Adaptive and higher-order BEM for the wave equation

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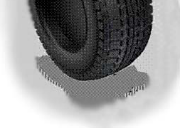
What's this talk about?

Time domain BEM for wave scattering off a knife's blade (screen problems).

convergence rates - theory and numerical experiments
(in DOF on a 2d screen)

- 0.5: h-version, uniform
- 0.77: h-version, adaptive
- 1.0: p-version, uniform
- $\beta/2$: h-version, β-graded, $\beta \in [1, 3)$

3d wave equation



$$
\begin{aligned}
u &= u(t,x) && \text{sound pressure} \\
\partial_t^2 u - \Delta u &= 0 && \text{in } \mathbb{R}_t \times \mathbb{R}_x^3 \setminus \Omega \\
u &= 0 && \text{for } t \leq 0 .
\end{aligned}
$$

Simple: boundary conditions $u = f$ on $\Gamma = \partial\Omega$.

Realistic: acoustic boundary conditions $\partial_\nu u - \alpha \partial_t u = f$ on $\Gamma = \partial\Omega$.

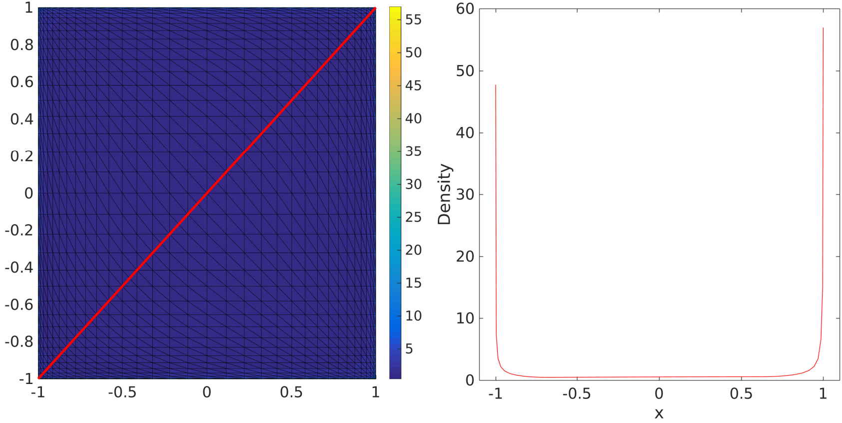
Some motivations for space or space-time refinements + adaptivity:

- Edge/corner/geometric singularities.
- Sharp travelling wave crests.

Variable Δt: Sauter-Veit, Veit-Merta-Zapletal-Lukas, Sauter-Schanz, ...
Variable Δx: Abboud

Screen problems: static graded meshes

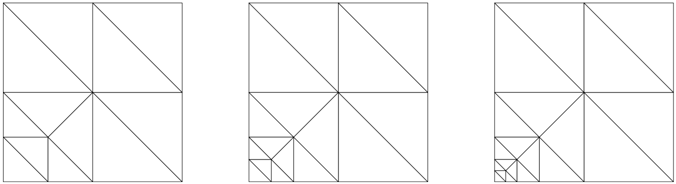
$\Omega^c = \mathbb{R}^3 \setminus ([-1,1]^2 \times \{0\})$, $\mathcal{V}\phi(x) = \sin(x)^5$ on $[-1,1]^2 \times \{0\}$.
solution near corner $r^{-0.703\ldots}$, near edge $r^{-\frac{1}{2}}$



Contents

$$
\begin{aligned}
u : \mathbb{R}_t \times \Omega_x \to \mathbb{R} \quad & \text{sound pressure} \\
\partial_t^2 u - \Delta u = \ 0 \qquad & \text{in } \mathbb{R}_t \times \Omega_x^c, \quad \Omega^c = \mathbb{R}^d \setminus \overline{\Omega} \\
u = \ 0 \qquad & \text{for } t \leq 0 \ .
\end{aligned}
$$

- TDBEM: basics
- Screen problems:
  singular expansions / graded meshes for edges and corners / tires
- Space adaptivity / towards space-time adaptivity



BEM for the wave equation

$$\begin{aligned} u : \mathbb{R}_t \times \Omega_x \to \mathbb{R} & \quad \text{sound pressure} \\ \partial_t^2 u - \Delta u = 0 & \quad \text{in } \mathbb{R}_t \times \Omega^c \\ u = 0 & \quad \text{for } t \leq 0 \ . \end{aligned}$$

simple: Dirichlet boundary conditions $u = f$ on $\Gamma = \partial\Omega$:

$$\rightsquigarrow \quad f(t,x) = \mathcal{V}\phi(t,x) = \int_\Gamma \frac{\phi(y, t - |x-y|)}{4\pi|x-y|} \, ds_y$$

BEM for the wave equation

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Function spaces: space-time Sobolev spaces

$$f = \mathcal{V}\phi(t,x) = \int_\Gamma \frac{\phi(y, t-|x-y|)}{4\pi|x-y|}\, ds_y$$

space–time anisotropic Sobolev spaces $H^r_\sigma(\mathbb{R}^+, H^s(\Gamma))$, $\sigma > 0$:

