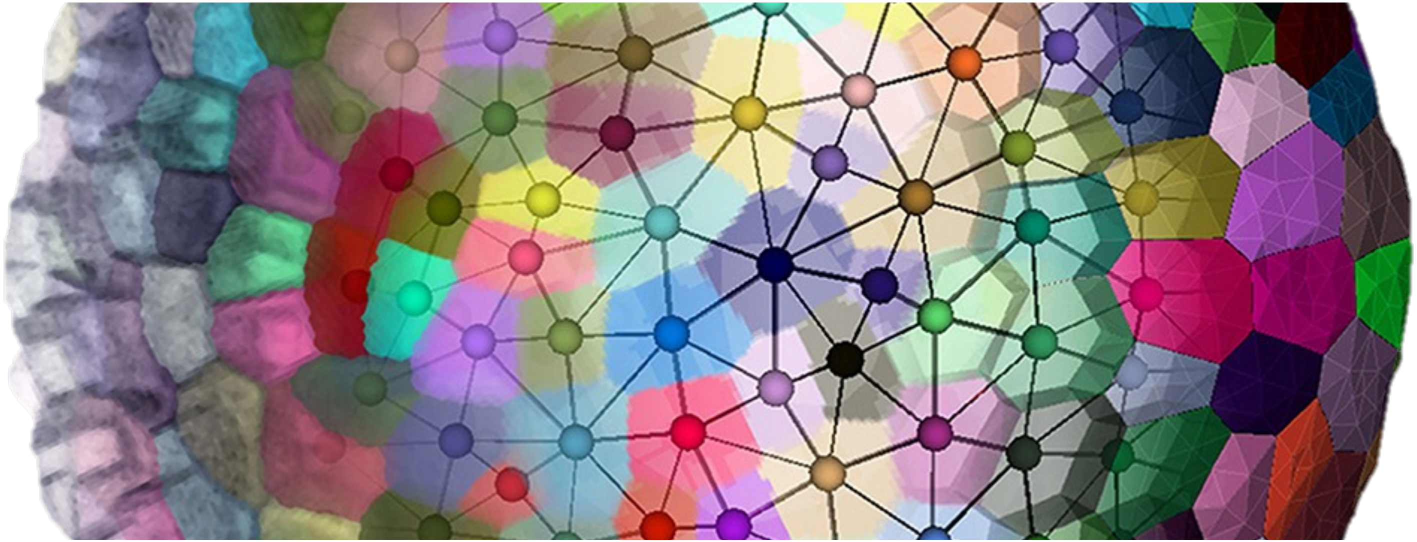

Plant Tissues as Topological Complexes:
Reconstruction & Other Combinatorial Challenges

Guillaume CERUTTI

30/09/2025

Combinatorics & Life Sciences

Mosaic
Morphogenesis: Simulations & Analysis *in silico*

Data-driven models of plant tissue morphogenesis

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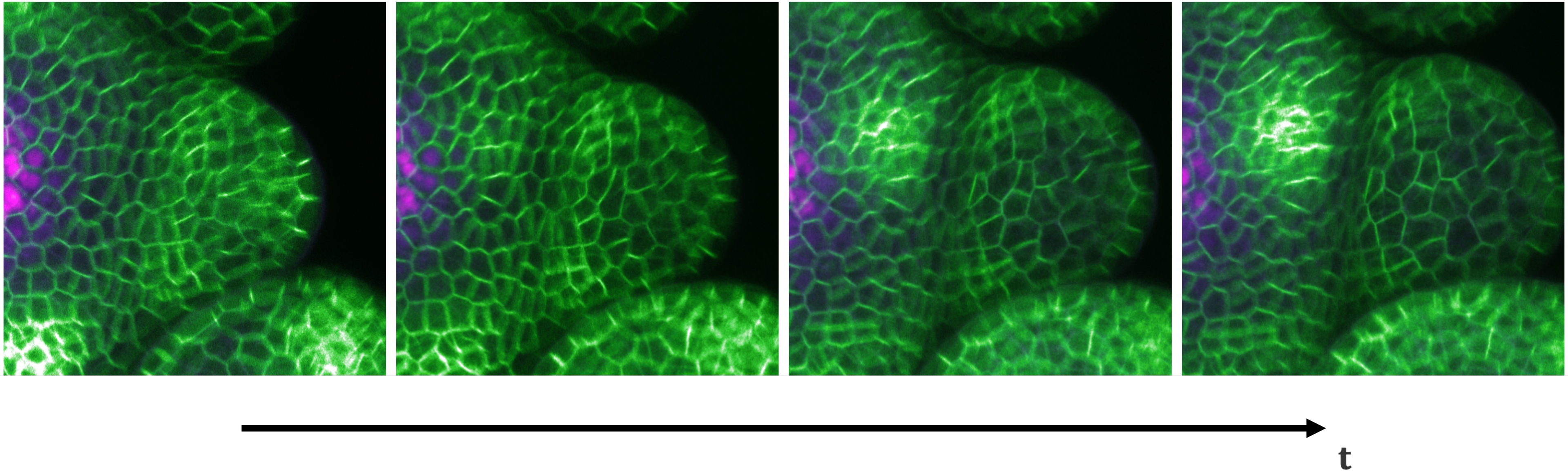

Data courtesy: C. Galvan-Ampudia (RDP)

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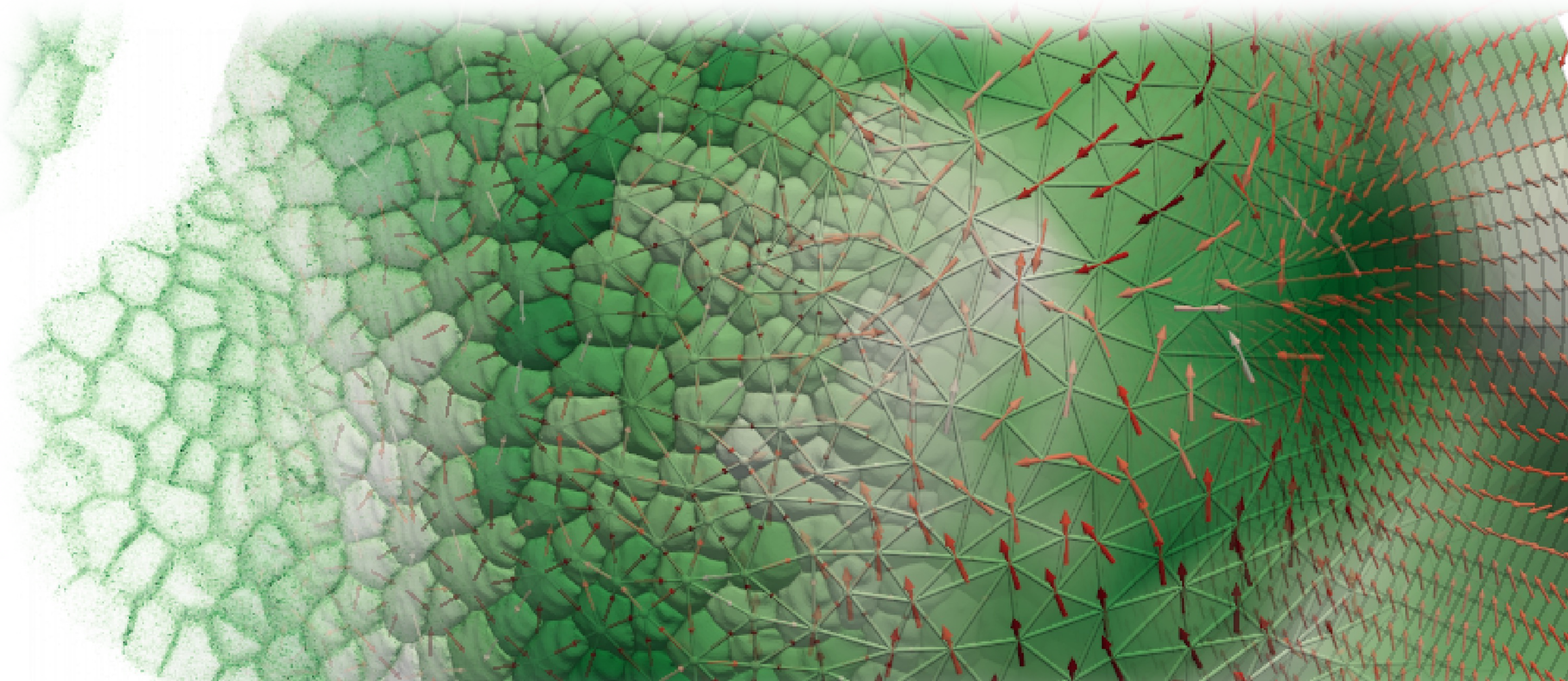

From experimental data to simulation-ready tissues

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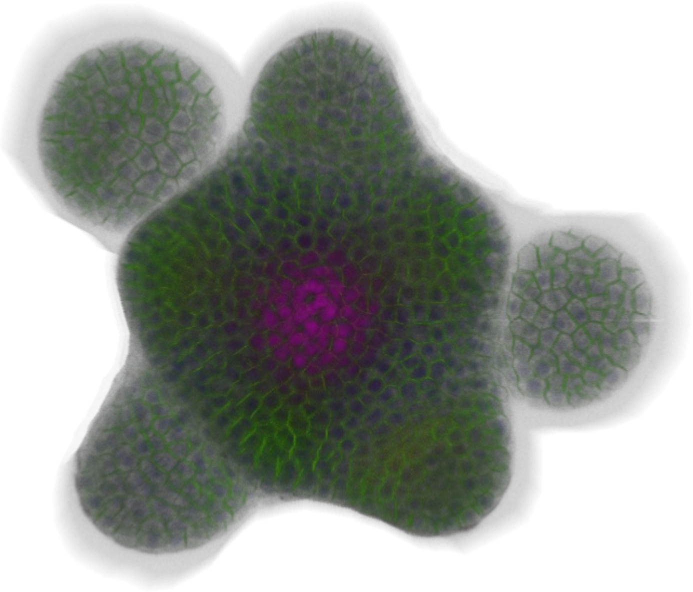

3D Confocal Images

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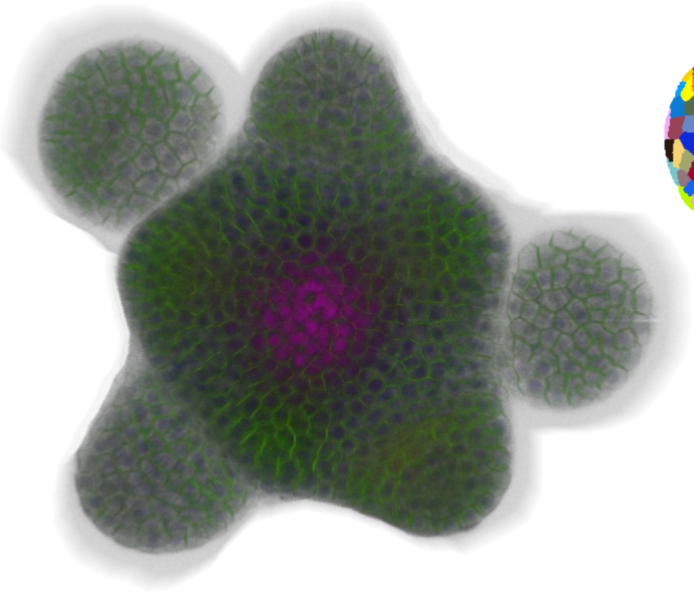

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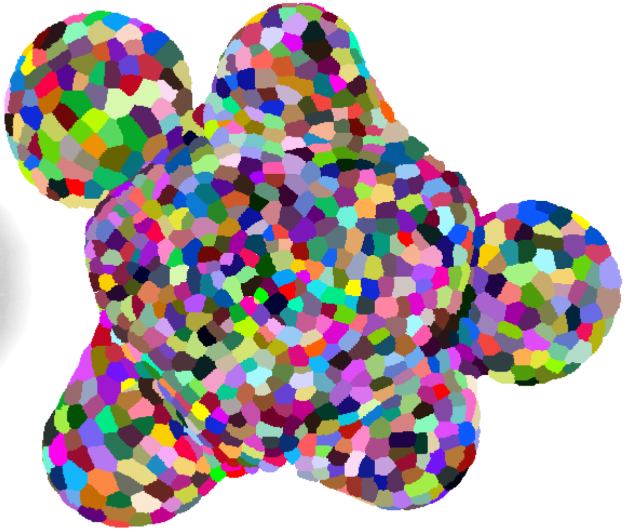

3D Watershed Segmentation

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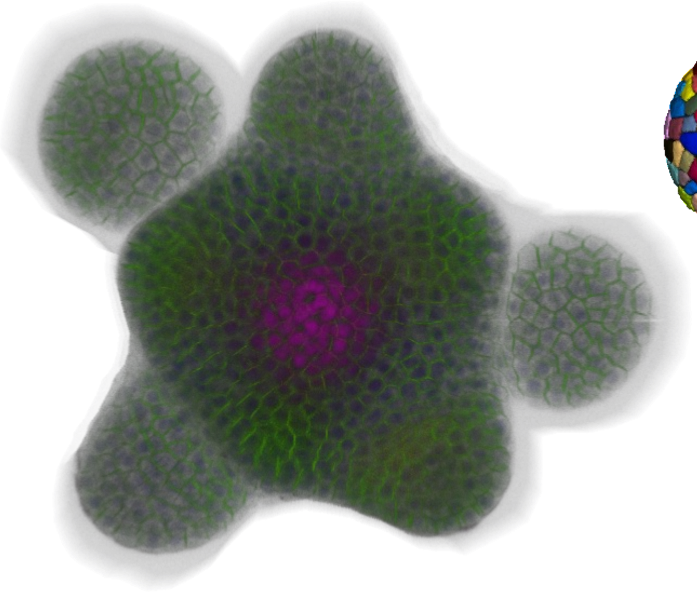

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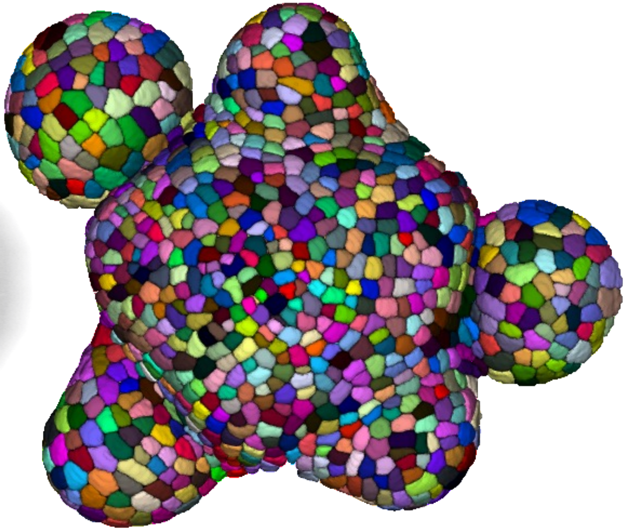

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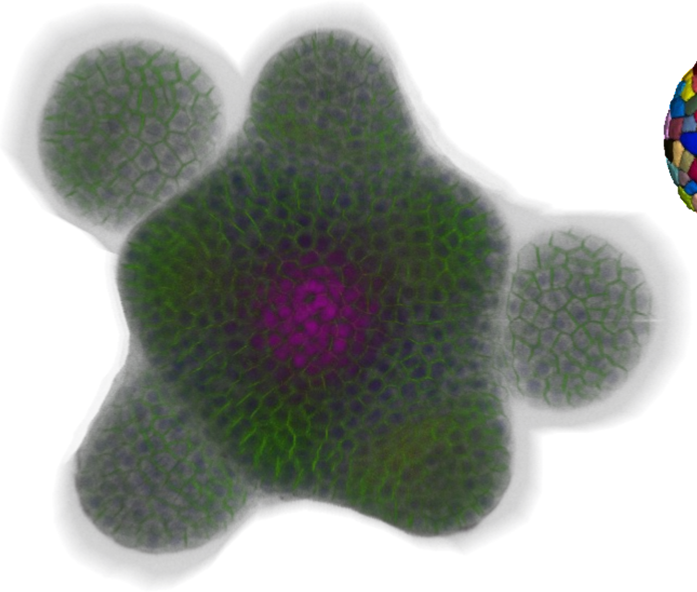

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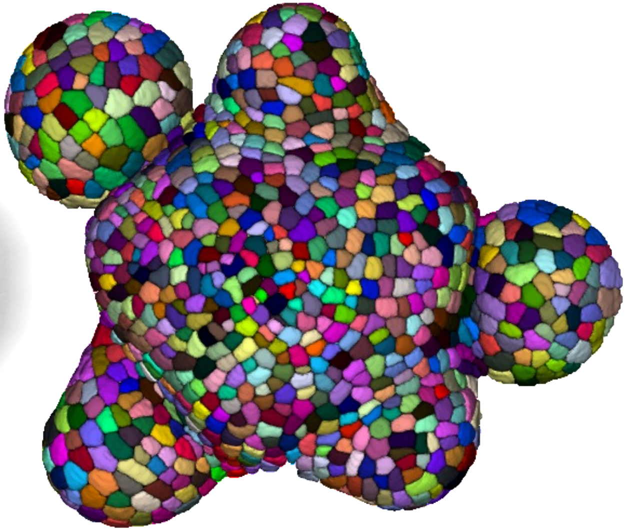


