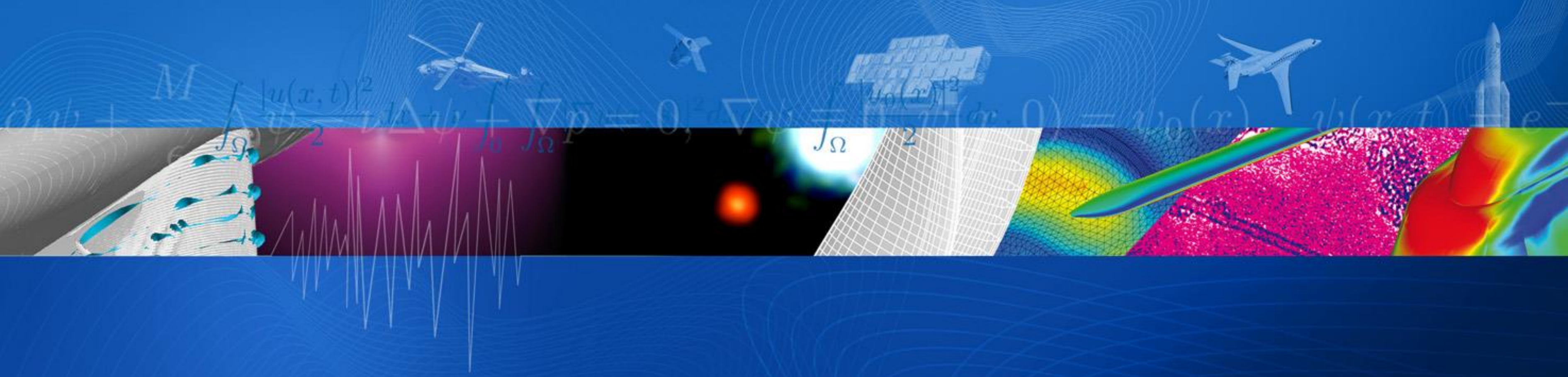

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Mathematical tools for impedance models of sound-absorbing materials in aeronautics

Estelle Piot

12/12/2017

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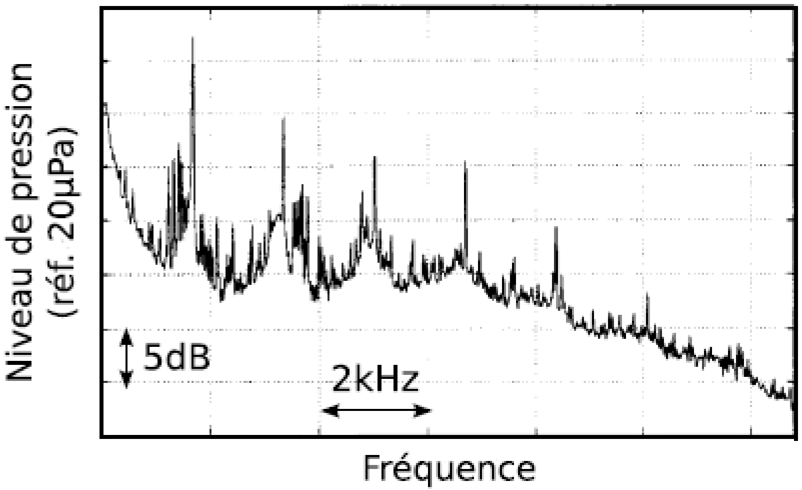
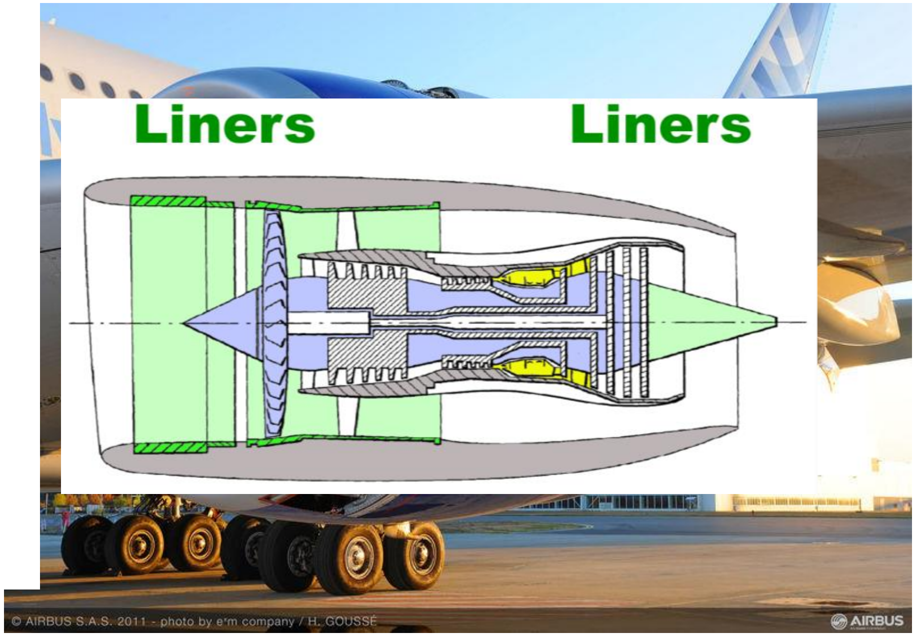
Outline

- Acoustic liners in aeronautics: some context
- Mathematical issues for liner impedance models
  - Impedance and homogeneization ?
  - Impedance eduction ?
  - Time domain impedance boundary condition ?
- Perspectives

Acoustic liners in aeronautics: some context

- Turbofan noise:
  - Broadband noise (50 Hz – 10 kHz)
  - Tonal noise (fan blade frequency and its harmonics)





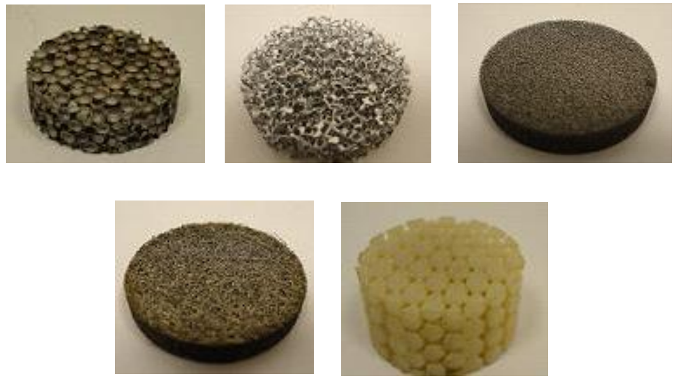
How to damp the noise ? ⇒ acoustic liners on the walls

Acoustic liners in aeronautics: some context

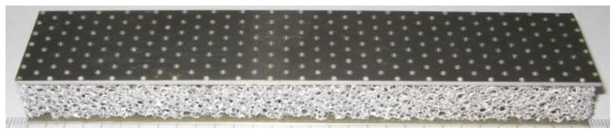
- What is an « acoustic liner » in aeronautics?