$H^r_\sigma(\mathbb{R}^+, H^s(\mathbb{R}^2))$ defined using Fourier–Laplace transform

$$\left\{\psi : \mathrm{supp}\ \psi \subset \overline{\mathbb{R}_+} \times \mathbb{R}^2,\ \int_{\mathbb{R}+i\sigma} d\omega \int_{\mathbb{R}^2} d\xi |\omega|^{2r}(|\omega|^2 + |\xi|^2)^s |\mathcal{F}\psi(\omega,\xi)|^2 < \infty\right\}$$

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Space-time variational formulation of Dirichlet problem:
Find $\phi \in H^1_\sigma(\mathbb{R}^+, H^{-\frac{1}{2}}(\Gamma))$ such that $\forall \psi \in H^1_\sigma(\mathbb{R}^+, H^{-\frac{1}{2}}(\Gamma))$:

$$\langle \mathcal{V}\partial_t\phi, \psi\rangle = \langle \partial_t f, \psi\rangle, \quad \langle\cdot,\cdot\rangle = \int_0^\infty e^{-2\sigma t}\int_\Gamma \cdot\, d\Gamma_x dt$$

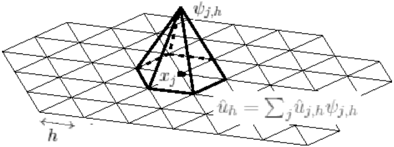
The solution ϕ exists for $f \in H^2_\sigma(\mathbb{R}^+, H^{\frac{1}{2}}(\Gamma))$.

Key: $\mathcal{V}\partial_t$ coercive with loss (Bamberger – Ha Duong '86)

$$\|\phi\|^2_{1,-\frac{1}{2},\Gamma} \gtrsim \langle \mathcal{V}\partial_t\phi, \phi\rangle \gtrsim \|\phi\|^2_{0,-\frac{1}{2},\Gamma}$$

Discretization

- $\Gamma = \cup_{i=1}^{M} \Gamma_i$ (quasi-uniform) triangulation
- V_h^p piecewise polynomial functions of degree p on $\Gamma = \cup_{i=1}^{M} \Gamma_i$ (continuous if $p \geq 1$)
- $[0, T) = \cup_{n=1}^{L} [t_{n-1}, t_n)$, $t_n = n(\Delta t)$
- $V_{\Delta t}^q$ piecewise polynomial functions of degree q in time (continuous and vanishing at $t = 0$ if $q \geq 1$)
- tensor products in space-time: $V_{h,\Delta t}^{p,q} = V_h^p \otimes V_{\Delta t}^q$



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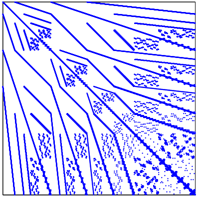
Time domain BEM: Find $\phi_{h,\Delta t} \in V_{h,\Delta t}^{p,q}$ such that $\forall \psi_{h,\Delta t} \in V_{h,\Delta t}^{p,q}$:

$$\langle \mathcal{V} \partial_t \phi_{h,\Delta t}, \psi_{h,\Delta t} \rangle = \langle \partial_t f, \psi_{h,\Delta t} \rangle$$

Sparse matrix, almost block lower triangular (causality).
Solve by time stepping scheme (HG – Stark '16).

$$\begin{array}{c} \longleftarrow t_n \longrightarrow \\ \begin{pmatrix} \mathbf{V}^0 & \mathbf{V}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{V}^1 & \mathbf{V}^0 & \mathbf{V}^{-1} & \mathbf{0} \\ \mathbf{V}^2 & \mathbf{V}^1 & \mathbf{V}^0 & \mathbf{V}^{-1} \\ \mathbf{V}^3 & \mathbf{V}^2 & \mathbf{V}^1 & \mathbf{V}^0 \end{pmatrix} \end{array} \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \\ \mathbf{c}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \mathbf{f}_4 \end{pmatrix} \updownarrow t_m$$



$\Omega^c = \mathbb{R}^3 \setminus ([-1,1]^2 \times \{0\})$, $\mathcal{V}\phi(t,x) = \sin(t)^5$ on $[-1,1]^2 \times \{0\}$.
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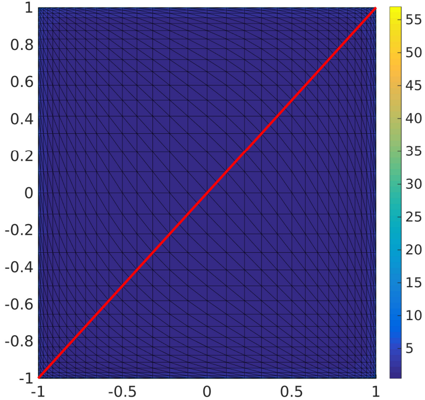

