. Claeys and R. Hiptmair, *First kind boundary integral formulation for the Hodge-Helmholtz equation*, SAM ETHZ report available.

Questions
• Discrete inf-sup condition holding uniformly at low frequency (forthcoming)
• Numerical experiments (work in progress with E.Demaldent CEA List)

• Transmission problems ?
• Effective stabilisation strategy at low frequency ?
• Precise connections with other existing approaches ?
• Compuation of nonbounding cylces ?
• Formulation remaining well conditionned for wide frequency range ?

IABEM 2018

Symposium of the International Association for Boundary Element Methods

What ?

International conference focused on boundary integral equations

Both theory and application oriented

Where ?

University Pierre-et-Marie Curie (Paris 6)

When ?

June 26-28, 2018

Website

`https://project.inria.fr/iabem2018/`

Thank you

for your attention