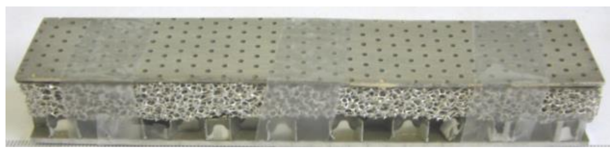
Combination of a **perforated plate** and honeycomb cells (**cavities**)

⇒ behaves as an Helmholtz resonator



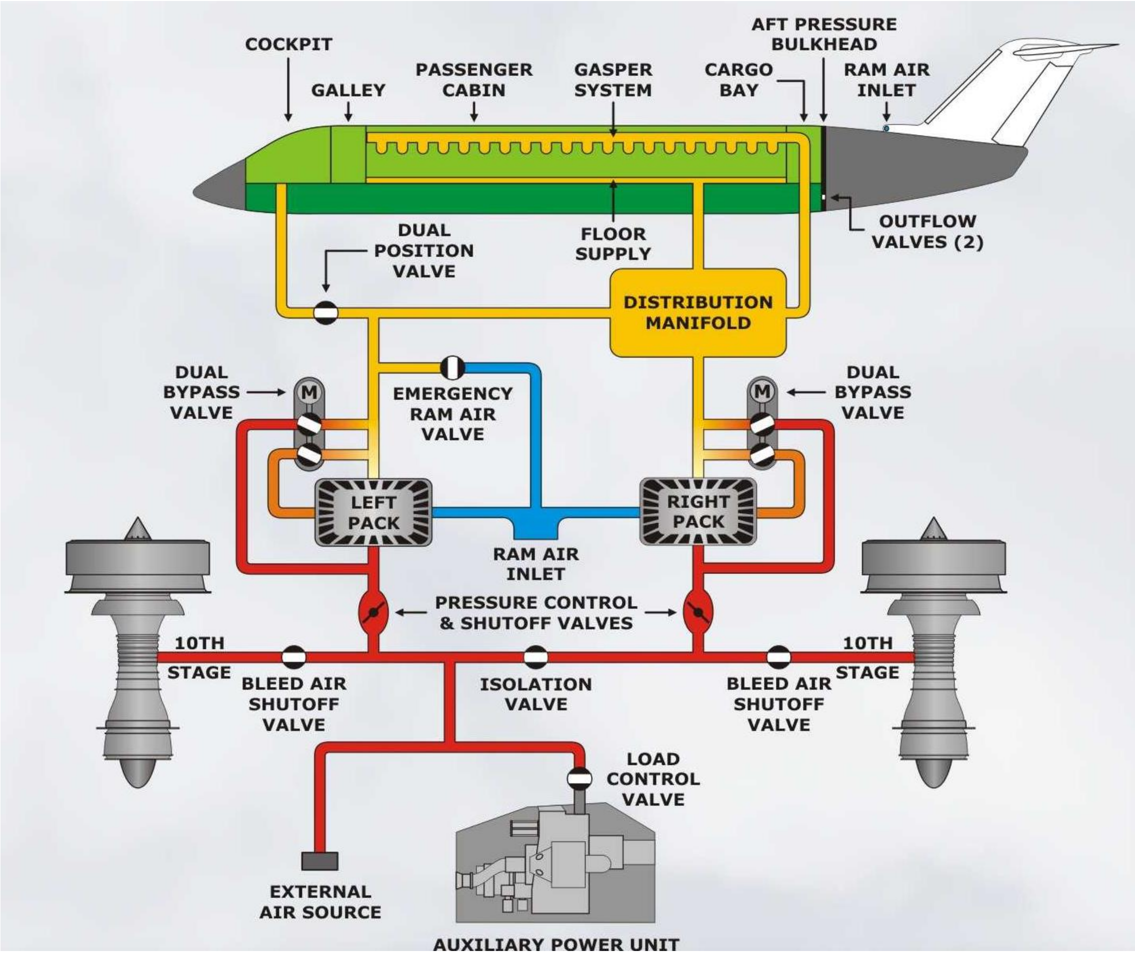
Porous material





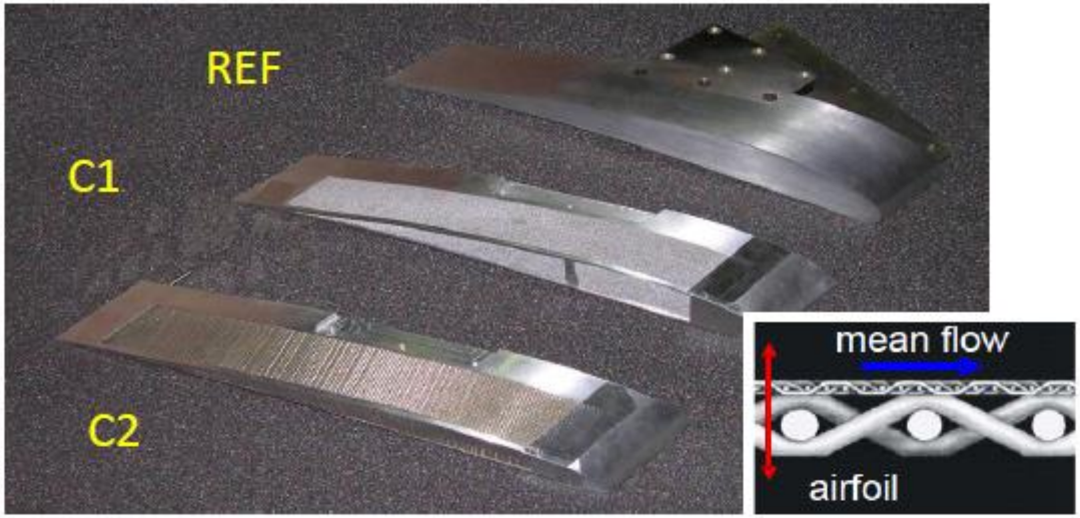
Acoustic liners in aeronautics: some context

- Noise reduction in air conditioning systems



Acoustic liners in aeronautics: some context

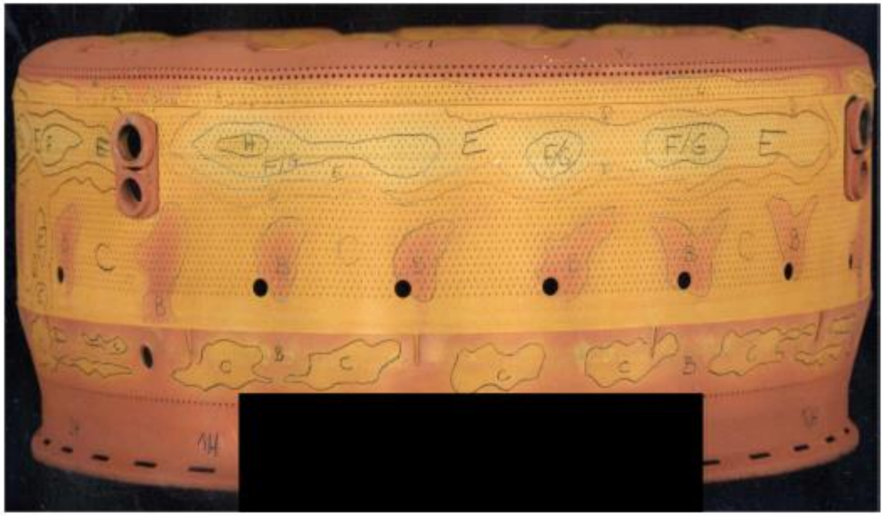
- Airframe noise reduction by modification of the noise source (flap/slat tip vortex shedding)