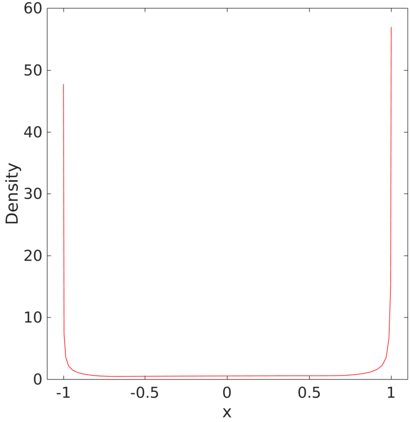

Screen problems: edge and corner singularities

Helmholtz: (Kondratiev, Dauge, ...)
Solution behaves like

- $r^{\gamma-1}$ near corner, γ=0.29 for square screen
- $r^{-\frac{1}{2}}$ near edge.

von Petersdorff '89, +Stephan '90: precise tensor product decomposition, BEM on graded meshes
$\Longrightarrow$ optimal approximation on graded meshes.

Screen problems: edge and corner singularities

Theorem ($r^{\gamma-1}$ in corner, $r^{-\frac{1}{2}}$ at edges, coeffs depend on ω)

Let $\mathcal{V}_\omega \psi_\omega = f_\omega \in H^2(\Gamma)$. *Then*

$$\psi_\omega = \psi_{0,\omega} + \chi_\omega(r) r^{\gamma-1} \alpha_\omega(\theta) + \tilde{\chi}_\omega(\theta) b_{1,\omega}(r) r^{-1} (\sin(\theta))^{-\frac{1}{2}} + \tilde{\chi}_\omega(\frac{\pi}{2} - \theta) b_{2,\omega}(r) r^{-1} (\cos(\theta))^{-\frac{1}{2}}$$

where $\psi_{0,\omega} \in H^{1-\epsilon}(\Gamma)$, $\alpha_\omega(\theta) \in H^{1-\epsilon}[0, \frac{\pi}{2}]$, $b_{i,\omega} = c_{i,\omega} r^\gamma + d_{i,\omega}(r)$, $r^{-\frac{1}{2}} d_{i,\omega}(r) \in H^1(\mathbb{R}^+)$, $r^{-\frac{3}{2}} d_{i,\omega}(r) \in L_2(\mathbb{R}^+)$, $c_{i,\omega} \in \mathbb{R}$.
(r, θ) *polar coordinates around (0,0),* $\chi_\omega, \ \tilde{\chi}_\omega \in C_c^\infty$, $= 1$ *near* 0.

γ eigenvalue: $\gamma \approx 0.2966$ for square

Screen problems: edge and corner singularities

Previous work on wave equation:

- Plamenevskii et al. since '99:
  singular expansions near corners and edges of polygon
- Müller – Schwab '15: $2d$ FEM on graded meshes

Theorem

a) Decomposition of solution in singular / regular parts with same singular exponents $\gamma - 1, -\frac{1}{2}$ as in elliptic case.

b) Error of best approximation in $H^0_\sigma(\mathbb{R}^+, H^{-\frac{1}{2}}(\Gamma)) = \mathcal{O}(h^{\min\{\frac{\beta}{2},\frac{3}{2}\}-\varepsilon})$.

Screen problems: edge and corner singularities

Theorem

a) Decomposition of solution in singular / regular parts with same singular exponents $\gamma - 1, -\frac{1}{2}$ as in elliptic case.

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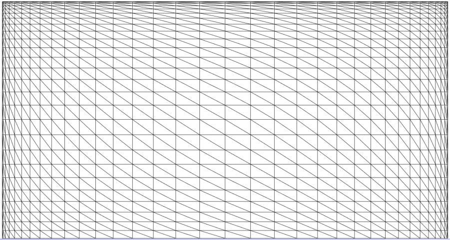
$$x_j = 1 - \left(\frac{j}{N}\right)^\beta,\ j = 1, \ldots, N\ .$$



A priori estimate for Dirichlet, $b(\phi, \psi) = \langle \mathcal{V}\dot{\phi}, \psi\rangle = \langle \dot{f}, \psi\rangle$

Theorem 2 (a priori estimate)

If $\phi \in H_\sigma^{1,-\frac{1}{2}}(\mathbb{R}_+, \Gamma)$: $\|\phi - \phi_{h,\Delta t}\|_{0,-\frac{1}{2}} \lesssim \inf_{\psi_{h,\Delta t}} (1 + \frac{1}{\Delta t})\|\phi - \psi_{h,\Delta t}\|_{1,-\frac{1}{2}}$.