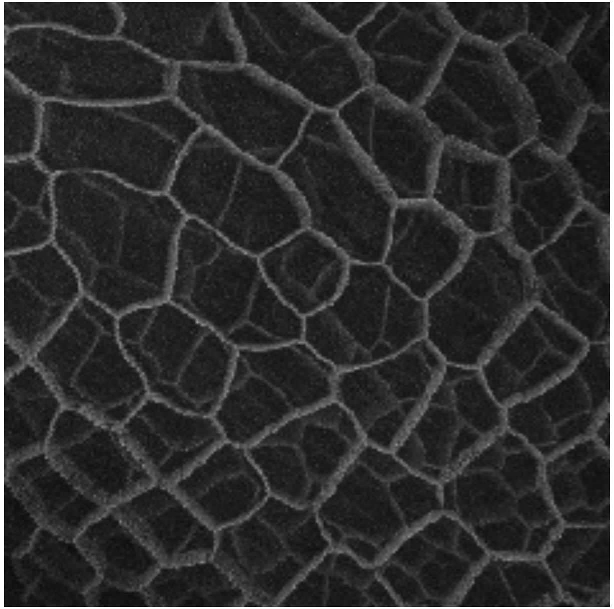
3D Watershed Segmentation

Realistic templates for numerical simulations (tissue mechanics FEM, discrete network ODEs)

Comparison of simulation outputs with quantitative measures computed on experimental data

Data courtesy: C. Galvan-Ampudia (RDP)

Multicellular Tissue = Dual of a Simplicial Complex

Data courtesy: J. Braat (RDP)

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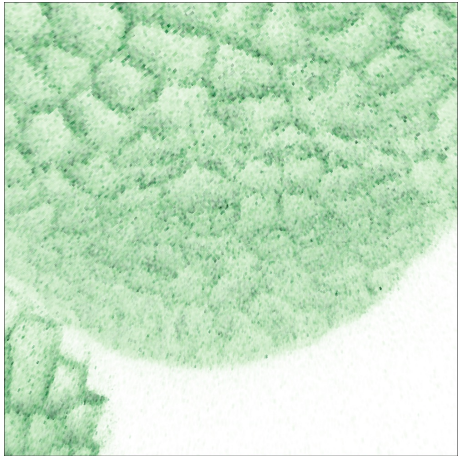
A minimalistic representation of tissue dynamics



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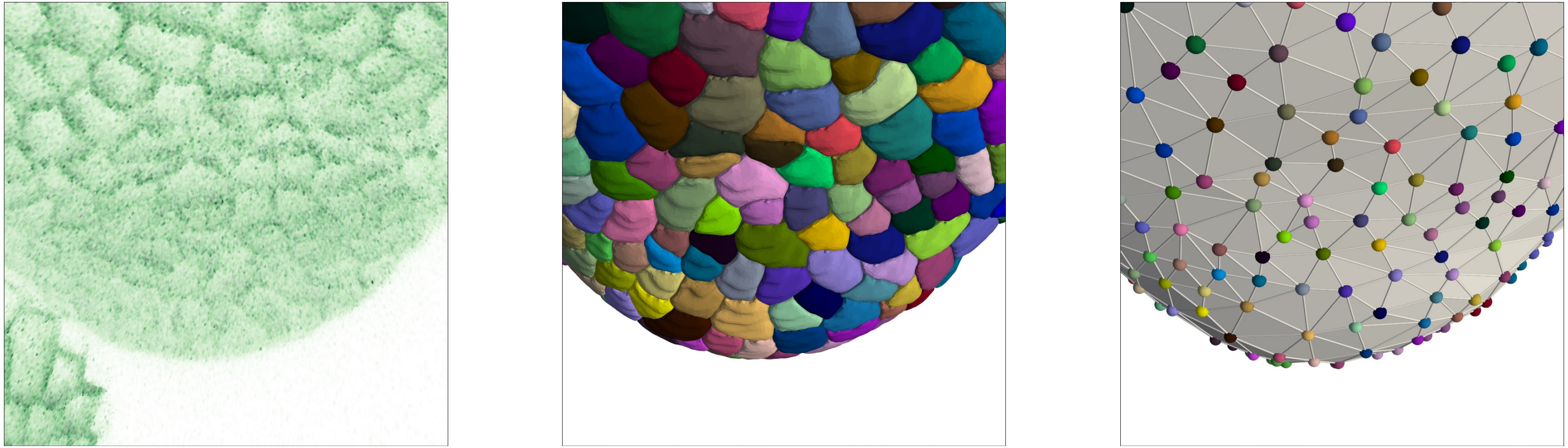

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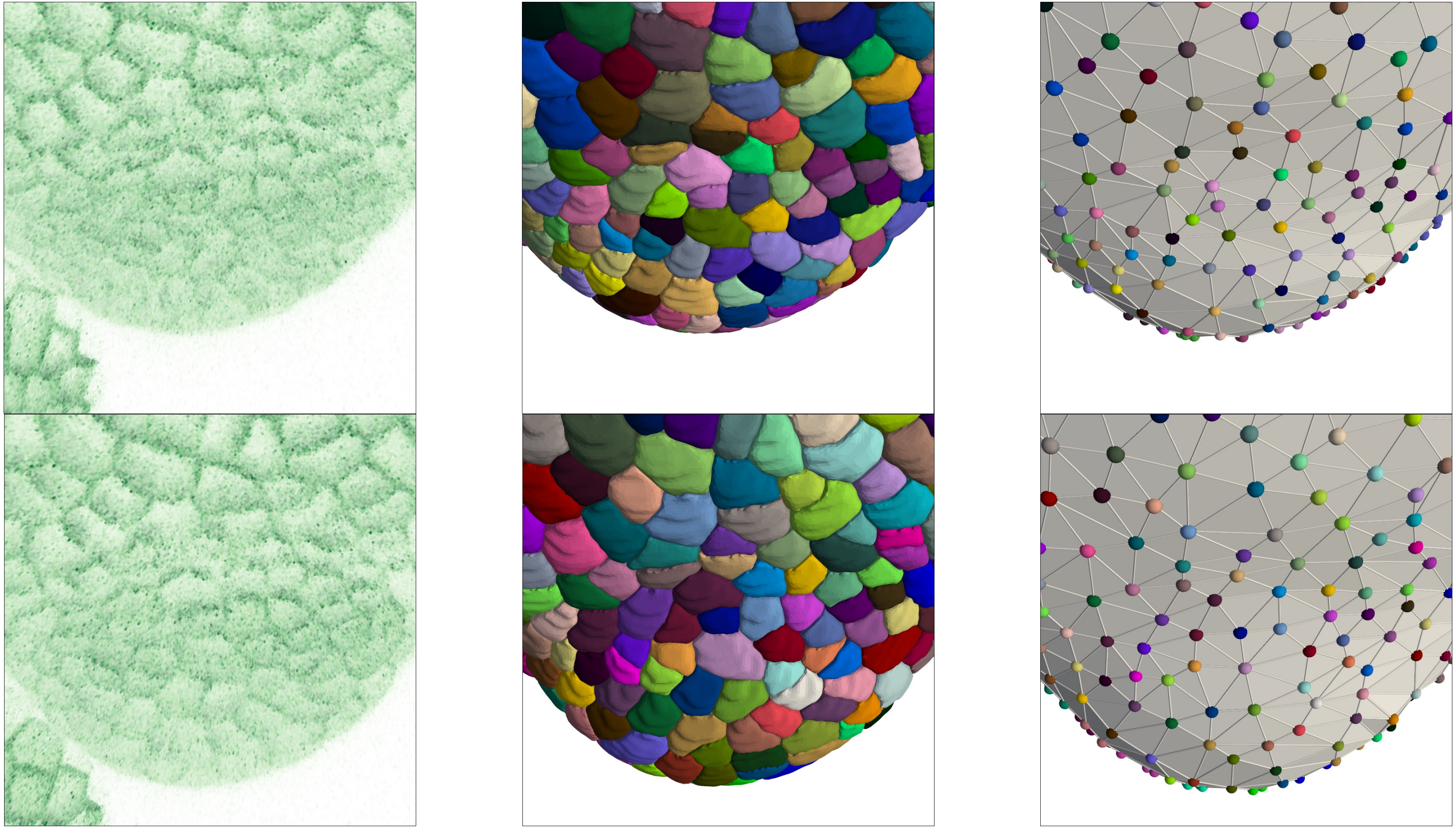

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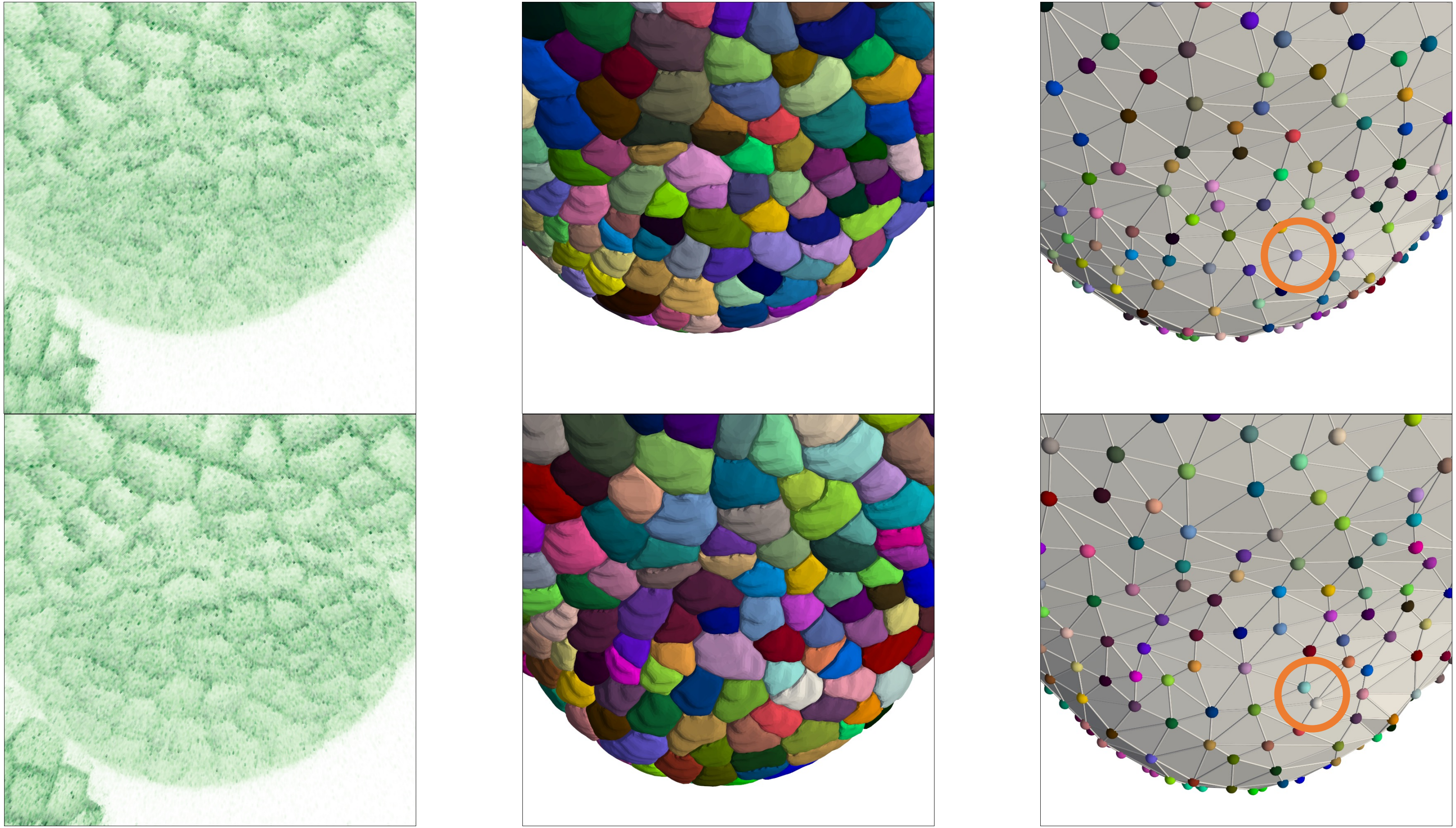

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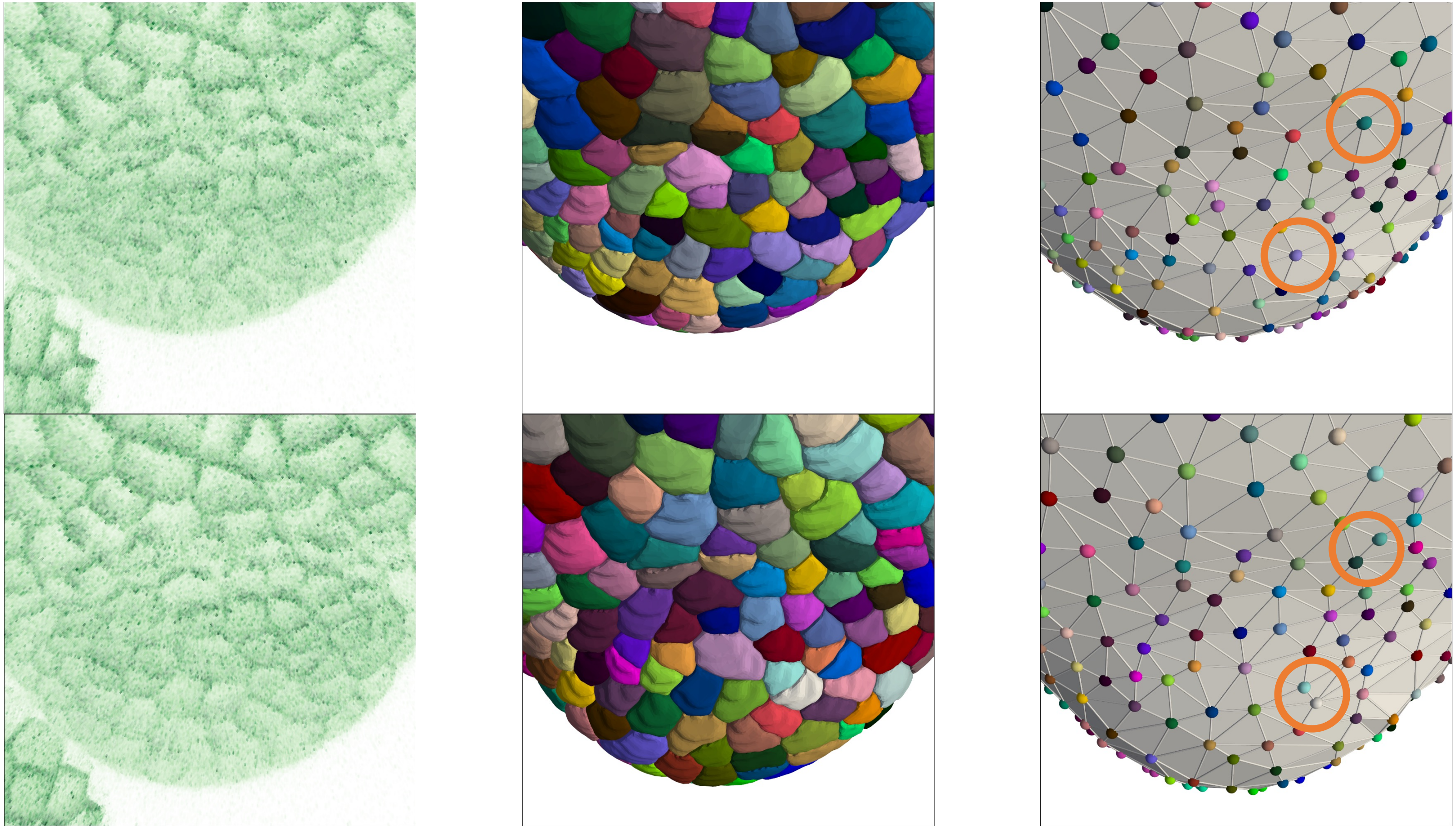