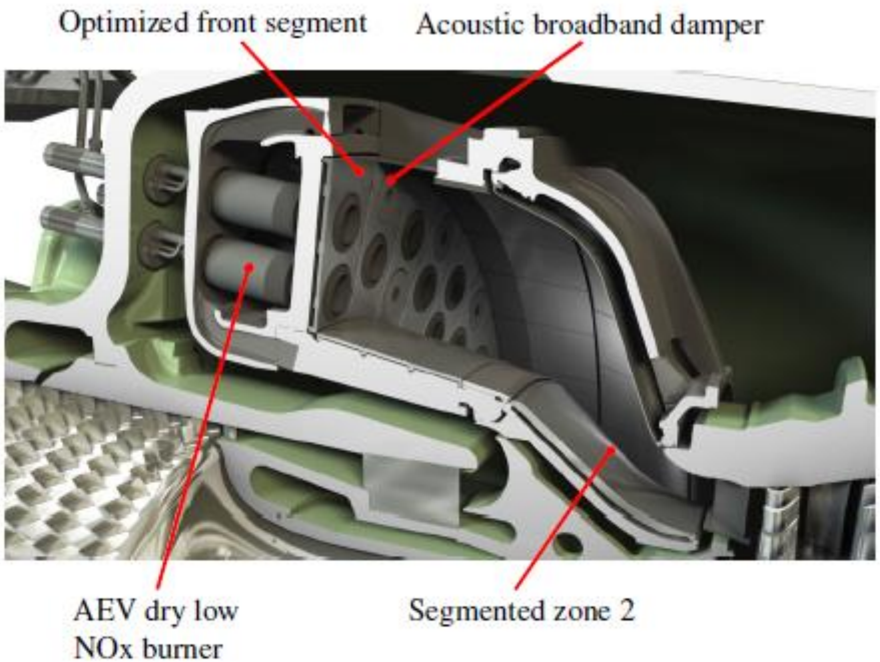
Coin de volet poreux OPENAIR (DASSAULT-EADS IW)

Acoustic liners in aeronautics: some context

- Perforated walls or acoustic dampers in combustion chambers



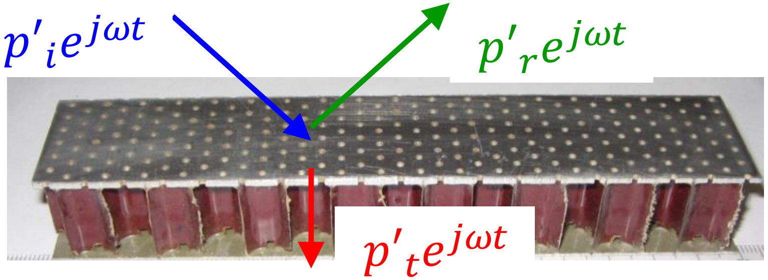
courtesy SAFRAN Helicopters Engine



from Bothien et al., *ASME Turbo Expo 2013,* GT2013-95693

Acoustic liners in aeronautics: some context

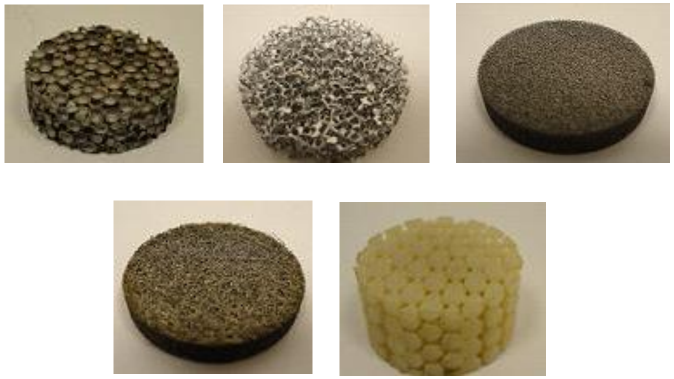
- What is an « acoustic liner » in aeronautics?



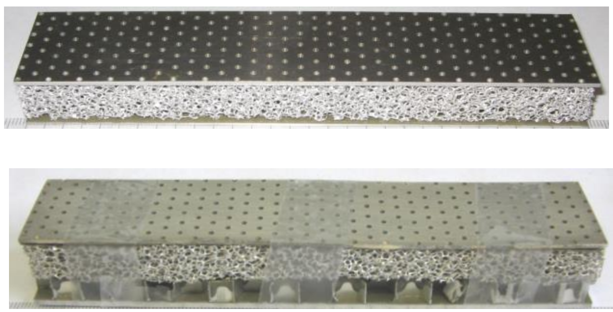
Localized reaction

Wall **impedance**: $$Z(\omega) = \frac{p'}{\boldsymbol{v}'.\boldsymbol{n}}$$

$$Z(\omega) = R(\omega) + jX(\omega)$$

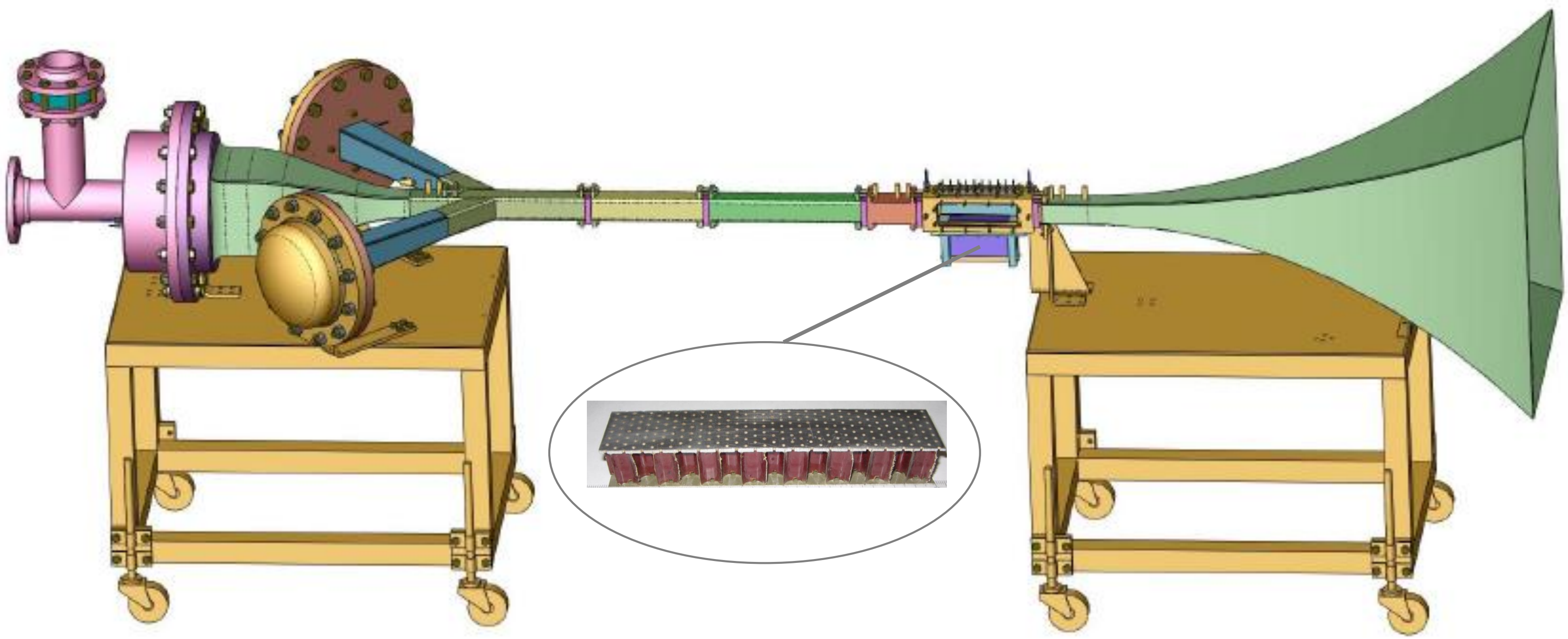


Non-localized reaction



Acoustic liners in aeronautics: some context

- A long history of experimental works at ONERA Toulouse:



B2A aero-thermo-acoustics flow duct and **LDV** acoustic velocity measurements

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  - Impedance eduction ?
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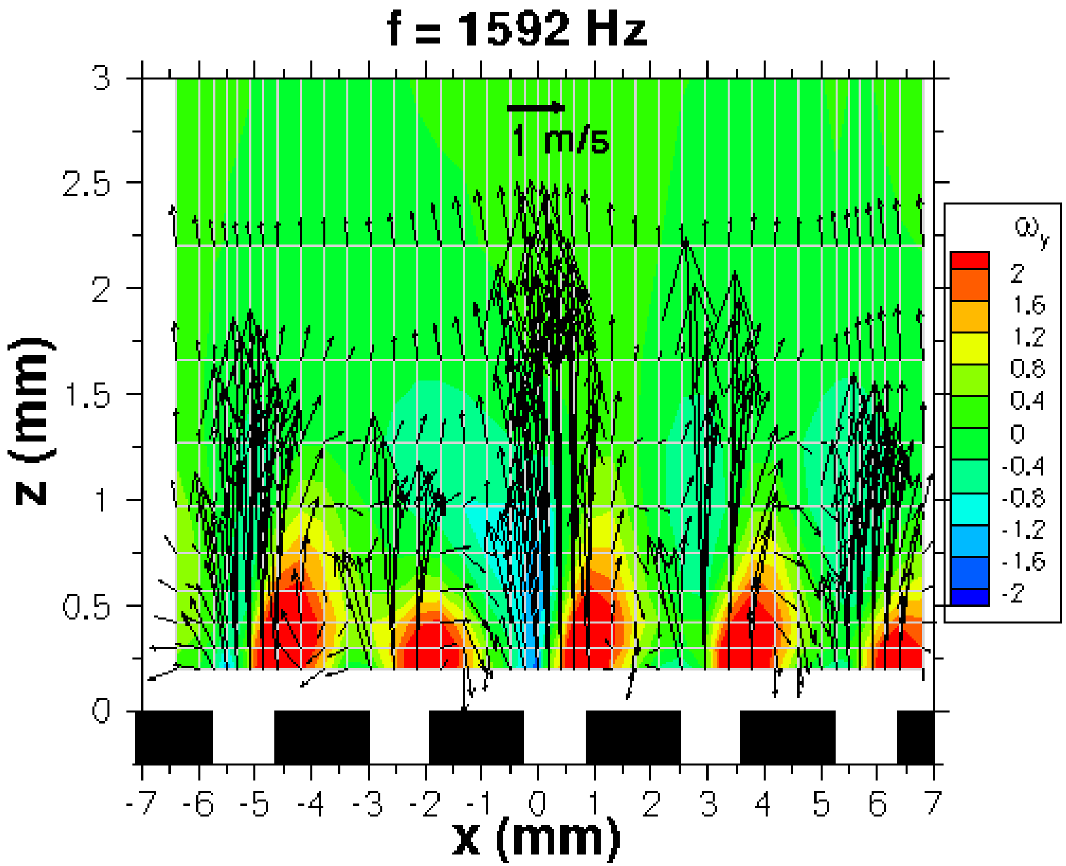
Impedance and homogeneization

- Some experimental results to visualize the issue: acoustic velocity field near the lined wall

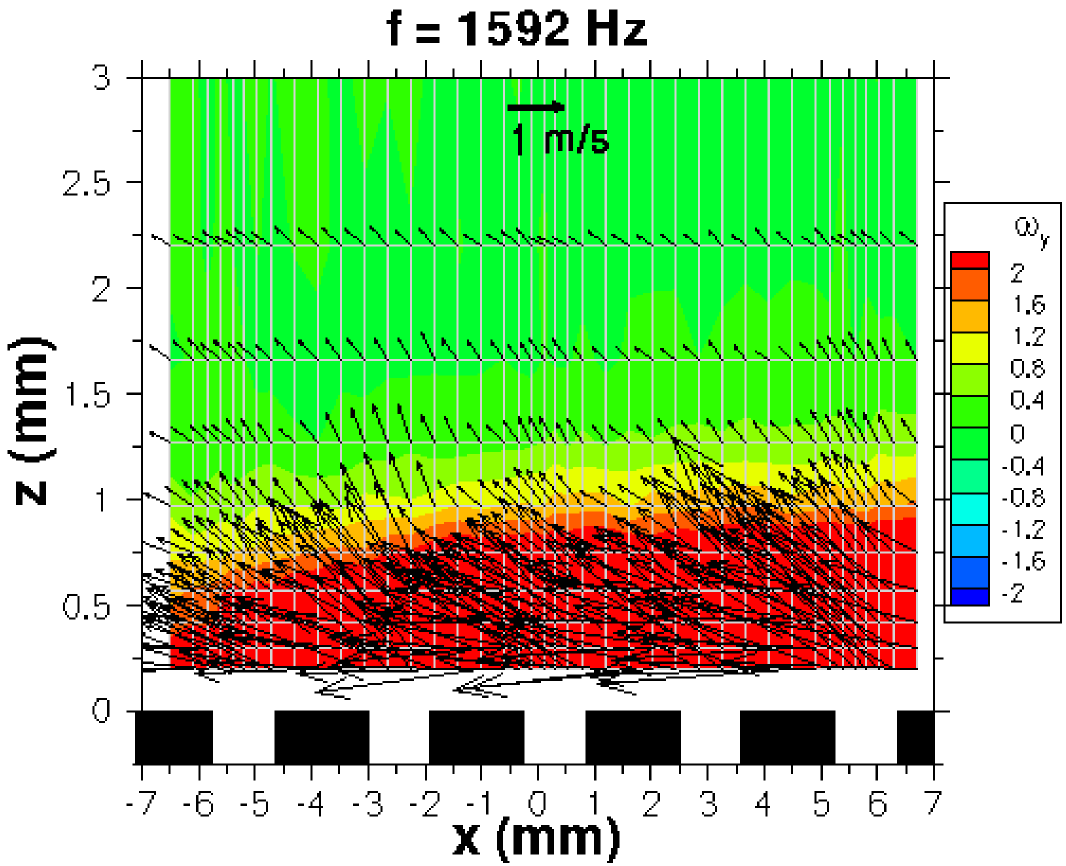


Mean flow M_b=0.05

Plane incident sound wave at SPL=132 dB

Impedance and homogeneization

- Some experimental results to visualize the issue: acoustic velocity field near the lined wall