Proof

$$\|\phi_{h,\Delta t} - \psi_{h,\Delta t}\|^2_{0,-1/2} \lesssim b(\phi_{h,\Delta t} - \phi, \phi_{h,\Delta t} - \psi_{h,\Delta t}) + b(\phi - \psi_{h,\Delta t}, \phi_{h,\Delta t} - \psi_{h,\Delta t})$$

$$b(\phi_{h,\Delta t} - \phi, \phi_{h,\Delta t} - \psi_{h,\Delta t}) = 0 \qquad \text{Galerkin orthogonality}$$

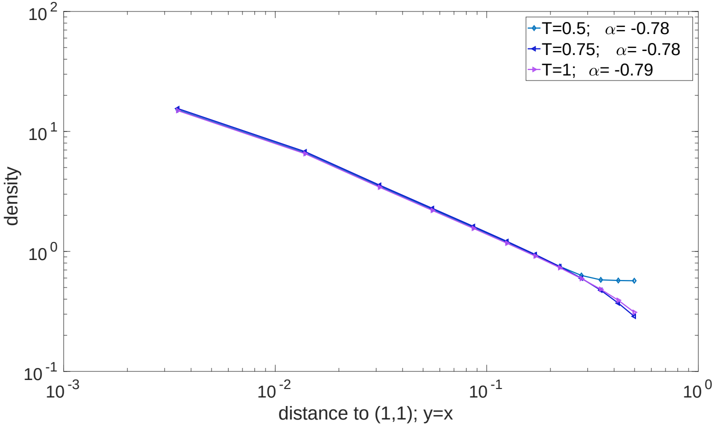
$$\begin{aligned}
b(\phi - \psi_{h,\Delta t}, \phi_{h,\Delta t} - \psi_{h,\Delta t}) &\le \|\mathcal{V}\frac{\partial}{\partial t}(\phi - \psi_{h,\Delta t})\|_{-1,+1/2} \cdot \|\phi_{h,\Delta t} - \psi_{h,\Delta t}\|_{1,-1/2} \\
&\lesssim \|\phi - \psi_{h,\Delta t}\|_{1,-1/2} \cdot \|\phi_{h,\Delta t} - \psi_{h,\Delta t}\|_{1,-1/2} \\
&\lesssim \frac{1}{\Delta t}\|\phi_{h,\Delta t} - \psi_{h,\Delta t}\|_{0,-1/2}\|\phi - \psi_{h,\Delta t}\|_{1,-1/2} \\
\Longrightarrow \|\phi - \phi_{h,\Delta t}\|_{0,-\frac{1}{2}} &\lesssim \|\phi - \psi_{h,\Delta t}\|_{0,-1/2} + \|\phi_{h,\Delta t} - \psi_{h,\Delta t}\|_{0,-1/2} \\
&\lesssim \left(1 + \frac{1}{\Delta t}\right)\|\phi - \psi_{h,\Delta t}\|_{1,-\frac{1}{2}}
\end{aligned}$$

Screen problems: Corner exponents for waves

$\Omega^c = \mathbb{R}^3 \setminus ([-1,1]^2 \times \{0\})$, $\mathcal{V}\phi = \sin(t)^5$ on $[-1,1]^2 \times \{0\}$, $0 < t < 1$.

corner exponent: $-0.78 \sim \gamma - 1 = -0.703$ as in elliptic case

Plot: $\phi(t,r)$ as function of r along $x = y$

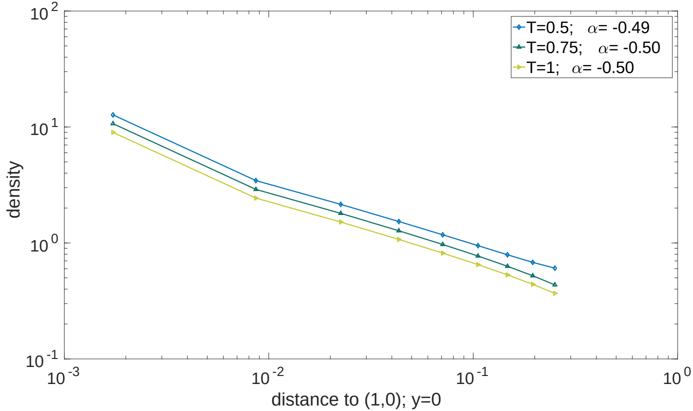


Screen problems: Edge exponents for waves

$\Omega^c = \mathbb{R}^3 \setminus ([-1,1]^2 \times \{0\})$, $\mathcal{V}\phi = \sin(t)^5$ on $[-1,1]^2 \times \{0\}$, $0 < t < 1$.

edge exponent: $-0.49 \sim -\frac{1}{2}$ as in elliptic case

Plot: $\phi(t, x, y = 0)$ as function of x



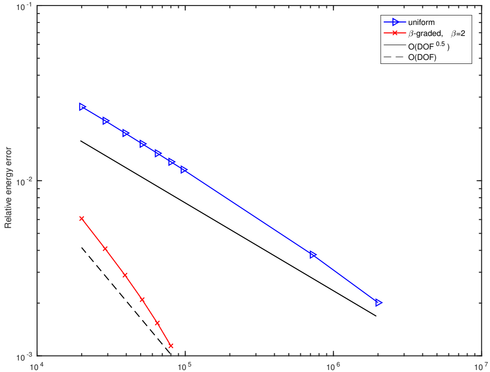
$\Omega^c = \mathbb{R}^3 \setminus ([-1,1]^2 \times \{0\})$, $\mathcal{V}\phi = \sin(t)^5$ on $[-1,1]^2 \times \{0\}$, $0 < t < 1$.