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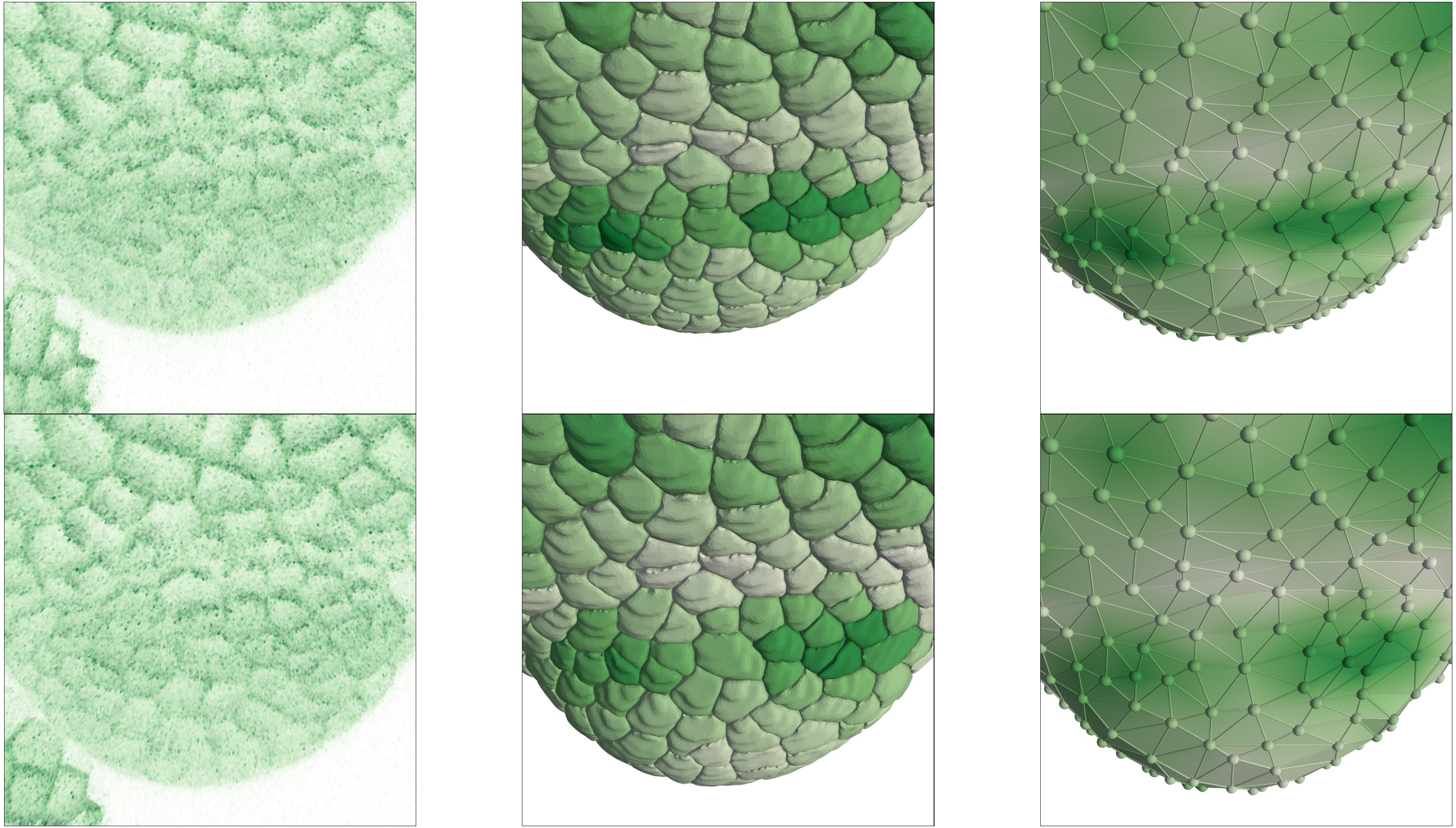

Tissue simplicial complex reconstruction problem

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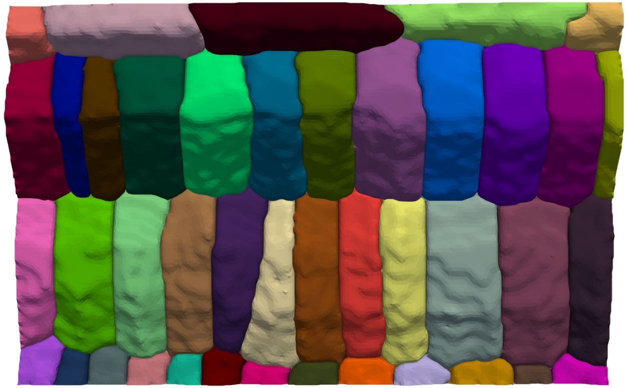
Given a tissue represented by a 2D / 3D labelled (segmented) image



Data courtesy: E. Gascon (Mosaic/RDP)

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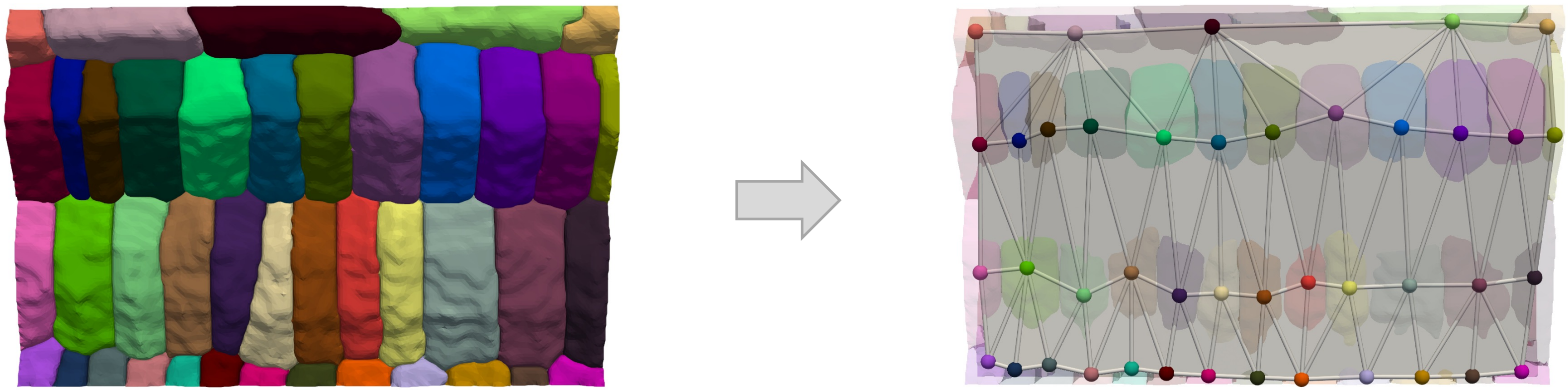

Build the set of simplices corresponding to the adjacency between tissue cells
(ensuring they form a valid triangulation / tetrahedralization)

Data courtesy: E. Gascon (Mosaic/RDP)

Simplicial Complexes: geometrical definitions

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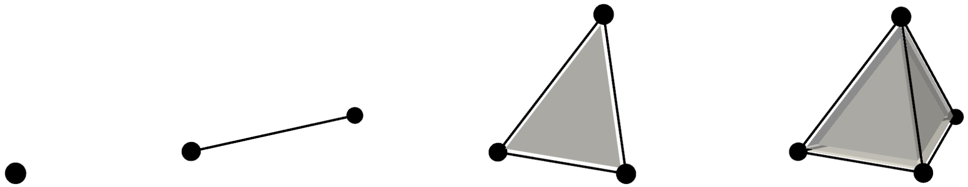
- n-simplicial complex : **partitioning** of a subset of $\mathbb{R}^d$ $(m \geq n)$ into **n-simplices**

Convex hull of
$n+1$ geometrically
independent points

0-simplex | 1-simplex | 2-simplex | 3-simplex



vertex | edge | triangle | tetrahedron

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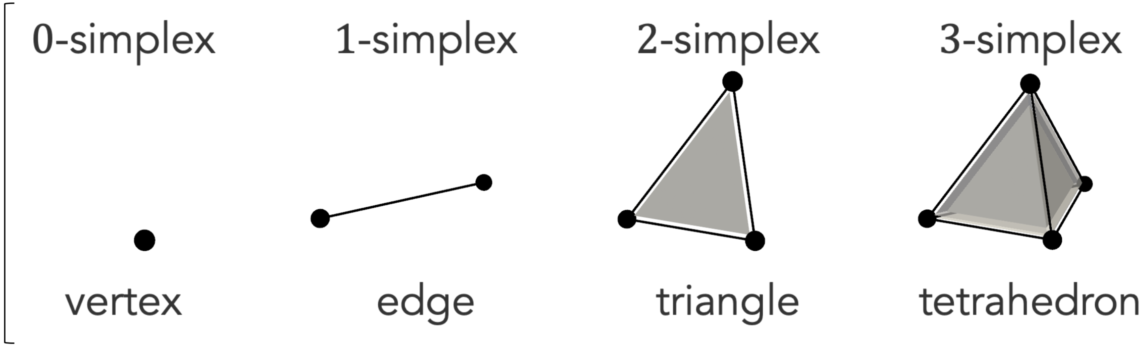

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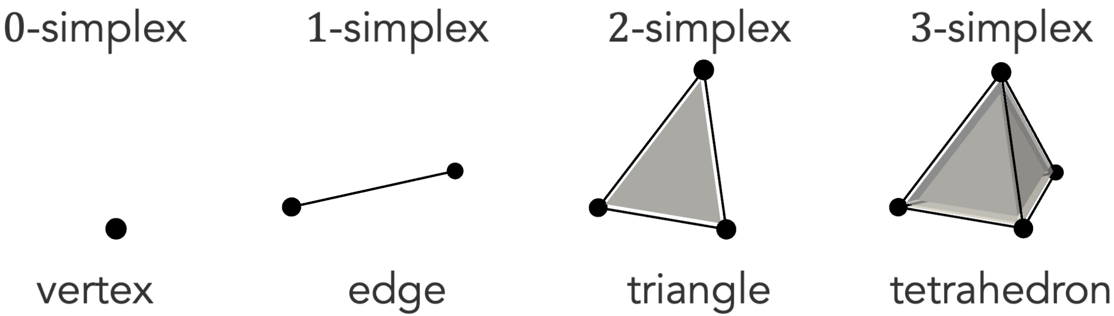

- Includes all the boundaries (**"faces"**) of its simplices (closed under taking faces)

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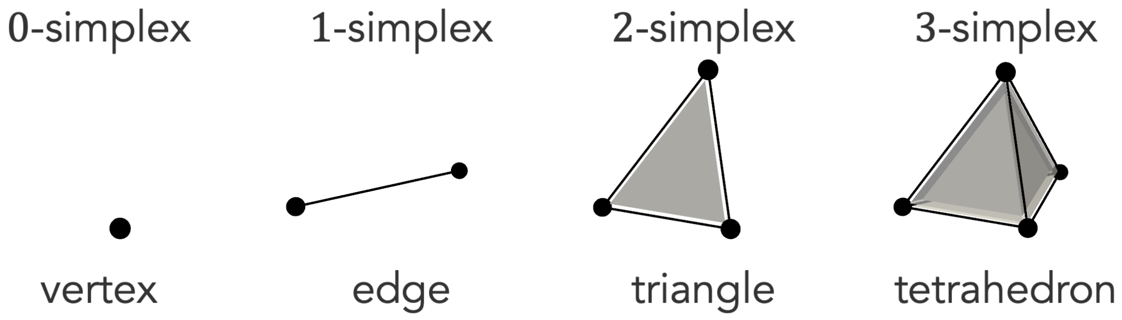

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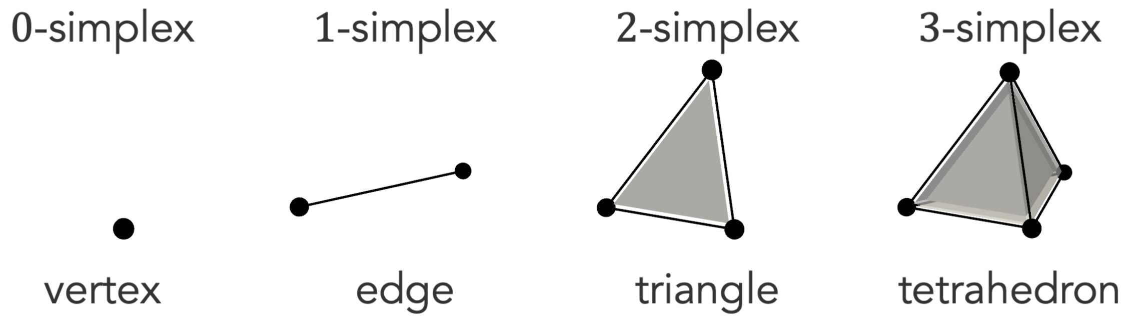

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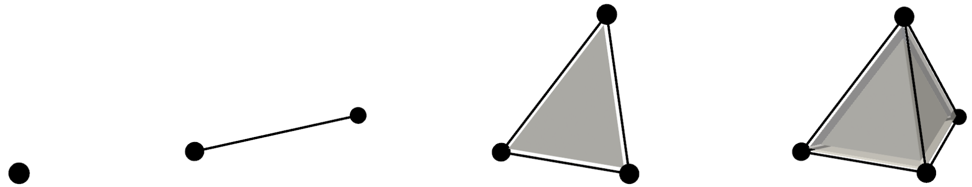

vertex	edge	triangle	tetrahedron

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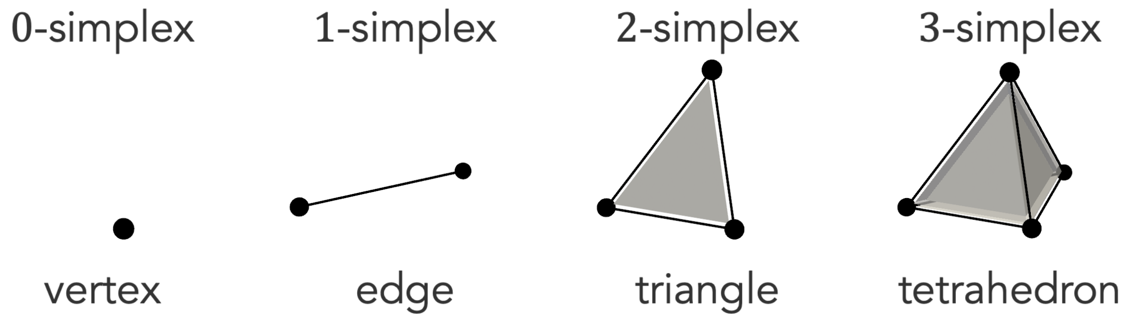

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Topological complexes from labelled tissue images

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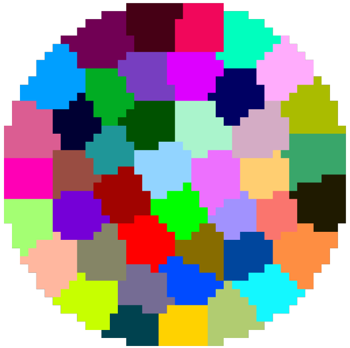
2D Tissue Image



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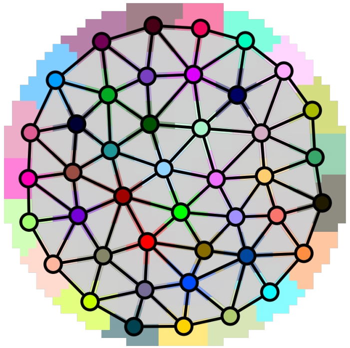

$d = 2, n = 2$

Topological complexes from labelled tissue images

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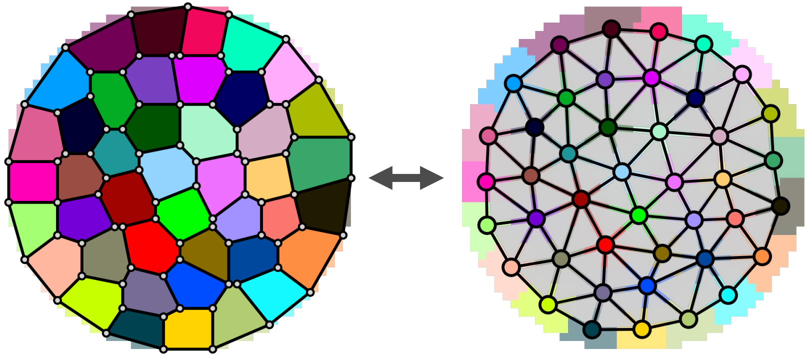

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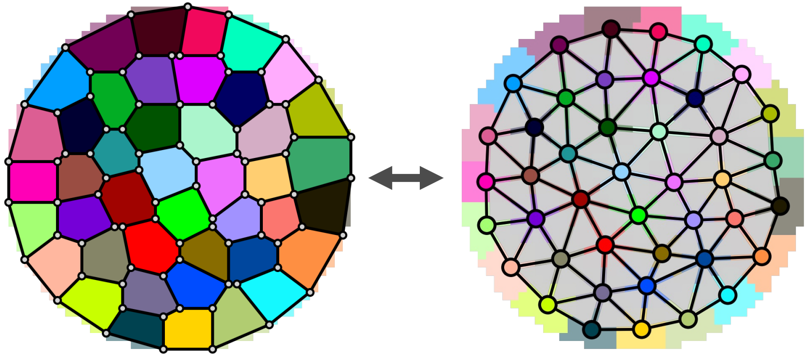

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3D Tissue Image

Topological complexes from labelled tissue images

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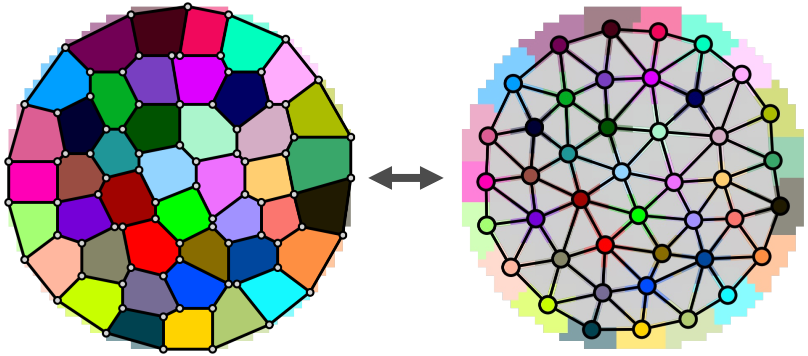