Mean flow M_b=0.3

Plane incident sound wave at SPL=132 dB

Impedance and homogeneization

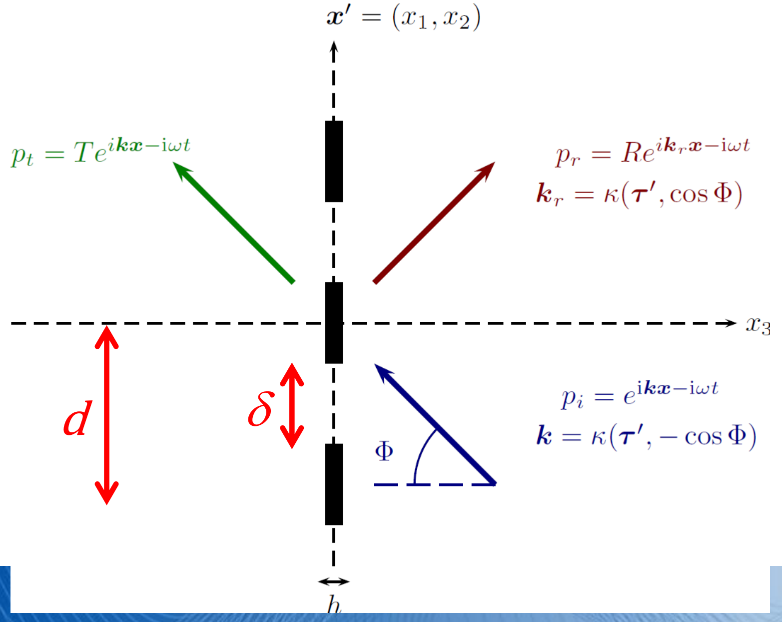
Wall impedance

$$Z = \frac{p'}{\boldsymbol{v}'.\boldsymbol{n}}$$

Impedance and homogeneization

- Collaboration with applied mathematics team at INSA Toulouse: A. Bendali, S. Tordeux, S. Laurens (post-doc), M'B. Fares (Cerfacs) : ANR APAM project 2009-2011

- 1st step: **linearized acoustics** framework (Helmholtz equation)
  - Scattering problem: plate with a lattice of perforations
  - « Known » result by the acoustics community : $Z_w = \frac{p'^{I} - p'^{II}}{v'^{I}.n} = \frac{p'^{I} - p'^{II}}{v'^{II}.n} = \frac{Z_h}{\sigma}$
  - How to obtain a mathematical justification? ➔ two-scale matched asymptotic expansions



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  - Two-scale matched asymptotic expansions

- Bendali et al. *SIAM Journal of Applied Mathematics*, 2013:
  - Key notion: **Rayleigh conductivity** $K_R = \frac{j\omega\rho Q}{P^I - P^{II}}$; defined in the **near field**
  - Matching rule with the **far field** solution: $\lim_{\infty} P^I = p'^I|_{paroi}$
  - « Well-posed » definition of the transmission impedance, through the notion of compliance, known at several orders
  - At $\mathcal{O}(\delta)$, $Z_w = \frac{j\omega A}{cK_R}$ ➔ agrees with the « empirical » definition, no influence of the lattice shape

Impedance and homogeneization

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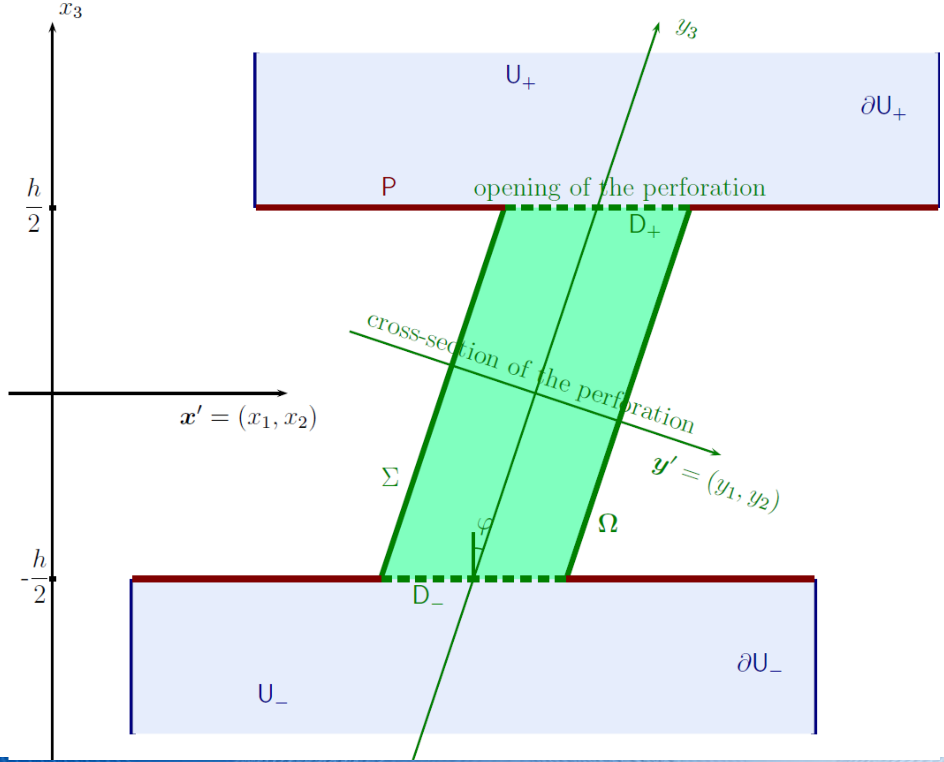
- 2nd step: modeling the **Rayleigh conductivity** of a perforation, in the framework of **linearized acoustics**



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- 2nd step: modeling the **Rayleigh conductivity** of a perforation, in the framework of **linearized acoustics**
  - « Known » result: untilted cylindrical aperture in a thin plate + empirical correction linked to the plate thickness

    ➔ widely used in the acoustics community (Howe's model, end correction of liner models …)
  - Laurens, Tordeux, Bendali, Fares& Kotiuga revisited the Rayleigh conductivity model for thick plates in *ESAIM, Math. Model. Numer. Anal, 2013 :*

    ➔ use of **Dirichlet and Kelvin energetics principles** to derive lower and upper bounds of K_R
  - Extension to tilted elliptical perforations and application to liners with bias flow in Laurens et al, *Journal of Fluid Mechanics, 2014*

Impedance and homogeneization

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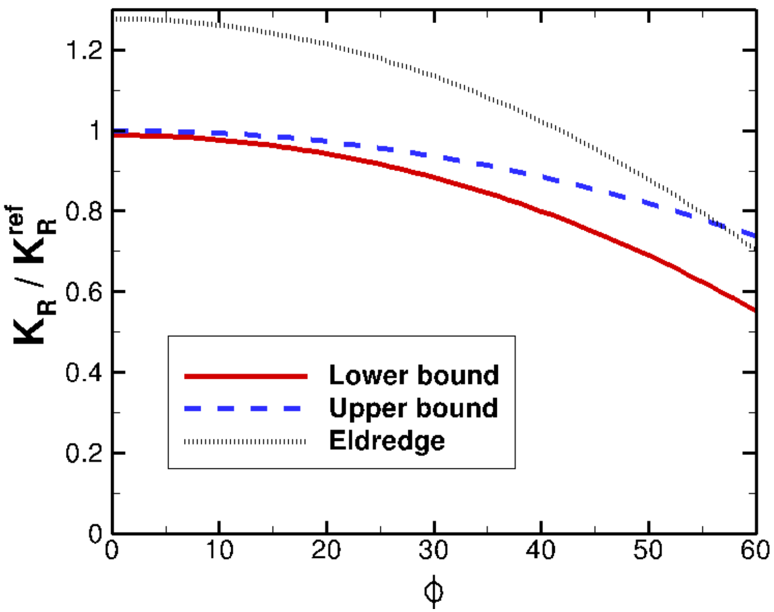
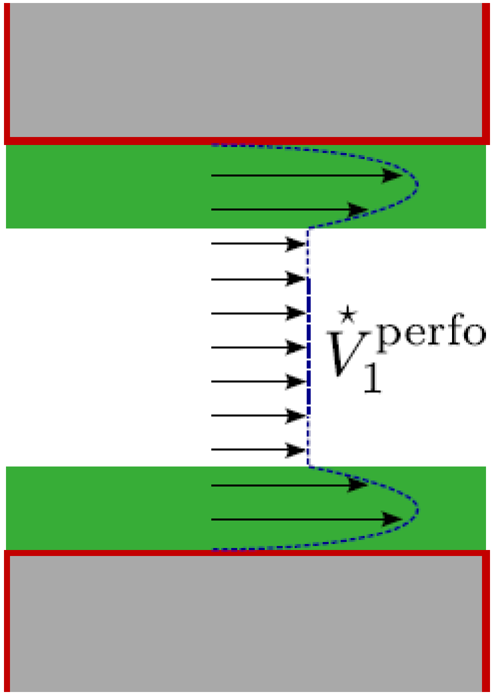