Convergence for fixed $\Delta t = 0.01$:

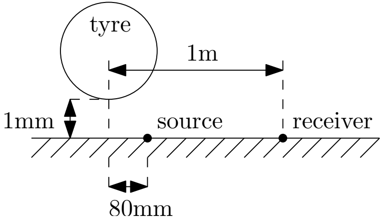
Energy norm^2 $= \langle \mathcal{V}\partial_t(\phi_h - \phi), \phi_h - \phi\rangle \sim h^2 \simeq DOF(\Gamma)^{-1}$ (2-graded)

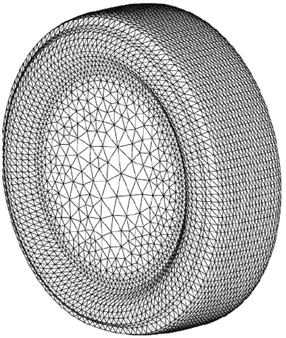
$\sim h \simeq DOF(\Gamma)^{-1/2}$ (uniform)

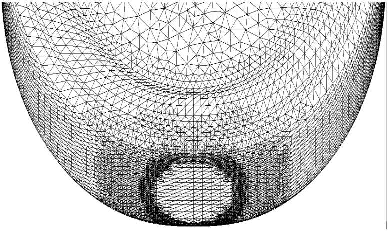
similar results for W and for Dirichlet-to-Neumann operator



Graded Meshes for Tyre (1)

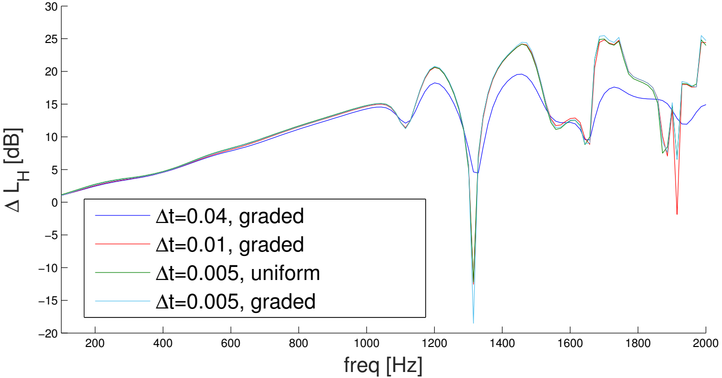






Graded Meshes for Tyre (2)

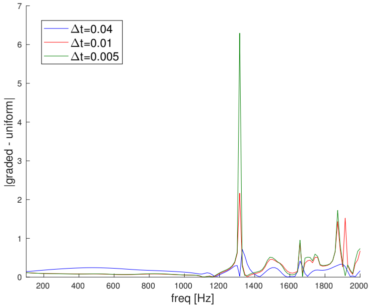
Amplification due to horn effect:

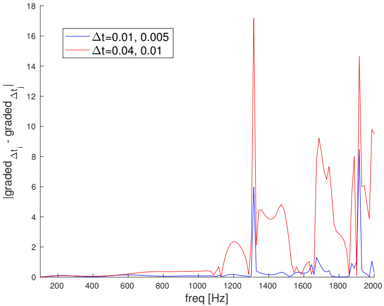


Grading with various Δt compared to uniform tire mesh.

Graded Meshes for Tyre (3)

Left: As $\Delta t \to 0$, difference between graded and uniform becomes larger.





Right: Relative effect of grading increases for small time steps.

Effects mostly seen in resonances.

Error Indicators

Theorem

Let $\phi_{h,\Delta t} \in H^0_\sigma(\mathbb{R}^+, H^{-\frac{1}{2}}(\Gamma))$ such that
$\mathcal{R} = \partial_t \ f - \mathcal{V}\partial_t \phi_{h,\Delta t} \in H^0_\sigma(\mathbb{R}^+, H^1(\Gamma)) \Longrightarrow$

$$\|\phi - \phi_{h,\Delta t}\|^2_{0,-\frac{1}{2}} \lesssim \sum_{i,\Delta} \max\{\Delta t, h_\Delta\} \ \|\mathcal{R}\|^2_{0,1,[t_i,t_{i+1})\times\Delta}$$

$$\max\{\Delta t, h\}\|\mathcal{R}\|^2_{0,1-\epsilon} \lesssim \|\phi - \phi_{h,\Delta t}\|^2_{2,-\frac{1}{2}}$$

The upper bound is independent of the approximation method: TDBEM, convolution quadrature, no assumption on mesh.

The lower bound holds on quasi-uniform meshes.