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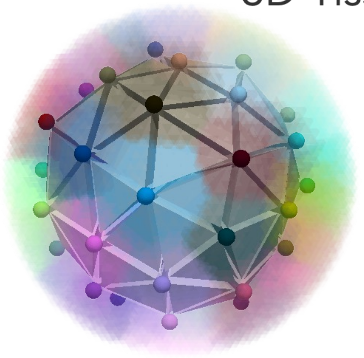

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Topological complexes from labelled tissue images

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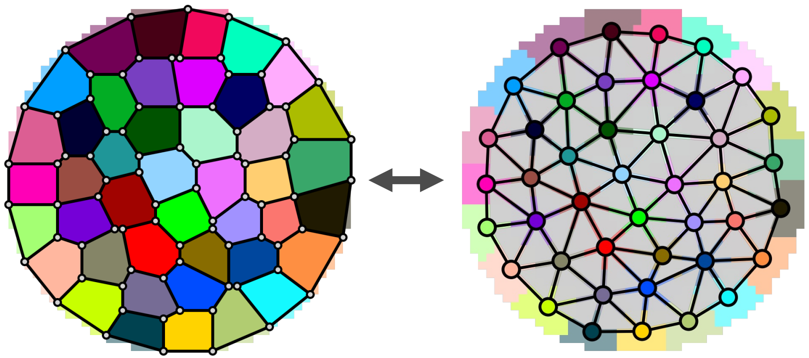

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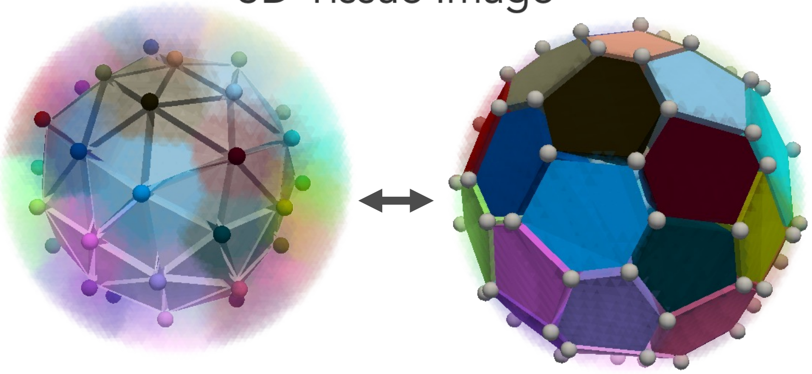

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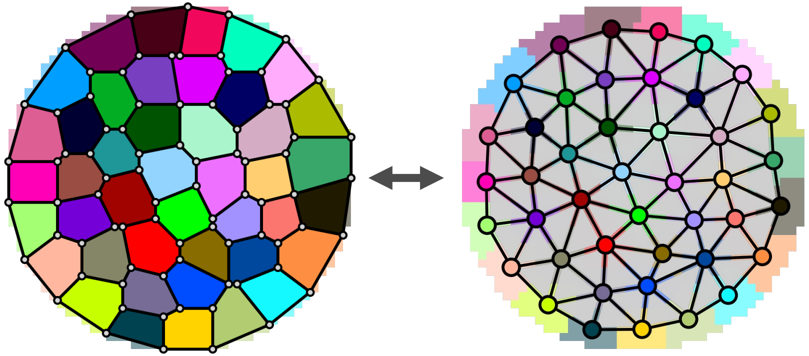

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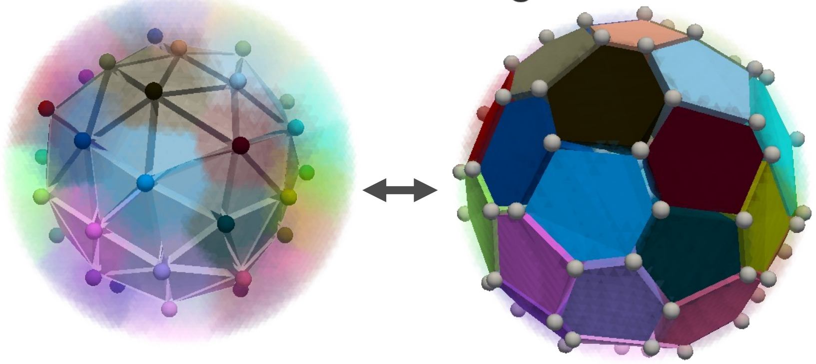

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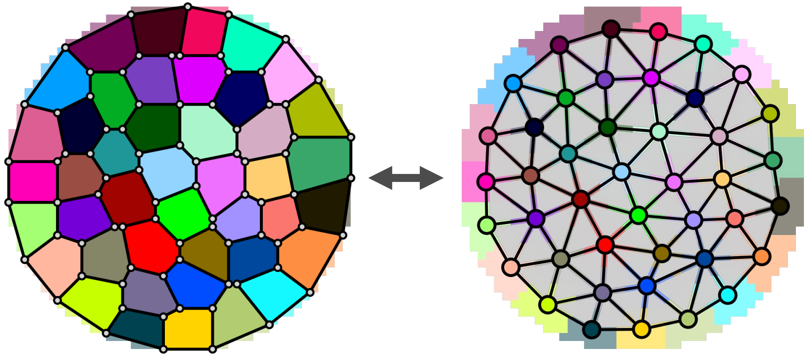

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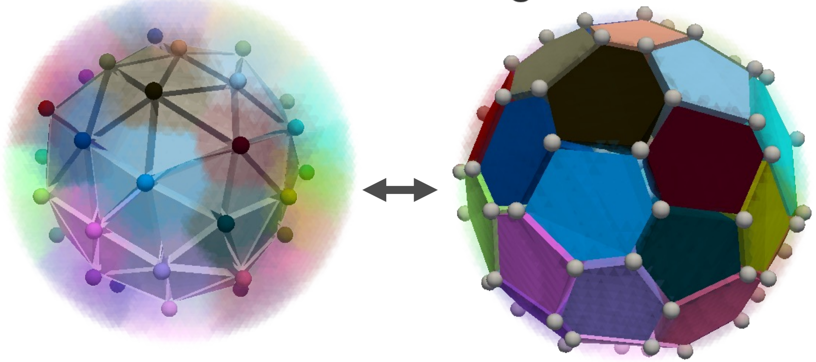

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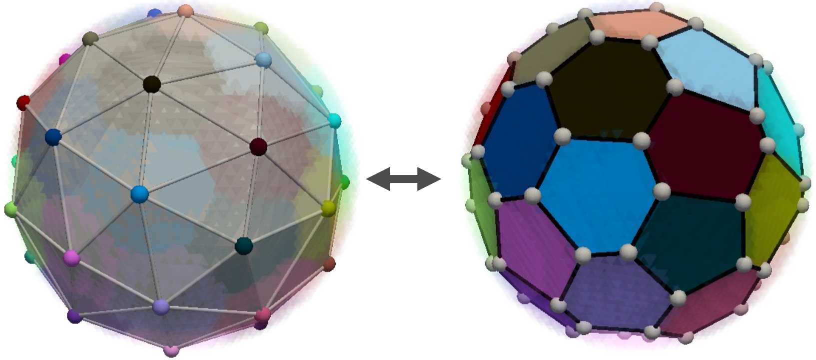

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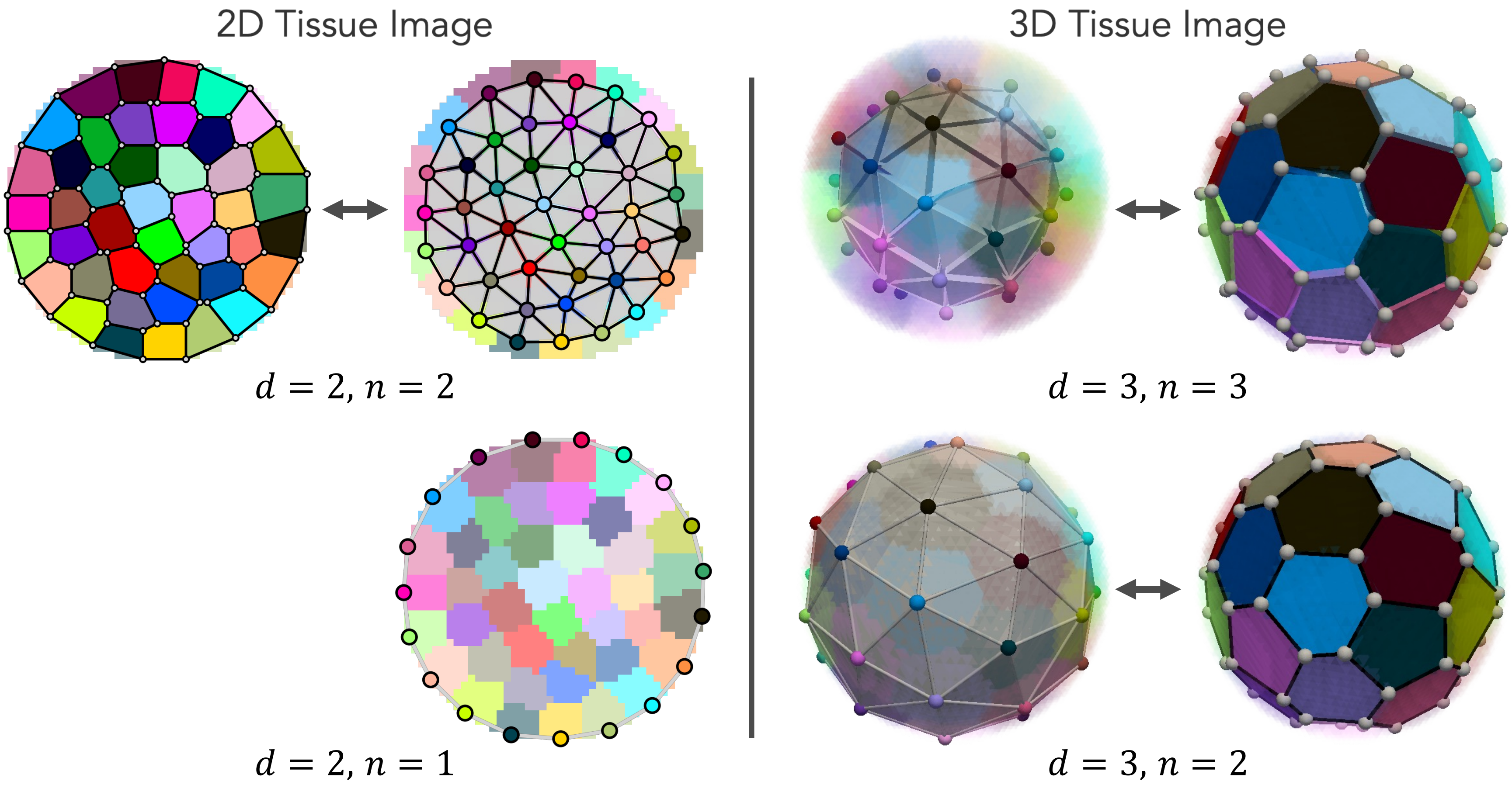

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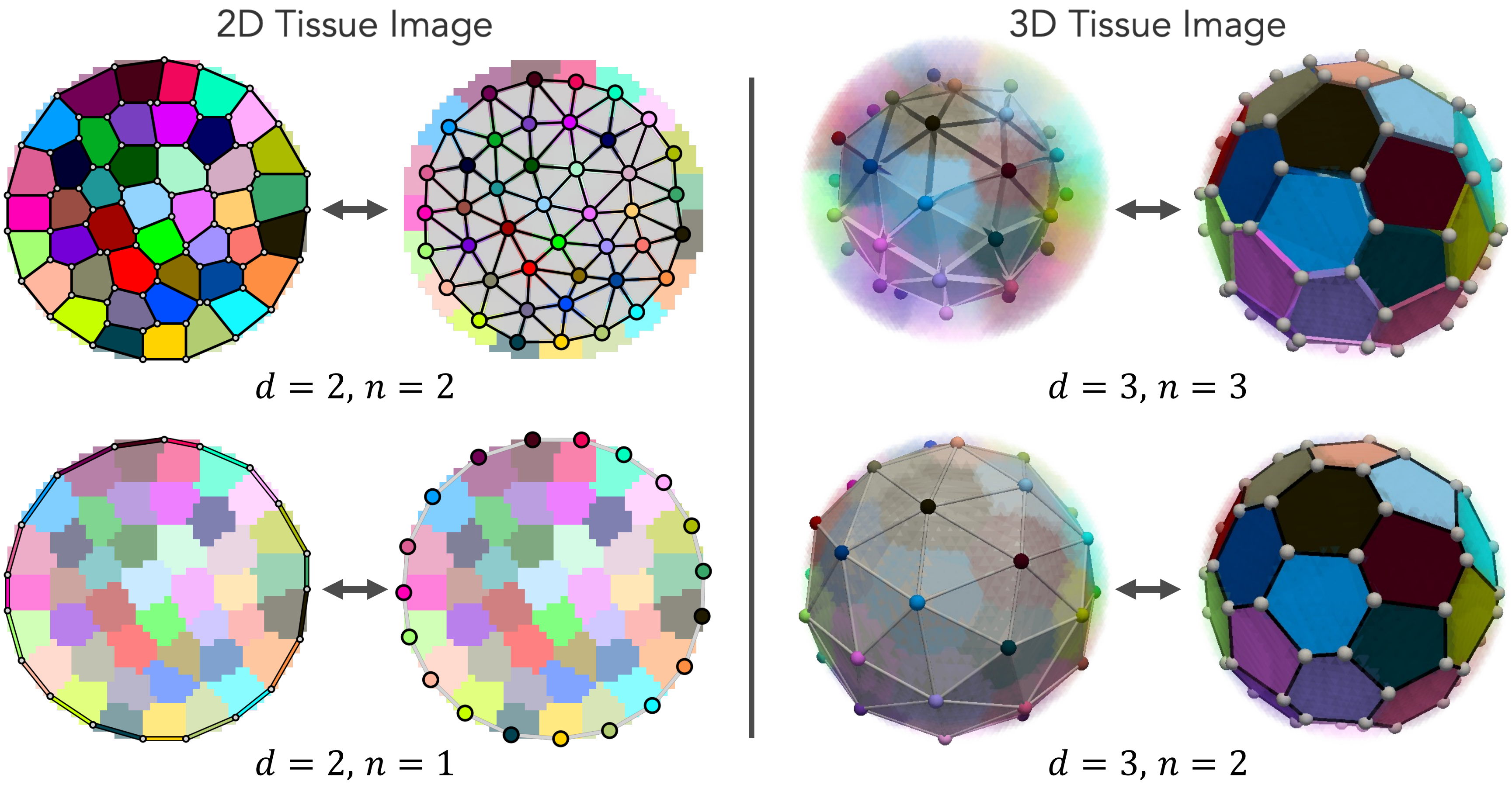

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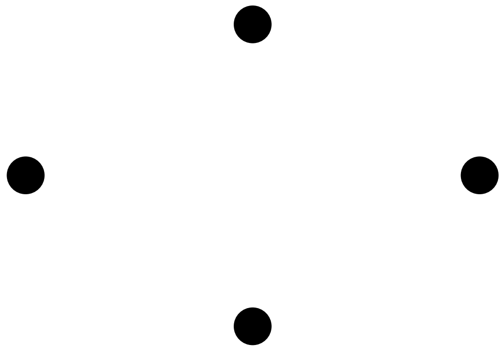

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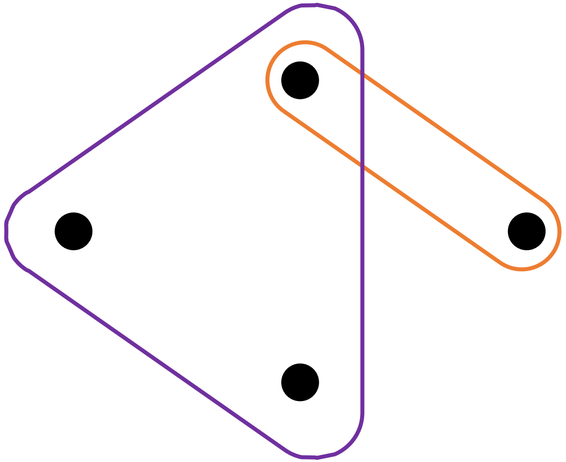

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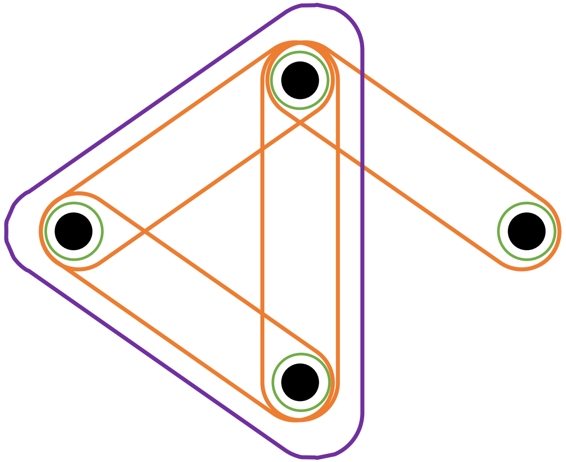