Impedance and homogeneization

- Next step: go beyond linearized acoustics
  - Taking into account of **viscous effects** ➔ V. Popie's PhD thesis (2012-2015), co-supervised with S. Tordeux

Viscous effects on the exterior wall of the aperture

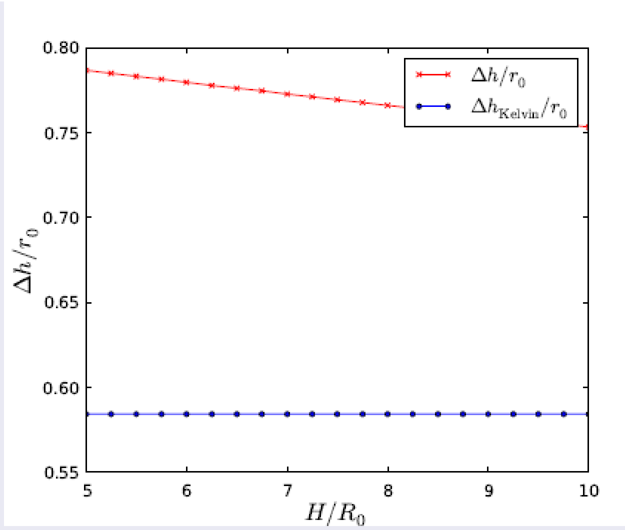


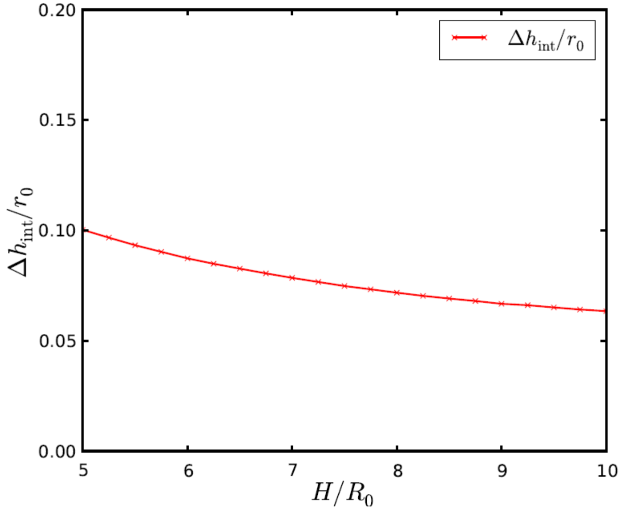
Viscous effects on the interior wall of the aperture

  - **Matched asymptotics expansion** yields the relation between the resistance of the perforated plate (ie real part of the impedance) and the viscous stress at the aperture wall

Impedance and homogeneization

- Next step: go beyond linearized acoustics
  - Taking into account of **viscous effects** ➔ V. Popie's PhD thesis (2012-2015), co-supervised with S. Tordeux





  - Computation of the **end corrections** (obtained from the **Rayleigh conductivity**) due to the viscous effects, thanks to the CESC code

- Mean flow effects? ➔ open question

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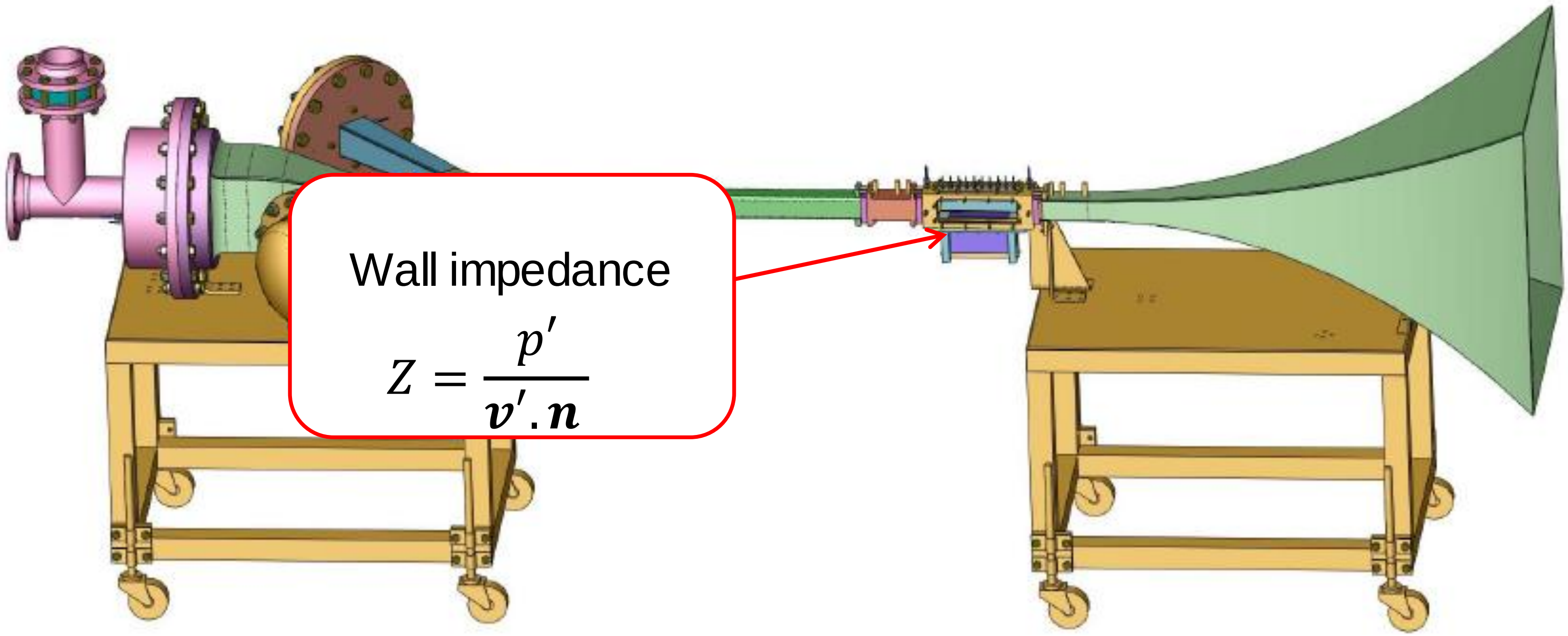
Impedance eduction