Error Indicators

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$$\max\{\Delta t, h\}\|\mathcal{R}\|^2_{0,1-\epsilon} \lesssim \|\phi - \phi_{h,\Delta t}\|^2_{2,-\frac{1}{2}}$$

Residual error indicators (RB):

$$\eta^2(\Delta, i) = \max\{\Delta t, h_\Delta\}\ \|\mathcal{R}\|^2_{0,1,[t_i,t_{i+1})\times\Delta}$$
$$= \int_{t_i}^{t_{i+1}} \int_\Delta \left\{\Delta t\left[\partial_t\dot{V}\phi(t,\mathbf{x}) - \partial_t\dot{f}(t,\mathbf{x})\right]^2 + h_\Delta\left[\nabla_\Gamma\dot{V}\phi(t,\mathbf{x}) - \nabla_\Gamma\dot{f}(t,\mathbf{x})\right]^2\right\}$$

Proof of upper bound

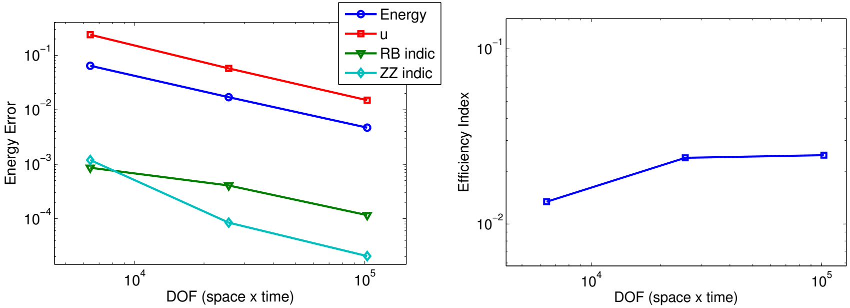
$b(\phi,\psi) = \int_0^\infty e^{-2\sigma t} \int_\Gamma V\dot{\phi}(t,x)\psi(t,x)\, d\Gamma_x\, dt$

$$\begin{aligned}
&\|\phi - \phi_{h,\Delta t}\|^2_{0,-\frac{1}{2},\Gamma} \\
&\quad \lesssim \int_0^\infty dt\; e^{-2\sigma t} \int_\Gamma d\Gamma\; V(\dot{\phi} - \dot{\phi}_{h,\Delta t})(\phi - \phi_{h,\Delta t}) \\
&\quad = \int_0^\infty dt\; e^{-2\sigma t} \int_\Gamma d\Gamma\; (\dot{f} - \mathcal{V}\dot{\phi}_{h,\Delta t})(\phi - \psi_{h,\Delta t}) \\
&\quad \lesssim\; \|\mathcal{R}\|_{0,\frac{1}{2},\Gamma}\, \|\phi - \psi_{h,\Delta t}\|_{0,-\frac{1}{2},\Gamma}\,.
\end{aligned}$$

- interpolation inequality: $\|\mathcal{R}\|^2_{0,\frac{1}{2},\Gamma} \lesssim \|\mathcal{R}\|_{0,1,\Gamma}\|\mathcal{R}\|_{0,0,\Gamma}\,.$
- residual orthogonal: $\mathcal{R} \perp \psi_{h,\Delta t}$ in $H^0_\sigma(\mathbb{R}^+, H^0(\Gamma))$.
- interpolation $\rightsquigarrow\; h, \Delta t$
  $\|\mathcal{R}\|_{0,0,\Gamma} \;\le \inf \|\mathcal{R} - \psi_{h,\Delta t}\|_{L^2([0,\widetilde{T}],L^2(\Gamma))}, \quad \psi_{h,\Delta t} \in V^{p,q}_{h,\Delta t}$

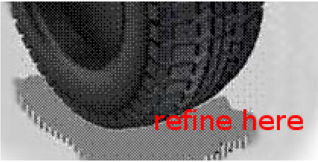
Adaptivity: Spherical Harmonic on the Uniform Sphere

$f(t, \mathbf{x}) = \sin^5(t) z^2$ on $\Gamma = \{x, y, z \mid x^2 + y^2 + z^2 = 1\}$, $0 < t < 2.5$.
Using a time step size $\Delta t = 0.1$, we look at RB and ZZ indicators on a uniform series of meshes.



- Indicators scale like actual error.

A First Adaptive Method: Singular Geometry

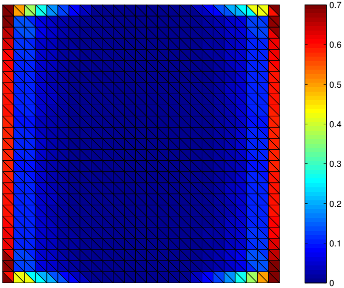


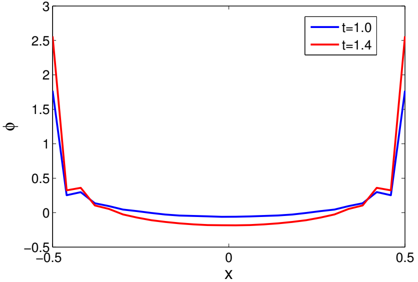
1. Start with coarse space-time grid: $(\Delta t)_i \simeq (\Delta x)_i \simeq h_0 \ \forall \Delta_i$
2. Solve discretisation of $\mathcal{V}\dot{\phi} = \dot{f}$.
3. Compute time-integrated error indicator $\eta(\Delta_i)$
4. $\sum_i \eta(\Delta_i) < \varepsilon \quad \Longrightarrow \quad$ STOP
5. $\eta(\Delta_i) > \delta\eta_{max} \quad \Longrightarrow \quad \Delta_i \to \Delta/4,\ (\Delta t)_i \to \frac{(\Delta t)_i}{2}$
6. GO TO 2.