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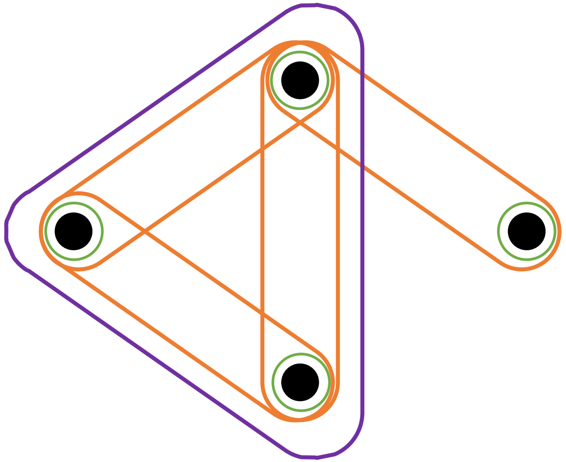

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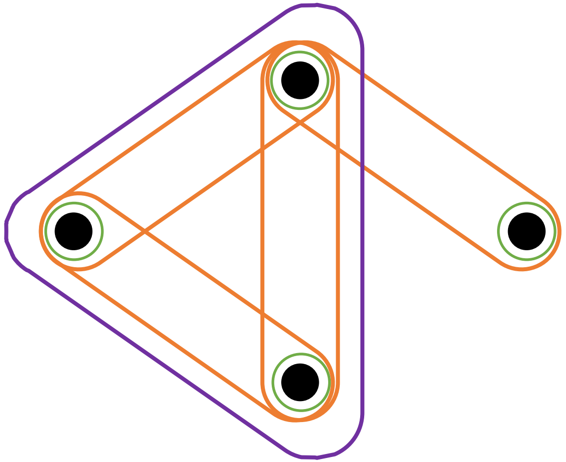

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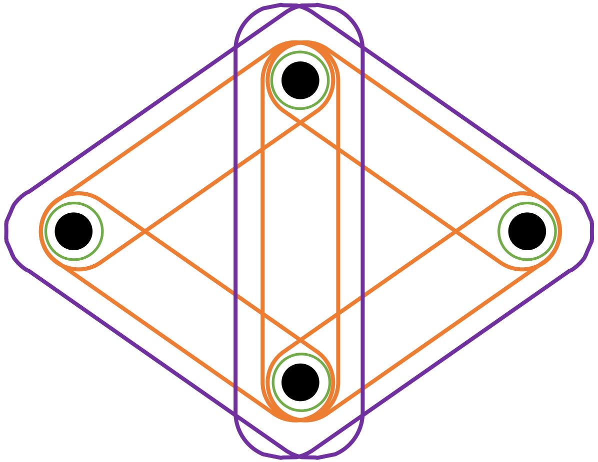

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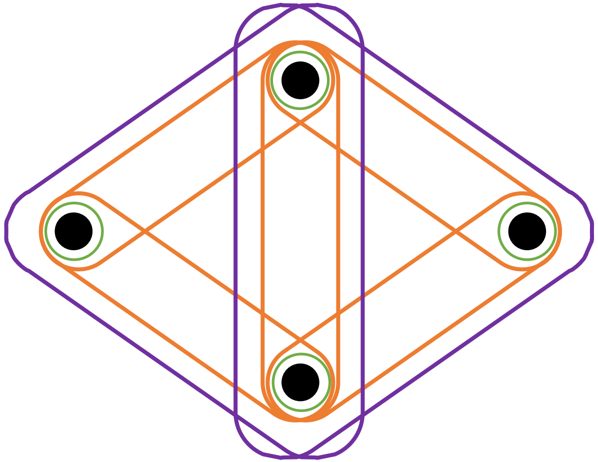

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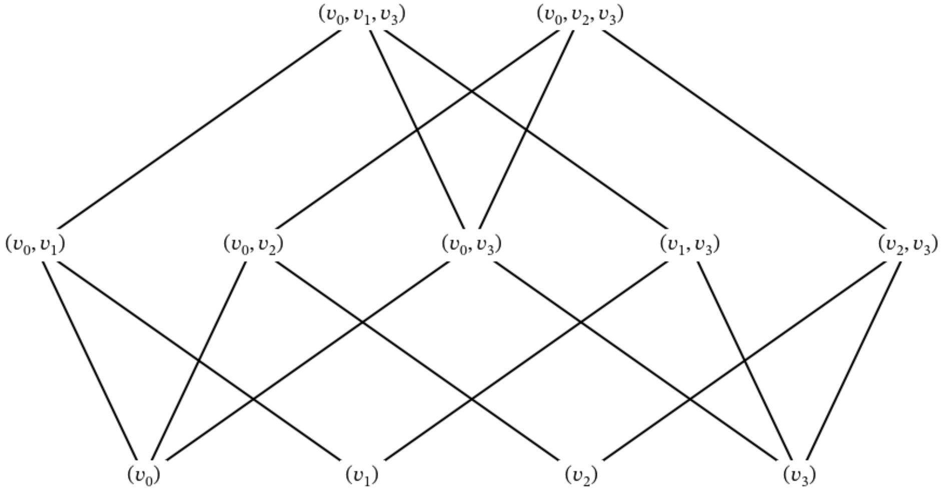

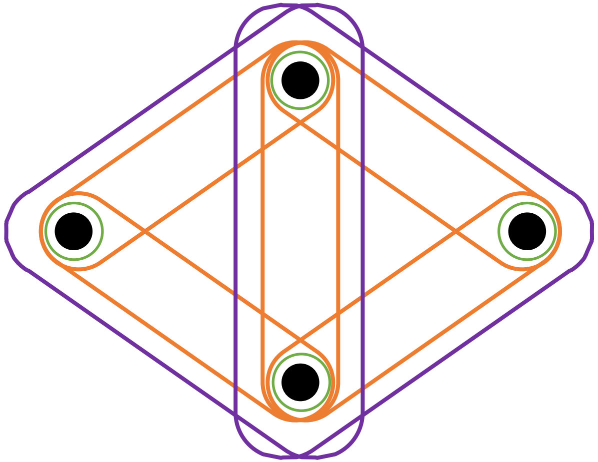

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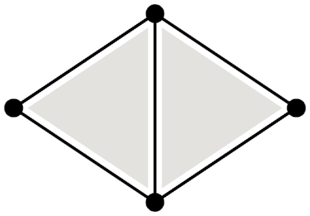

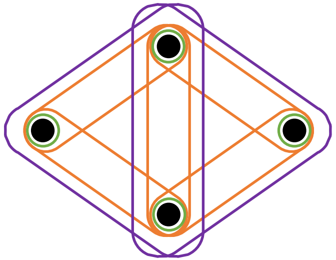

Combinatorial core: abstract simplicial complexes

- Every simplicial complex is a **geometric realization** of an abstract complex K
- Given a set of "vertices" V : collection of subsets of V, **closed under taking subsets**
  - Subset σ of size $k+1$ = k-simplex: **dimension** n of K as the max k in all simplices
  - A n-dimensional ASC K is **pure** if any $\sigma \in K$ is a subset of at least one n-simplex
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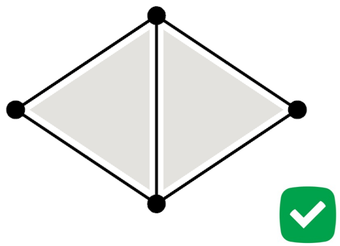

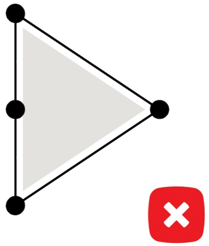

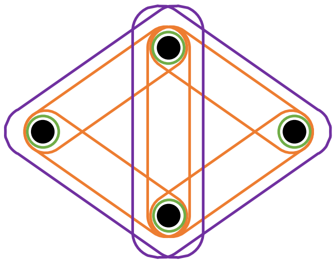

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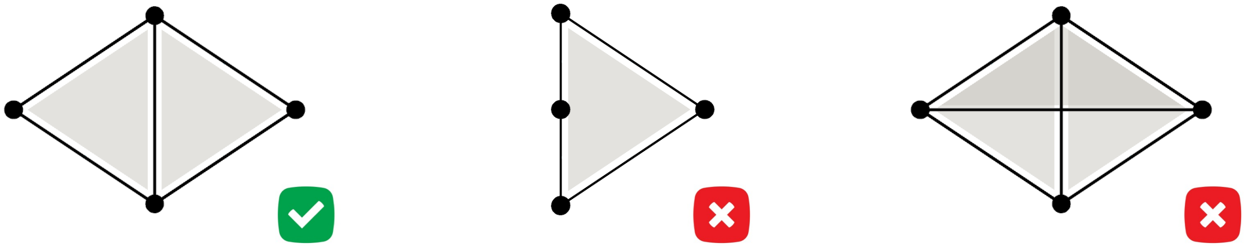

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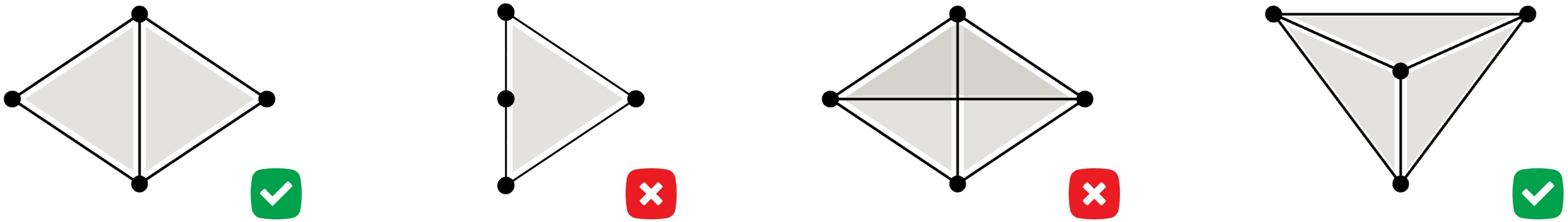

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Not all embeddings of an ASC lead to a geometric realization (invalid geometric SC)

Adjacency simplices from labelled image topology

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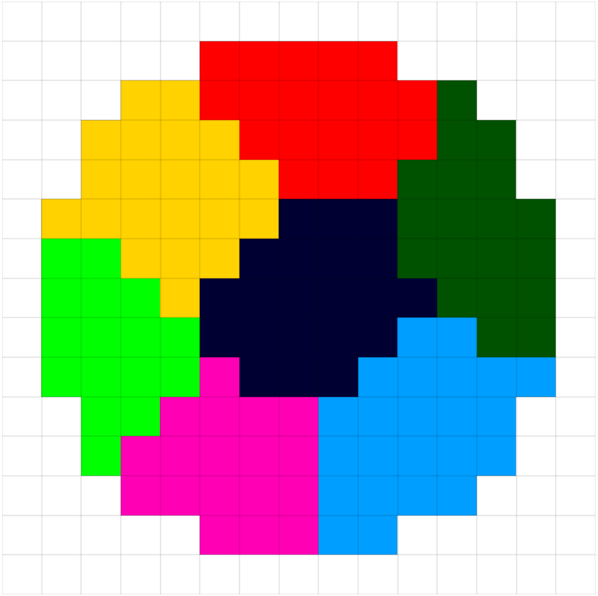
2D Tissue Image



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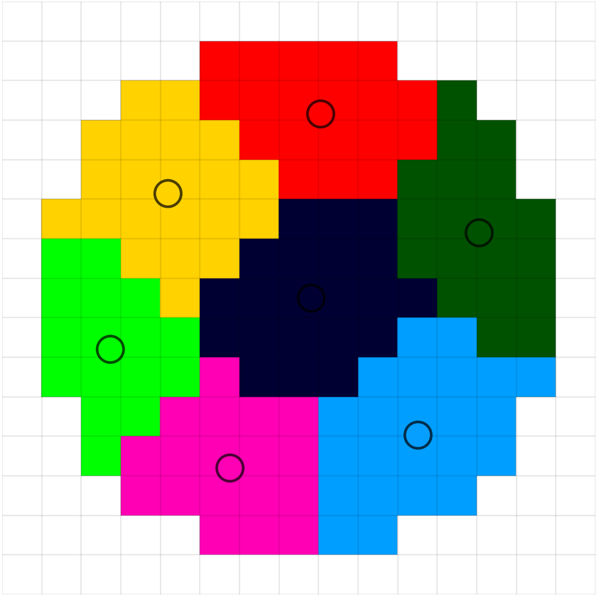

Vertices V and their geometric embedding $\mathbf{p}$ are (almost) imposed by image labels

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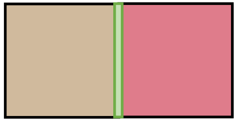
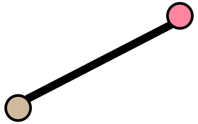
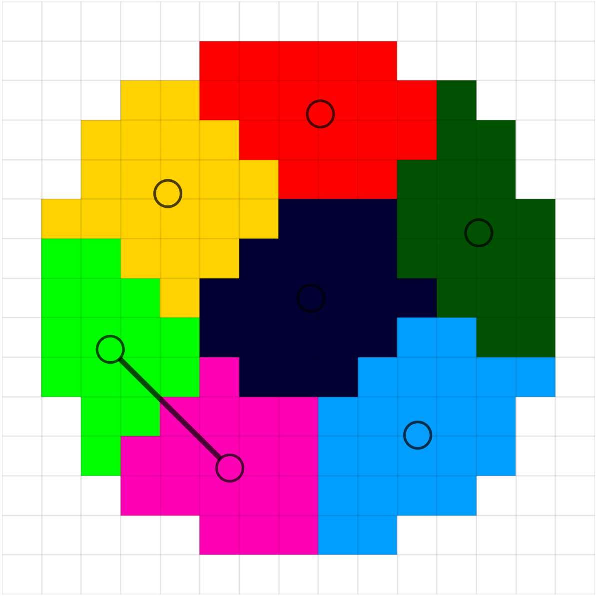
Linel (,)



1-simplex





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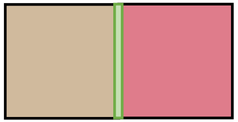

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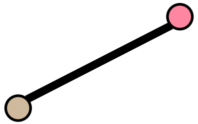

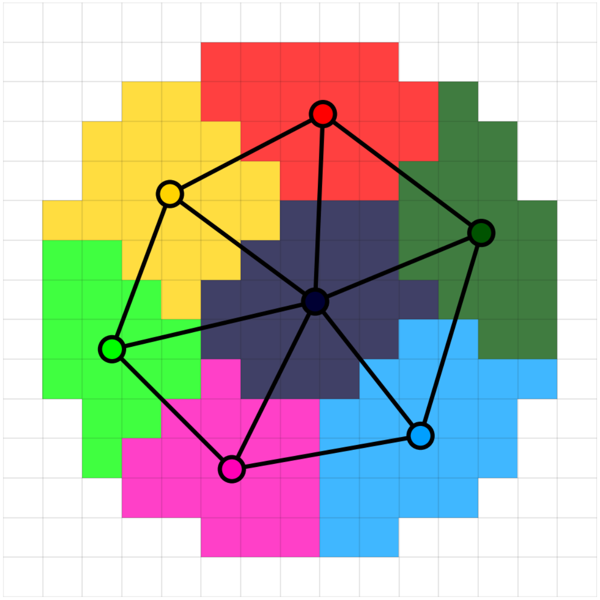

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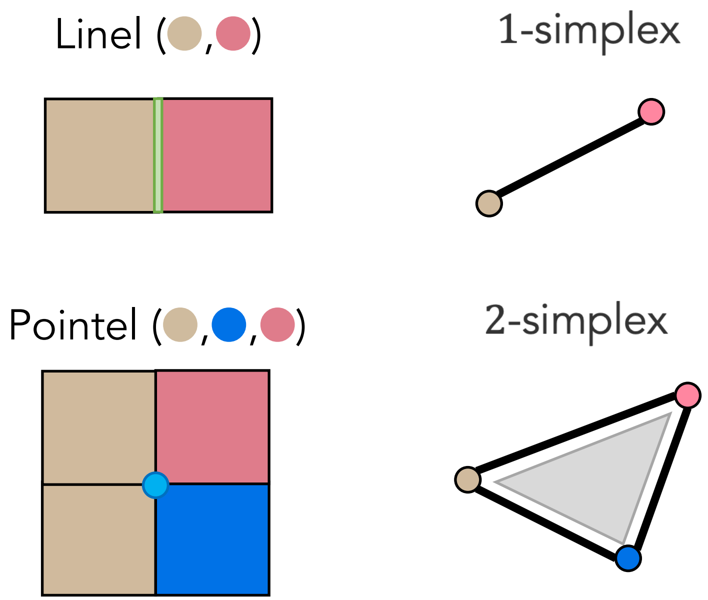