Wall impedance

$$Z = \frac{p'}{\boldsymbol{v}'.\boldsymbol{n}}$$

Impedance eduction

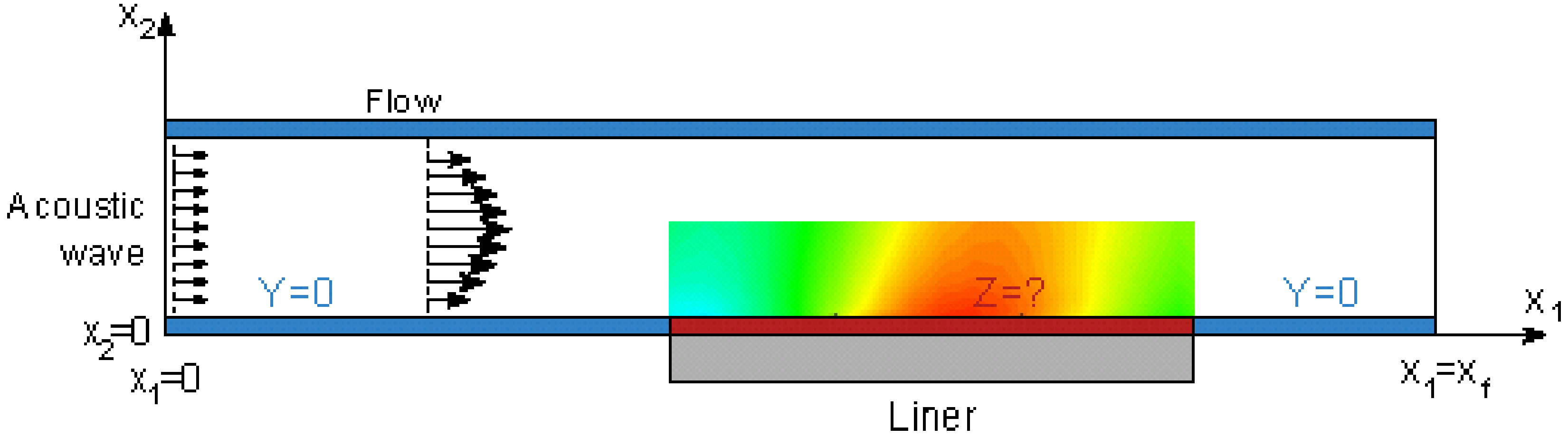


Wall impedance

$$Z = \frac{p'}{\boldsymbol{v}'.\boldsymbol{n}}$$

Impedance eduction

- Impedance eduction : principle



- Measurement of an « easy-to-measure » quantity
- Numerical simulation of the sound propagation in the lab configuration
- Eduction of the wall boundary condition (ie **impedance**) so that the simulations match the experiments

➔ **minimization of a cost function $\mathcal{J}(Z)$**

Impedance eduction

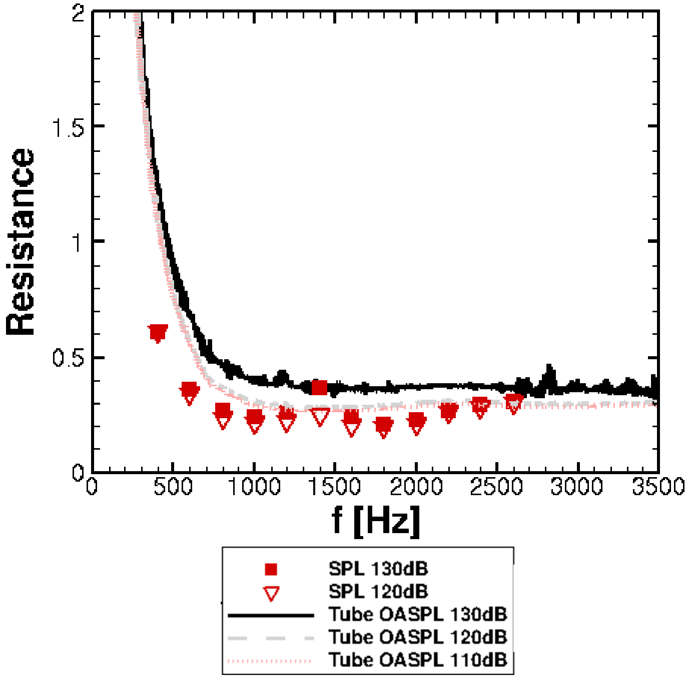
- ONERA method (see Primus et al. *Journal of Sound and Vibration*, 2013), use of the in-house solver **Elvin**

- Measurement of an « easy-to-measure » quantity
  - ➔ acoustic pressure at the wall opposite the liner
  - ➔ **acoustic velocity** in the test section, measured by LDV
- Numerical simulation of the sound propagation in the lab configuration
  - ➔ solving the **2D harmonic Linearized Euler Equations**
  - ➔ experimental mean flow is taken into account
  - ➔ Discontinuous Galerkin spatial discretization
- Eduction of the wall boundary condition (ie **impedance**) so that the simulations match the experiments
  - ➔ gradient-based optimisation process, use of the **adjoint equations**
  - ➔ genetic algorithms, use of **bayesian inference**

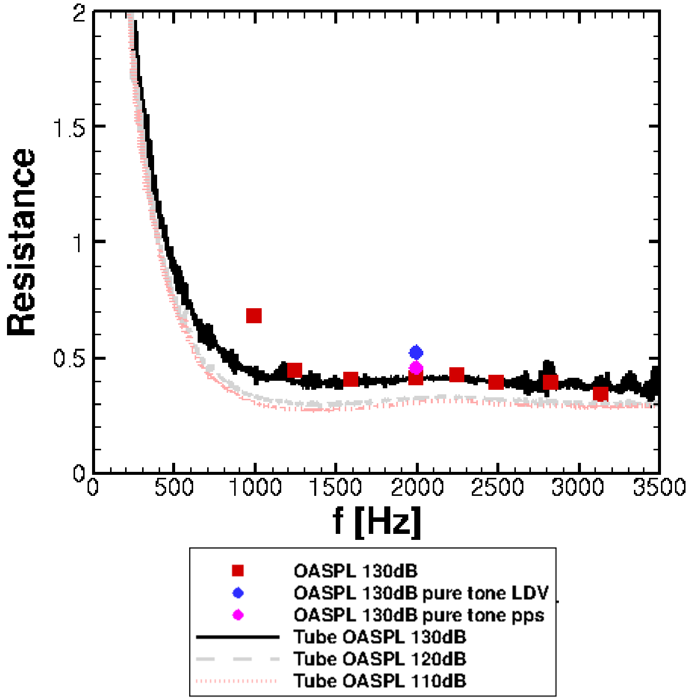
Impedance eduction

- Some results:

NASA method

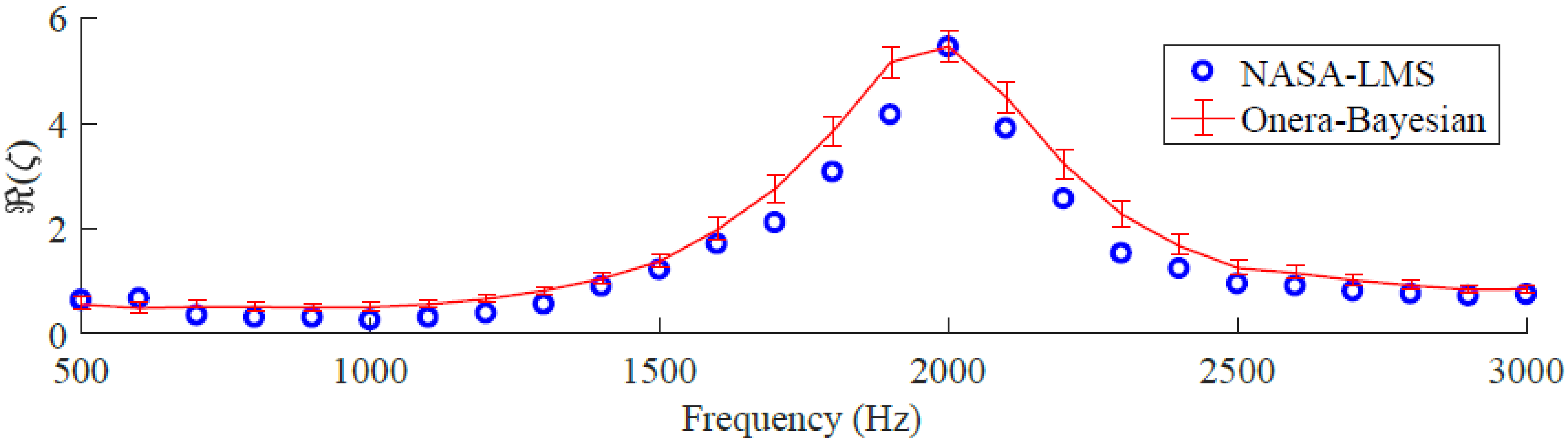


ONERA method



Impedance eduction

- Some results:



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Time-domain impedance boundary condition

- Impedance law: how to translate it from the frequency domain to the time domain?
  - Impedance in the frequency domain: $\forall \omega \in \mathbb{R}, p'(\omega) = Z(\omega)\ v'_n(\omega)$

➔ Causal linear time invariant (LTI) convolution operator :

$$\forall t > 0, p'(t) = (z * v'_n)(t)$$

- Admissibility criteria on $Z(\omega)$:
  - Causality ➔ use of the Laplace transform $s = j\omega$
  - Passivity : $\Re\big(Z(s)\big) \geq 0$
  - Reality : $Z(s)$ is hermitian

- Exemple of impedance model for a perforated plate (Kirby & Cummings 1998, Malmary et. al 2001) :

$$Z = \underbrace{\frac{\sqrt{2\nu\omega}}{\sigma c_0}\frac{h}{\delta}}_{\text{Viscous effects in the perforation}} + \underbrace{\left[26{,}16\left(\frac{h}{2\delta}\right)^{-0{,}169} - 20\right]\frac{v^*}{\sigma c_0}}_{\text{Grazing flow effect}} - \underbrace{0{,}645\frac{\omega h}{\sigma c_0}}_{\text{Empirical end correction}} + \underbrace{\frac{4}{3\pi}\frac{1-\sigma^2}{\sigma c_0 {C_D}^2}|\boldsymbol{v}'.\boldsymbol{n}|}_{\text{High SPL effect}} + \underbrace{j\frac{\omega}{\sigma c_0}\left[h + \frac{16\delta}{3\pi}\right]}_{\text{End correction}}$$

Time-domain impedance boundary condition

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- Generic shape of the physics-based models:

$$Z(s) = \underbrace{a_{\frac{1}{2}}\sqrt{s}}_{\text{Visco-thermal effects in the plate}} + \overbrace{a_0}^{\text{Grazing flow}} + \underbrace{a_1 s}_{\text{End correction}} + \coth\left[\left(\underbrace{b_{\frac{1}{2}}\sqrt{s}}_{\text{Visco-thermal effects in the cavity}} + \overbrace{b_1 s}^{\text{Cavity resonant effect}}\right) l_c\right] + \underbrace{a_{NL}|\boldsymbol{v}'.\boldsymbol{n}|}_{\text{High SPL}}$$

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- Generic shape of the physics-based models:

$$Z(s) = a_{\frac{1}{2}}\sqrt{s} + a_0 + a_1 s + \coth\left[\left(b_{\frac{1}{2}}\sqrt{s} + b_1 s\right) l_c\right]$$

- How to derive a **temporal realisation**? ➔ Florian Monteghetti's PhD (2015-…), co-supervised with D. Matignon (ISAE-Supaero).

Time-domain impedance boundary condition

- Impedance law: how to translate it from the frequency domain to the time domain?
  - Monteghetti et al. *Journal of the Acoustical Society of America*, 2016
  - Illustration of the **diffusive representation** of the impedance boundary condition

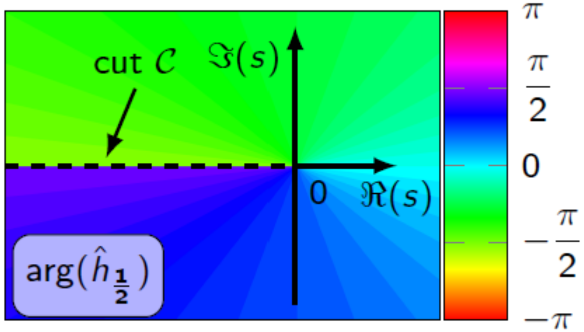
$$Z(s) = a_0 + a_{\frac{1}{2}} s \frac{1}{\sqrt{s}} + a_1 s$$

$$p' = a_0 v'_n + a_{\frac{1}{2}} h_{\frac{1}{2}} * \frac{dv'_n}{dt} + a_1 \frac{dv'_n}{dt}$$

$$p' = a_0 v'_n + a_{\frac{1}{2}} \int_0^\infty \mu(\xi) \left(e^{-\xi t} * \frac{dv'_n}{dt} \right) d\xi \quad + a_1 \frac{dv'_n}{dt}$$

$$p' = a_0 v'_n + a_{\frac{1}{2}} \int_0^\infty \mu(\xi) [-\xi \varphi_\xi + v'_n] d\xi \quad + a_1 \frac{dv'_n}{dt}$$

Inverse Laplace + residue theorem



Additional **ODEs** on **diffusive variables**: $$\frac{d\varphi_\xi}{dt} = -\xi \varphi_\xi + v'_n \quad + \text{initial condition}$$

Time-domain impedance boundary condition

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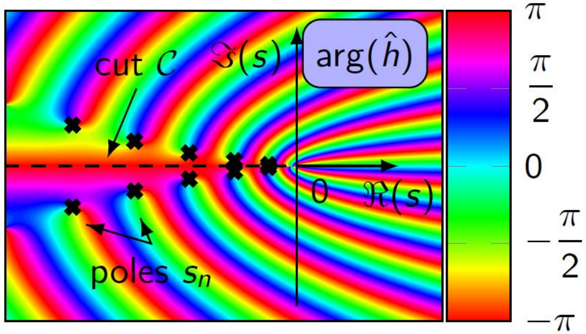
Inverse Laplace + residue theorem



Additional **ODEs** on N_φ **diffusive variables**: $$\frac{d\varphi_i}{dt} = -\xi_i \varphi_i + v'_n \quad + \text{initial condition}$$

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Perspectives

- Mathematical justification of the impedance notion in the presence of grazing flow
- Accurate impedance model for various aperture shapes and environmental conditions (flow, thermal gradient, vibrations…)
- Reliable and robust time-domain impedance boundary condition for numerical simulations

Questions ?

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