Adaptivity: Wave Scattering on the Screen (1)

$\mathcal{V}\phi = \sin^5(t)x^2$ on $\Gamma = [-0.5, 0.5]^2 \times \{z = 0\}$, $0 < t < 2.5$, $\Delta t = 0.1$.

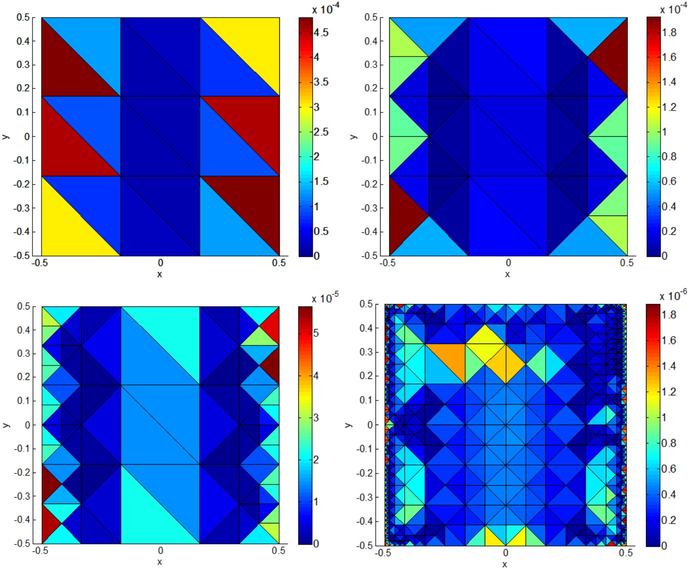
Compare residual indicators, energy, and sound pressure for uniform / adaptive mesh refinements.





- Uniform method: Density ϕ at $t = 1.0, 1.4$

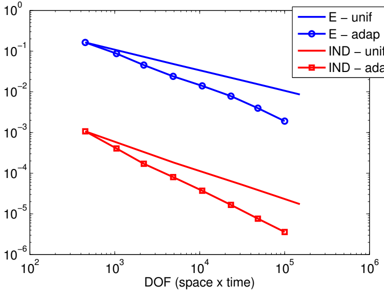
Adaptivity: Wave Scattering on the Screen (2)

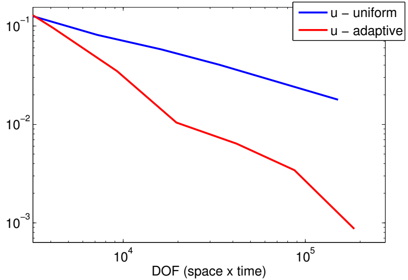


Adaptivity: Wave Scattering on the Screen (3)

$\mathcal{V}\phi = \sin^5(t)x^2$ on $\Gamma = [-0.5, 0.5]^2 \times \{z = 0\}$, $0 < t < 2.5$, $\Delta t = 0.1$.

Compare residual indicators, energy, and sound pressure for uniform / adaptive mesh refinements.

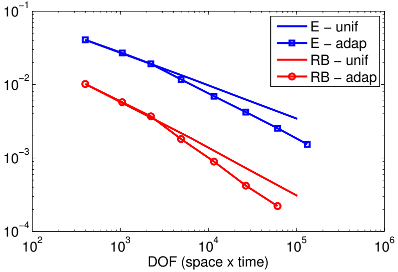


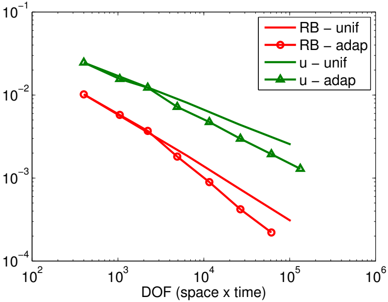


- Convergence rate 0.5 (uniform), 0.77 adaptive reproduces rates for time-independent BEM.

$\mathcal{V}\phi = \sin^5(t)$ on $\Gamma = 30 - 60 - 90$ Triangle, $0 < t < 2.5$.

Compare residual indicators, energy, and sound pressure for uniform / adaptive mesh refinements.