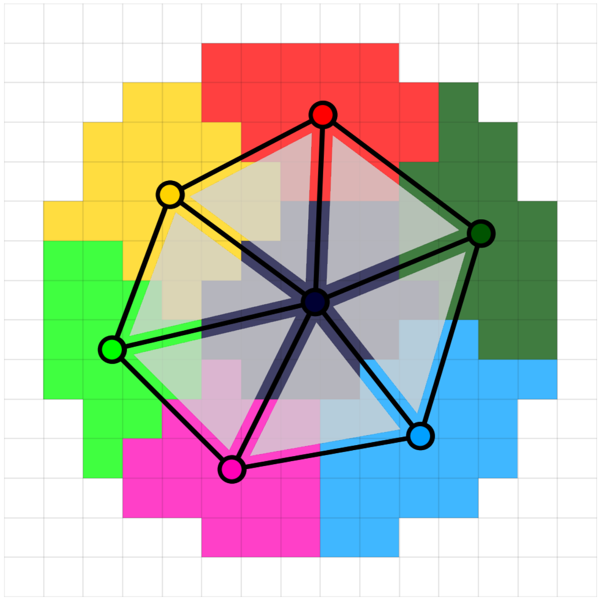

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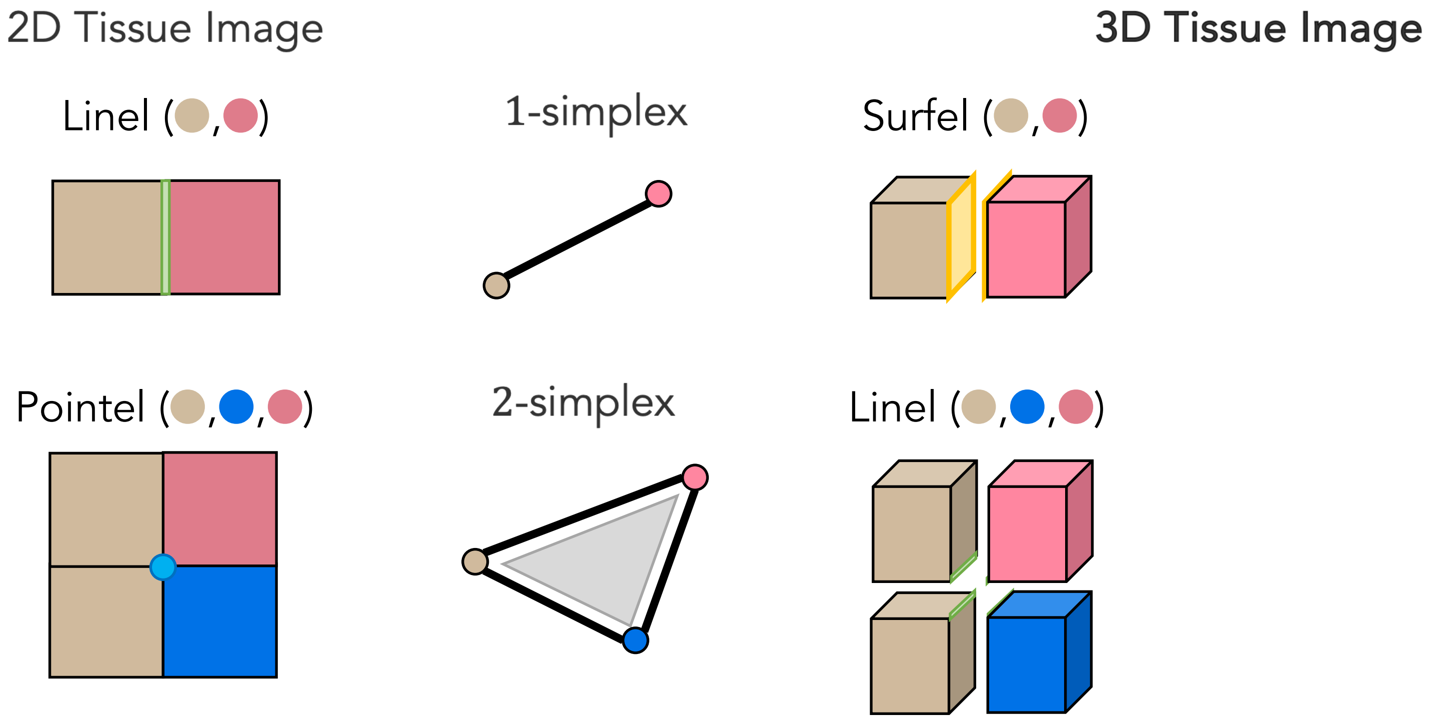

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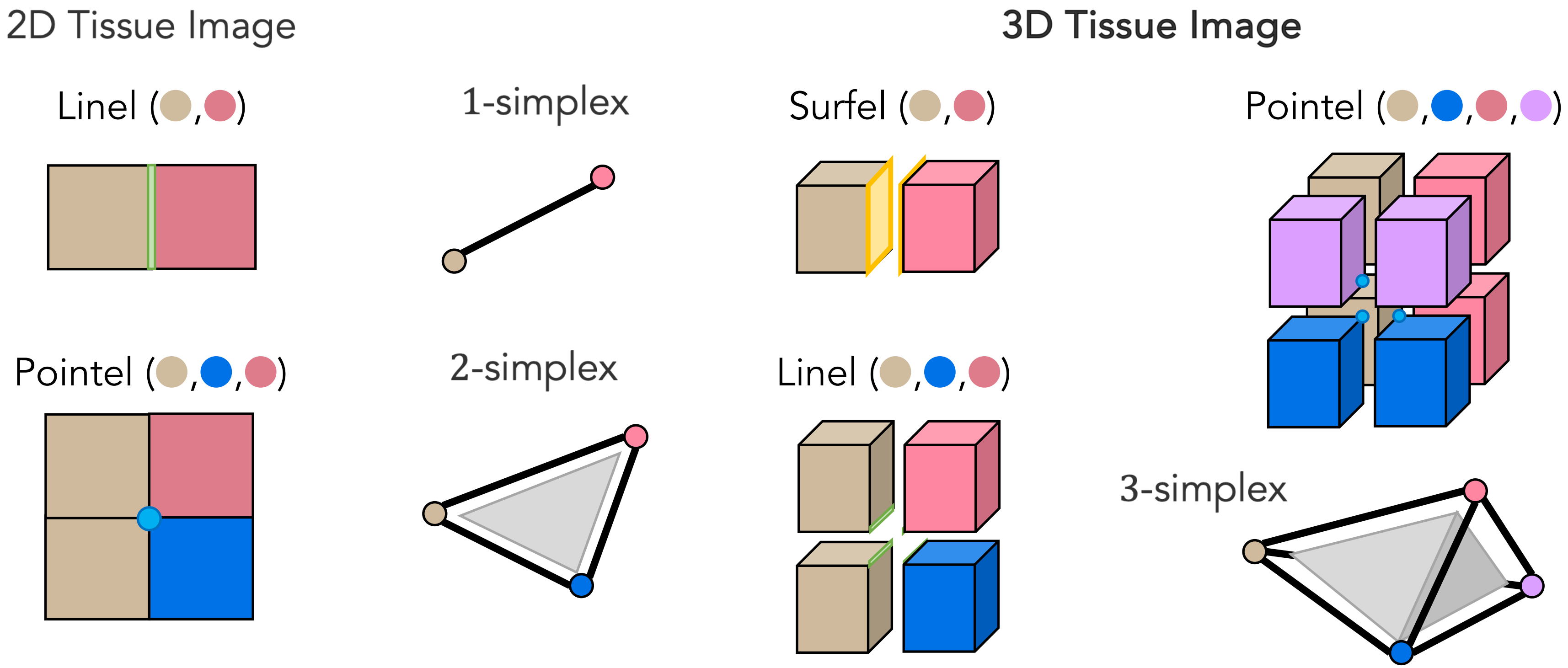

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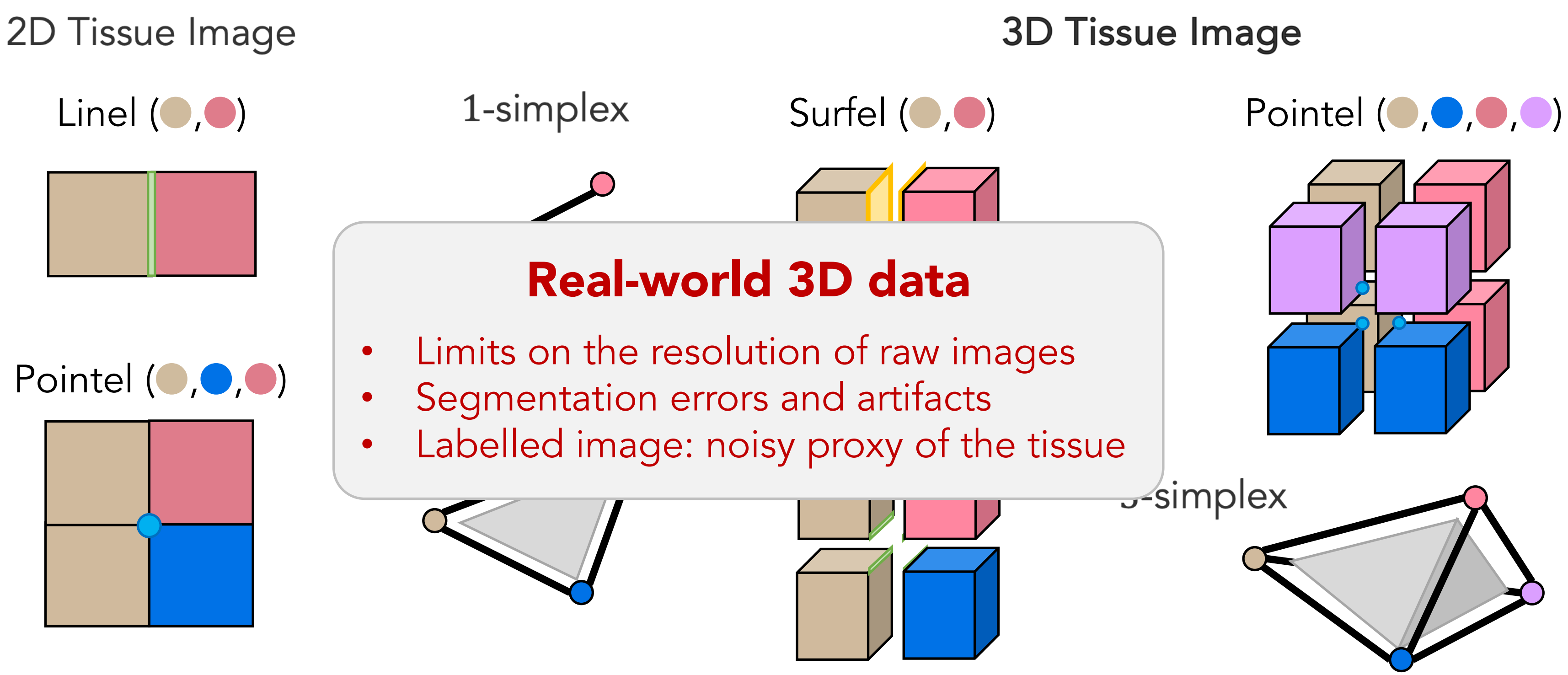

Real-world 3D data

- Limits on the resolution of raw images
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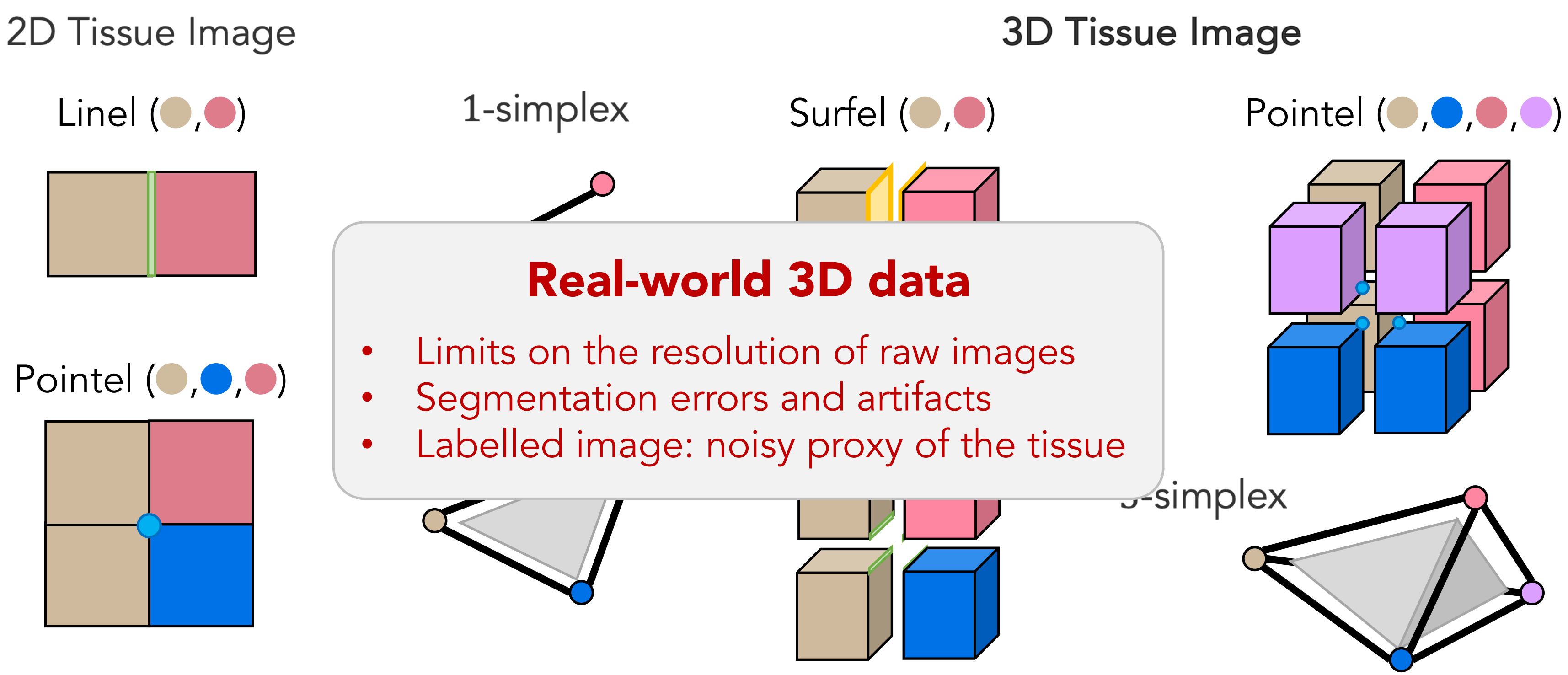

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Collection of image adjacency simplices: not necessarily a **valid simplicial complex**

Dealing with **non-simplicial** image adjacencies (4-label linels, 5+-label pointels)

Complex reconstruction: a combinatorial problem?

