A new second-order probability density function for the surface elevation in irregular seas

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Abstract:

To date, the predominant means for computing the probability density function (PDF) for the free surface elevation of a nonlinear, irregular water wave field, free of assumptions involving narrow-bandedness and small directionality, is the approximate Gram-Charlier series solution of Longuet-Higgins (1963, hereafter LH63). In this work we re-visit the derivation of this PDF to second order in the wave steepness, utilizing both moment and cumulant generating functions. We show that LH63's approximate solution based on the cumulant generating function, in fact, matches that derived from the moment generating function. Moreover, through a change of variables, it is shown that the approximation employed by LH63 is unnecessary, and the second-order PDF stemming from the cumulant generating function can be represented exactly in terms of the Airy function. The new second-order PDF predicts increased probability of extreme positive surface elevations typical of e.g. rogue waves, relative to both second- and third-order solutions of LH63. A semi-theoretical method is also proposed for remedying (non-physical) spurious oscillations that arise in the negative tail, based on the envelope of the Airy function with negative arguments. The new PDF is compared against those based on data sets generated from the second-order irregular wave theory of Madsen & Fuhrman (2012), as well as the fully-nonlinear, spectrally-accurate numerical wave model of Klahn et al. (2021a,b). Good accuracy is collectively demonstrated for directionally-spread irregular seas in both finite and infinite water depth, for a range of directional spreading.

References:

Klahn, M., Madsen, P. A. & Fuhrman, D. R. 2021a Simulation of three-dimensional nonlinear water waves using a pseudospectral volumetric method with an artificial boundary condition. *Int. J. Numer. Meth. Fluids* **93**, 1843–1870.

Klahn, M., Madsen, P. A. & Fuhrman, D. R. 2021b On the statistical properties of surface elevation, velocities and accelerations in multi-directional irregular water waves. *J. Fluid Mech.* **910**, A23.

Longuet-Higgins, M. S. 1963 The effect of non-linearities on statistical distributions in the theory of sea waves. *J. Fluid Mech.* **17**, 459–480.

Madsen, P. A. & Fuhrman, D. R. 2012 Third-order theory for multi-directional irregular waves. *J. Fluid Mech.* **698**, 304–334.