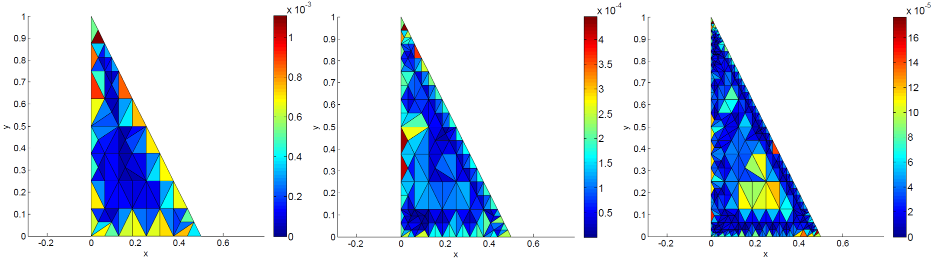




- Convergence rate 0.45 (uniform), 0.65 adaptive.

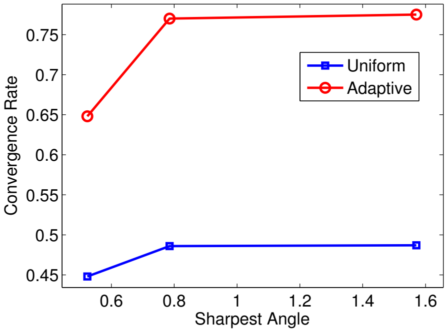
Adaptivity: Wave Scattering on Triangular Screen (2)

$\mathcal{V}\phi = \sin^5(t)$ on $\Gamma = 30 - 60 - 90$ Triangle, $0 < t < 2.5$.



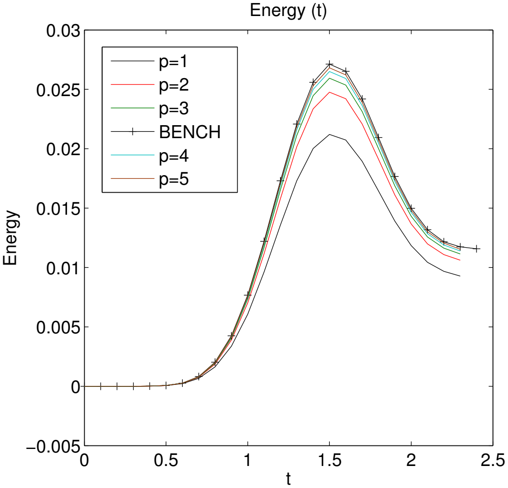
Adaptivity on Screens

$\mathcal{V}\phi = F$ on $\Gamma =$ Polygonal Screen, $0 < t < 2.5$.

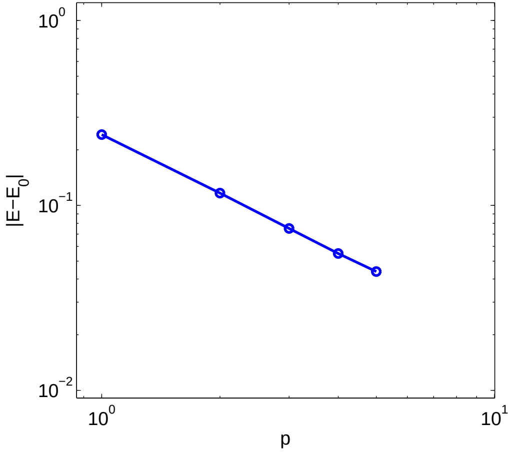


Convergence rate dominated by edge singularity.
Apparent rate decay for small angles due to preasymptotic regime?

p-version TDBEM



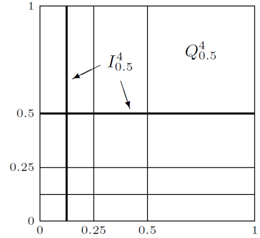
p-version TDBEM



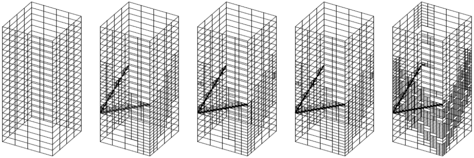
Convergence rate $= 1$, twice the rate of h-version.
Expected from a priori analysis: Error of best approximation $\lesssim \frac{h}{p^2}$

Work in Progress

p-version and hp-graded meshes:



Space-time adaptivity:



(a) mesh 1 (b) mesh 4 (c) mesh 7 (d) mesh 10 (e) mesh 13

In 2d: picture by Glaefke

Conclusions

Analysis & numerics: convergence rates for screen problems
(in DOF on a 2d screen)

- 0.5: h-version, uniform
- 0.77: h-version, adaptive (as for Δ)
- 1.0: p-version, uniform
- $\beta/2$: h-version, β-graded, $\beta \in [1, 3)$

in particular:

- Singular expansions & (optimal) graded meshes for edge and corner singularities.
- A posteriori estimate for numerical approximations without assumptions on mesh (TDBEM, CQ, ...)

H. Gimperlein, F. Meyer, C. Oezdemir, D. Stark, E. P. Stephan, Boundary elements with mesh refinements for the wave equation. preprint.

H. Gimperlein, C. Oezdemir, D. Stark, E. P. Stephan, A residual a posteriori error estimate for the time domain boundary element method. preprint.

Thank you very much