Data courtesy: J. Legrand, P. Das (Mosaic/RDP)

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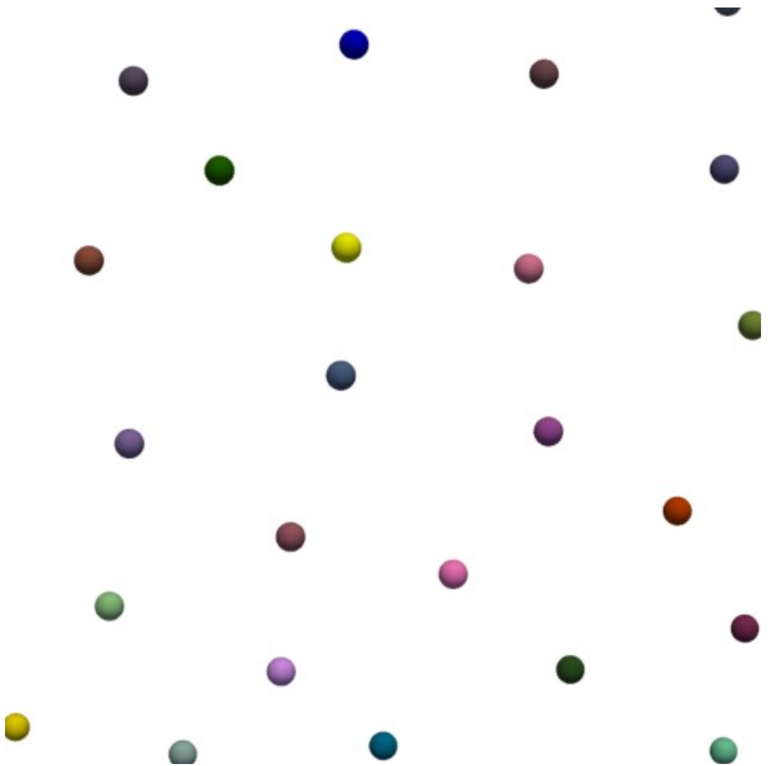
Given a set of space-embedded vertices $\{\mathbf{p}(v) \in \mathbb{R}^d, v \in V\}$ (e.g. image cell centers)



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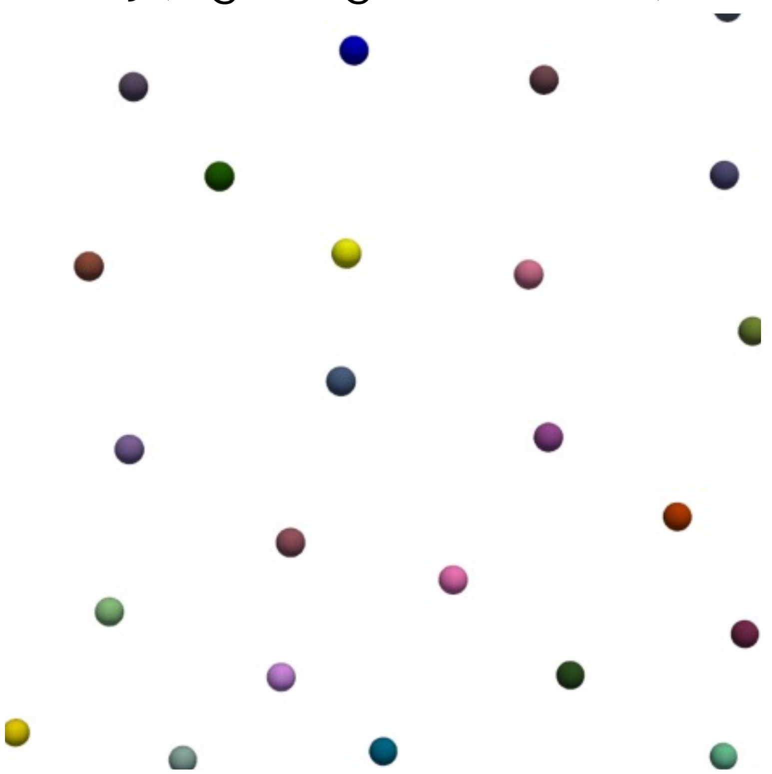

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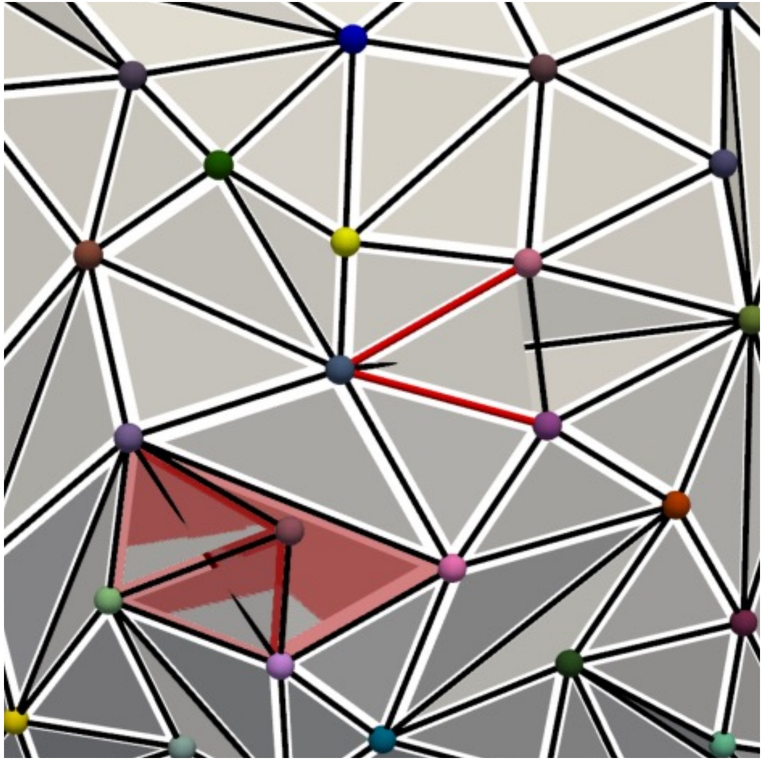

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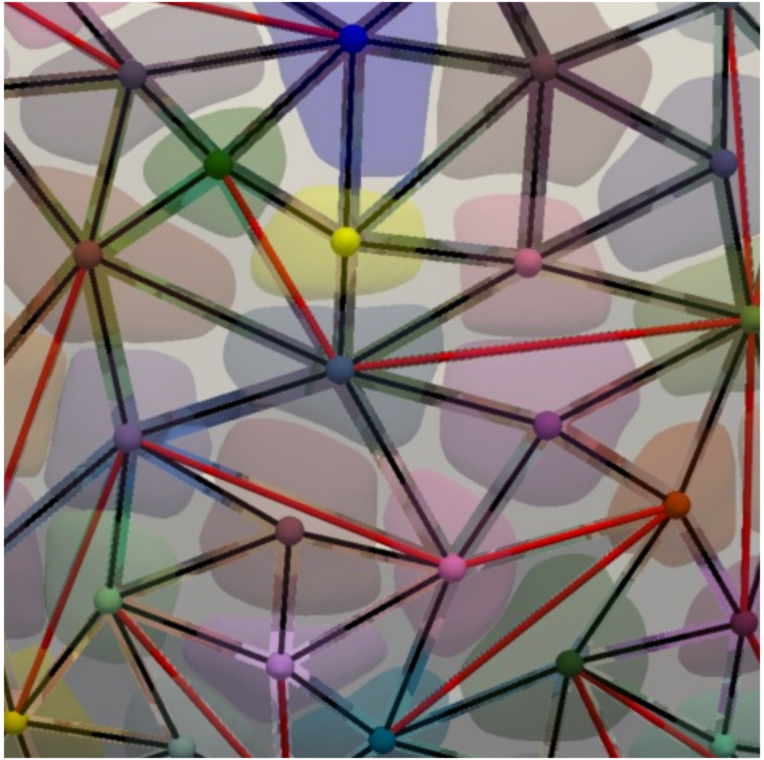

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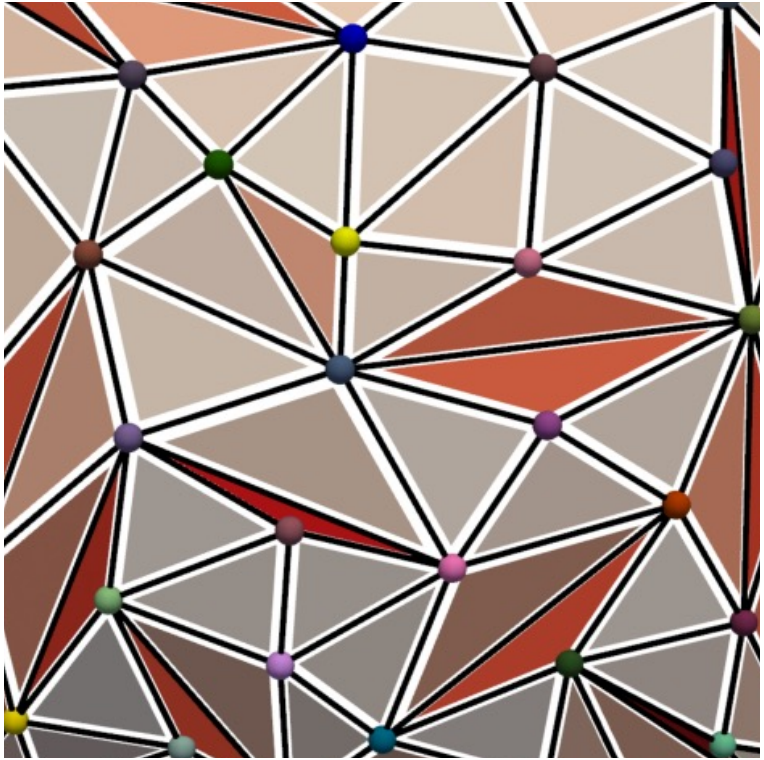

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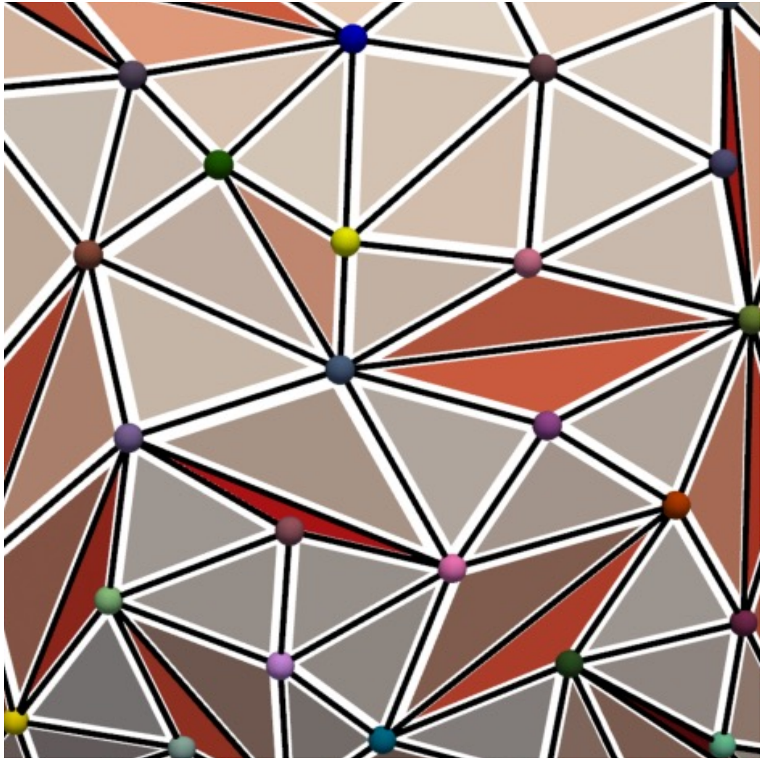
Constrained Optimization



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Walking the space of (abstract) simplicial complexes

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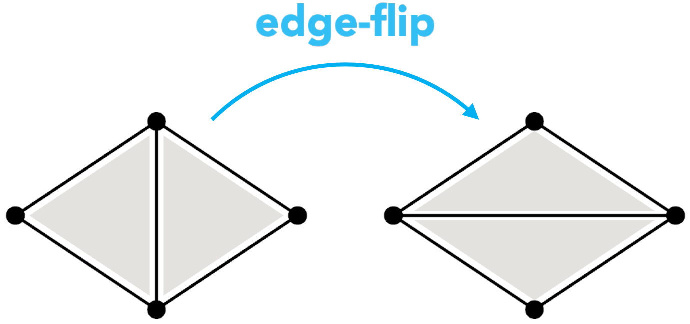

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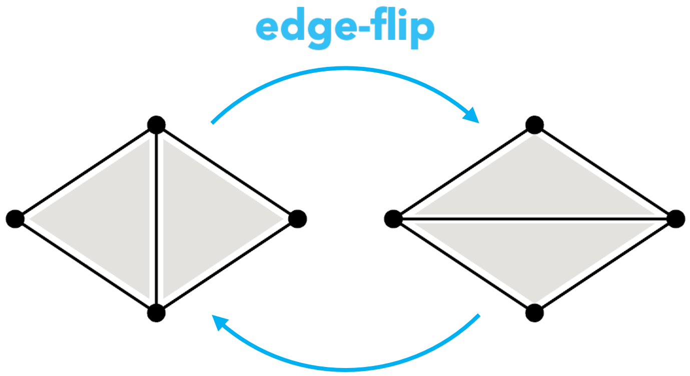

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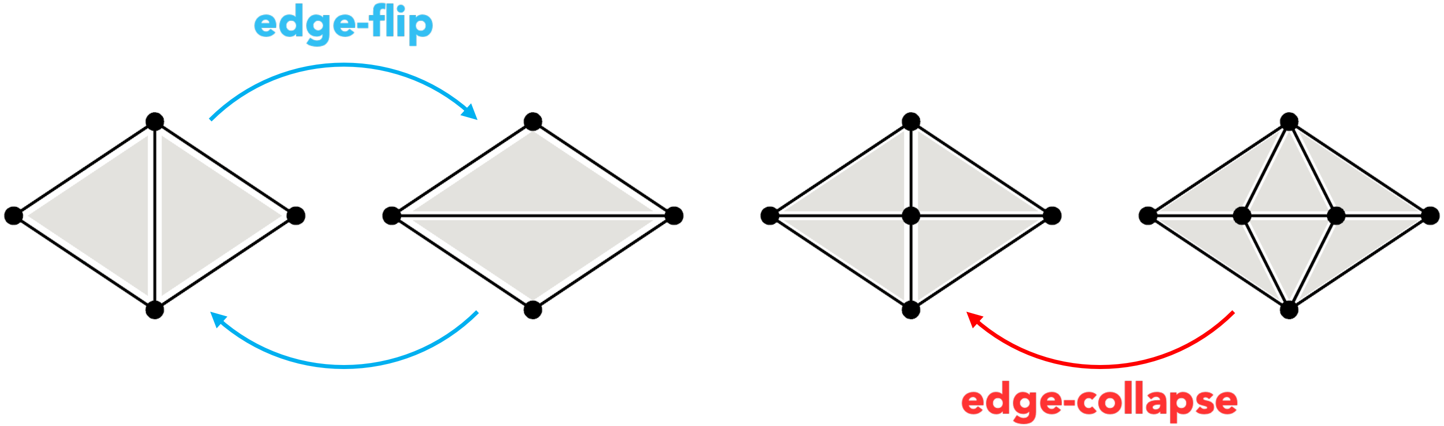

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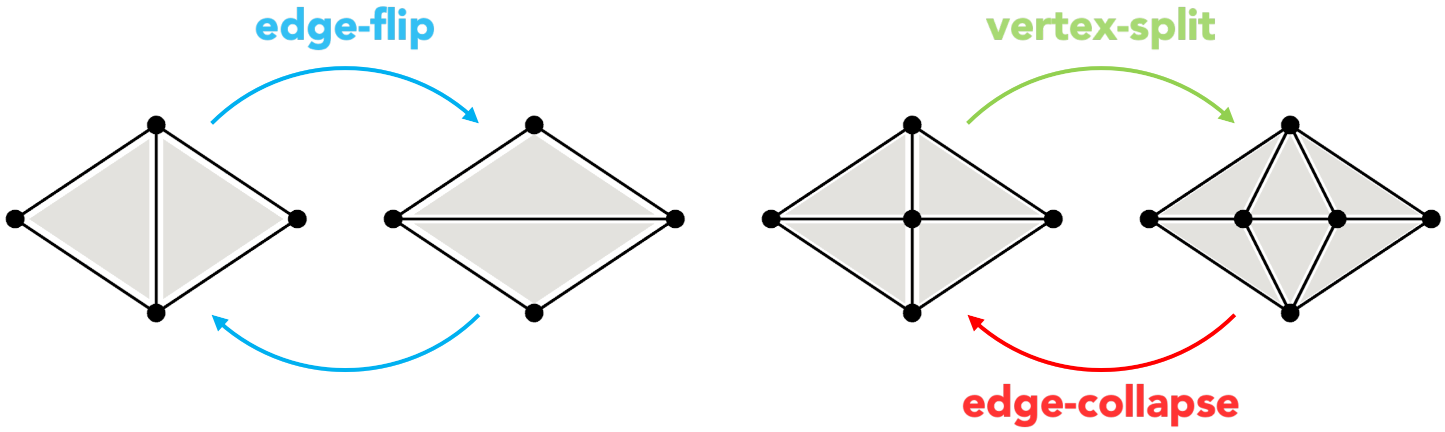

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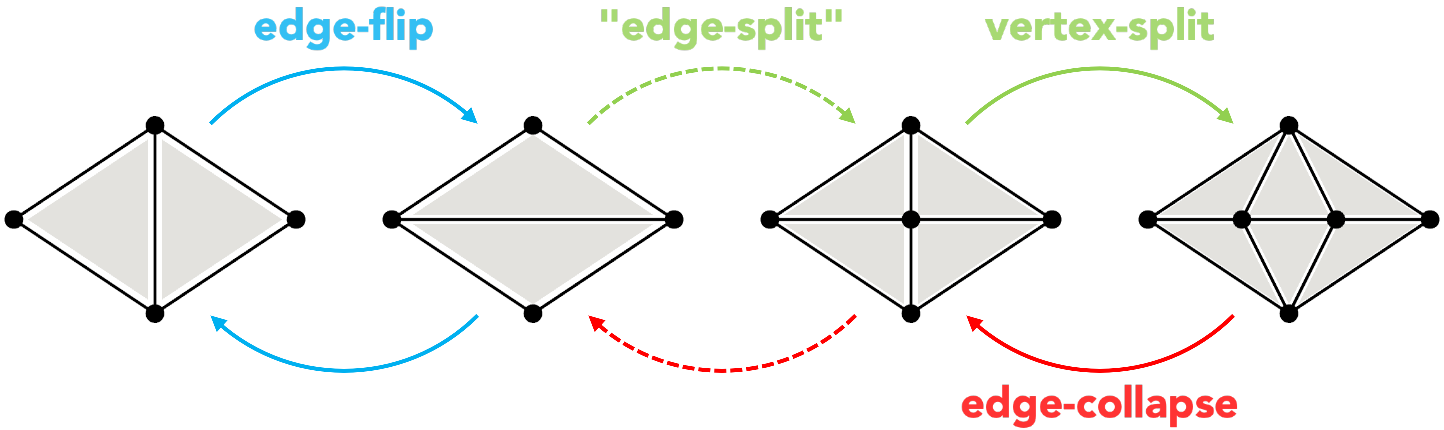

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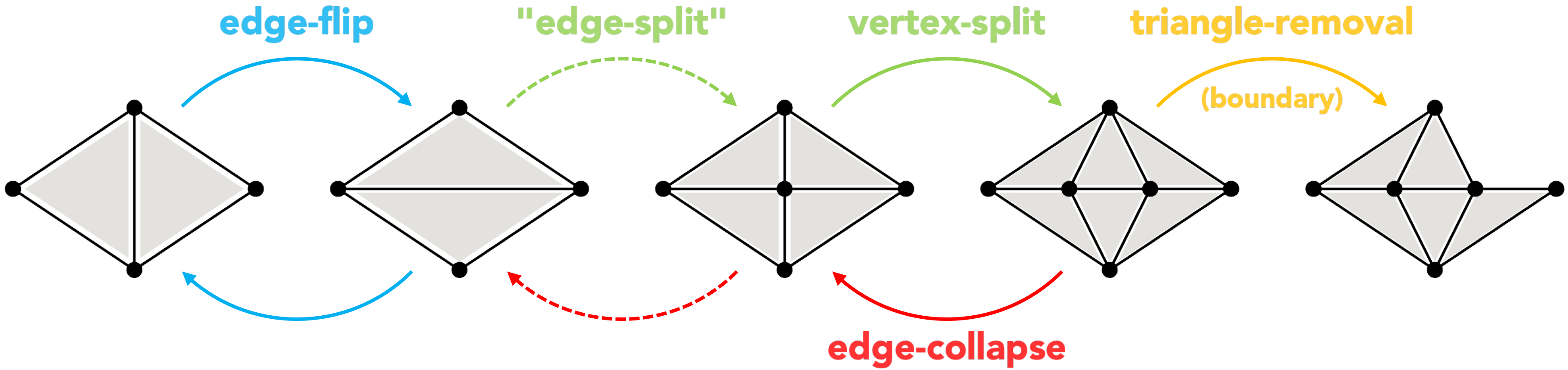

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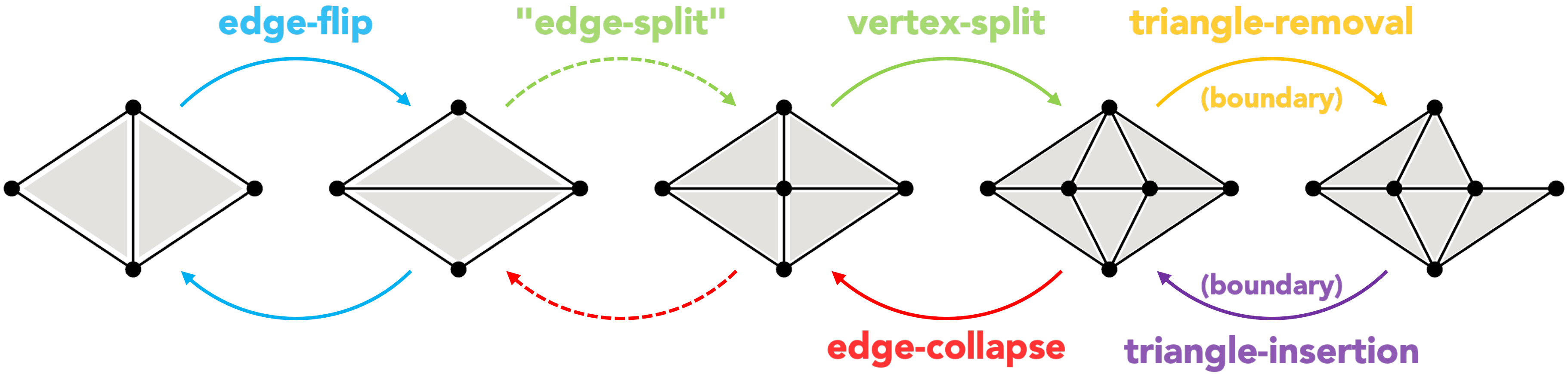

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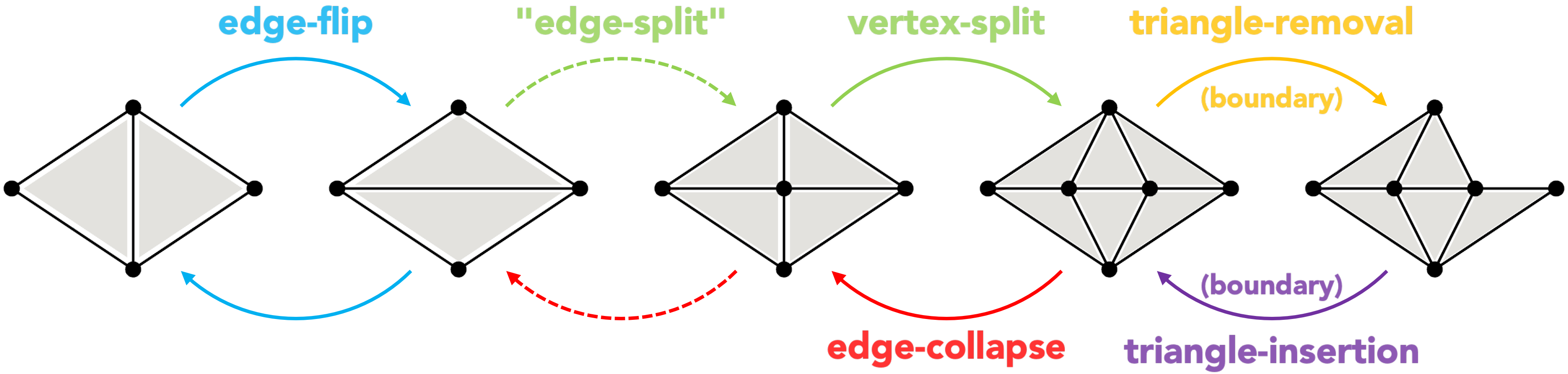

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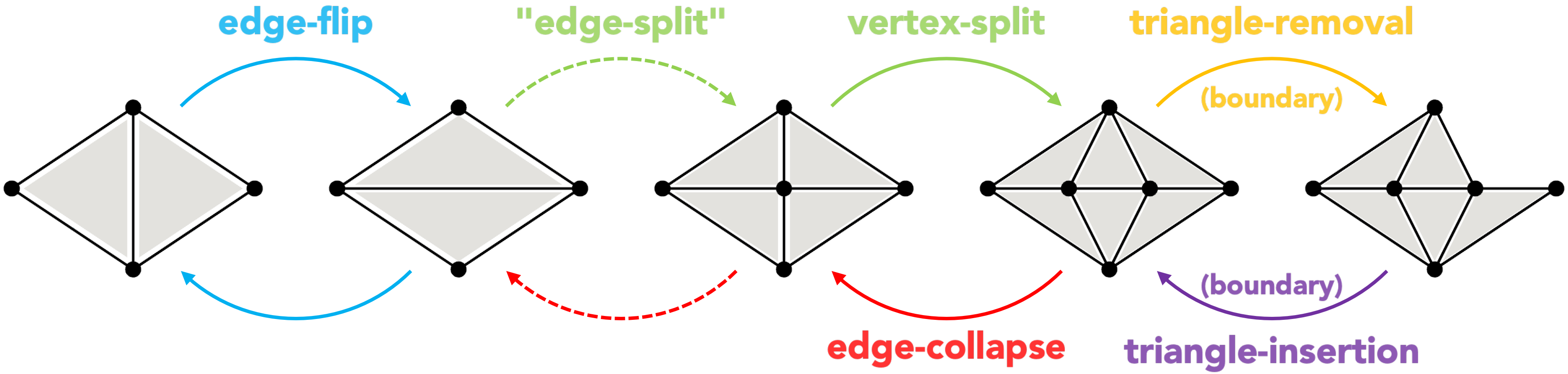

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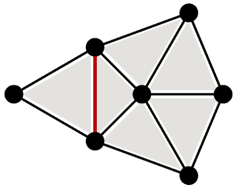

purity criterion

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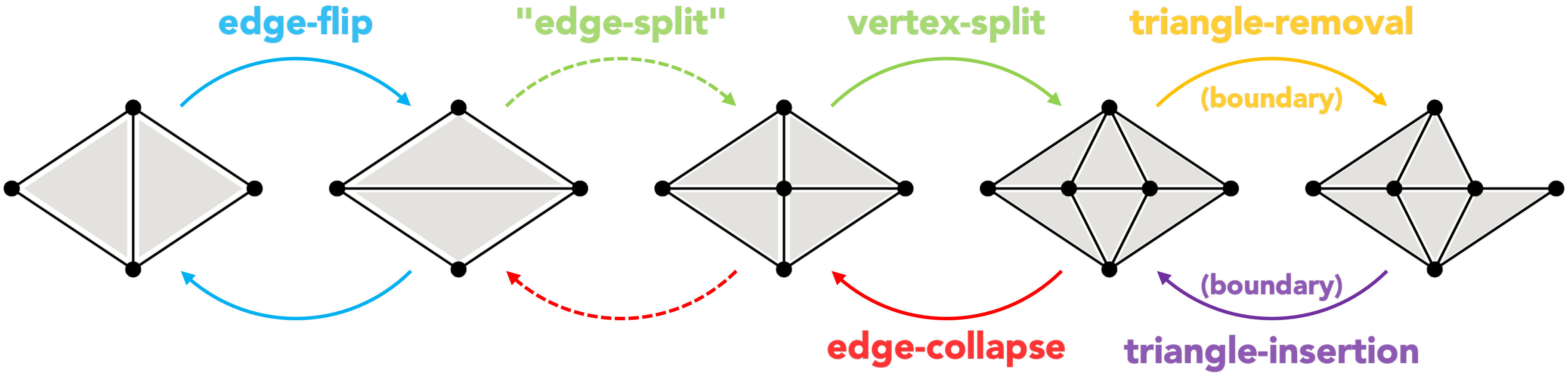

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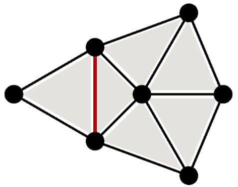

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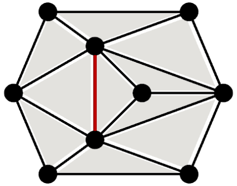

manifold criterion

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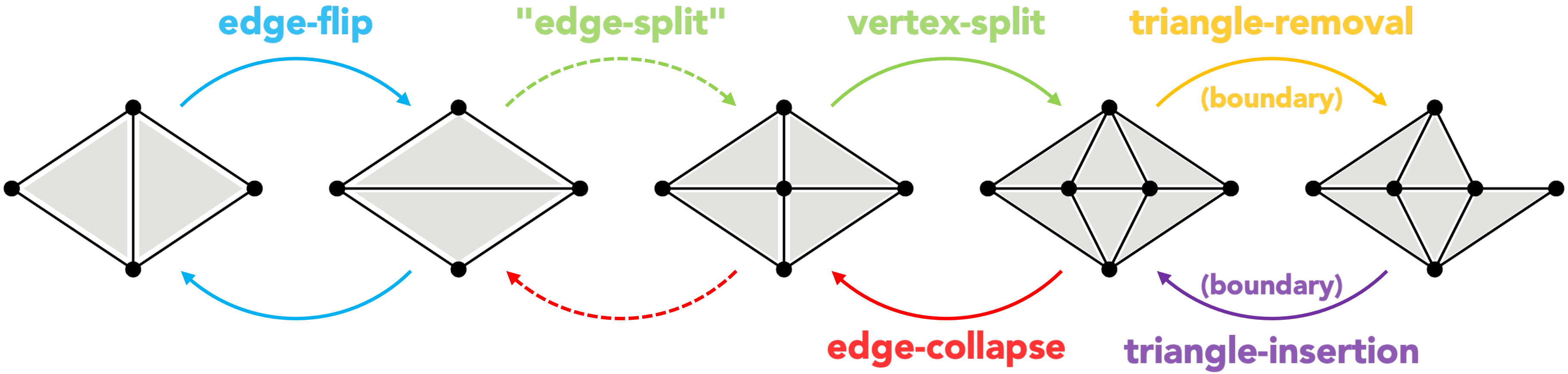

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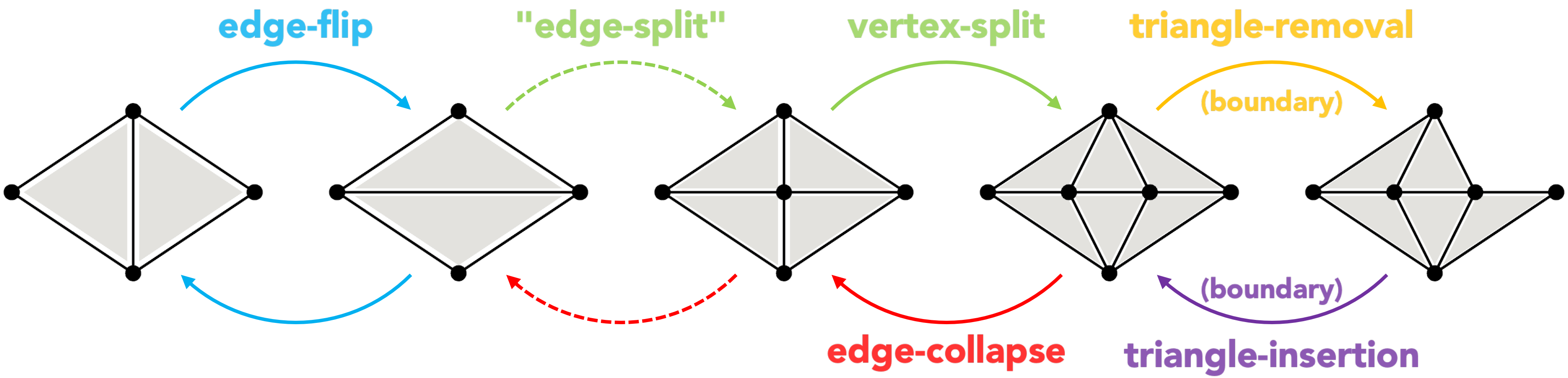

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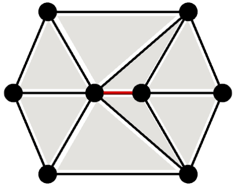

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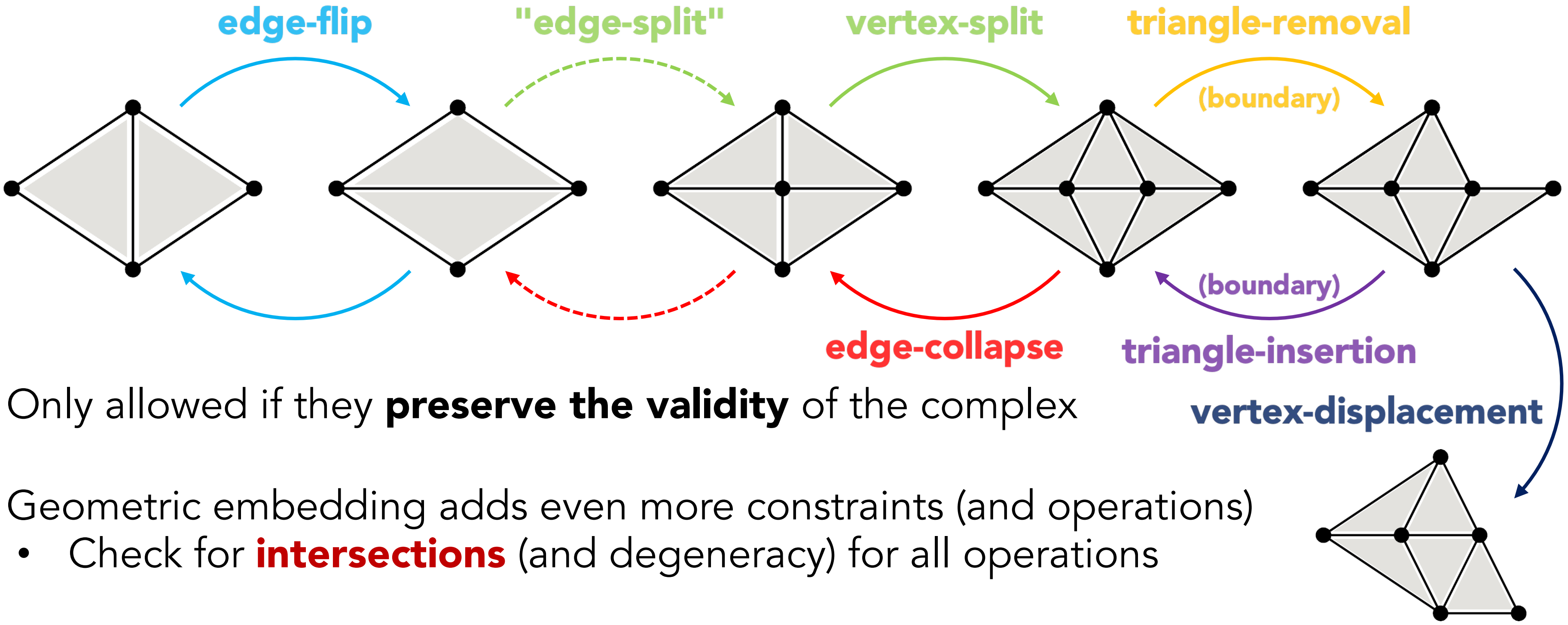

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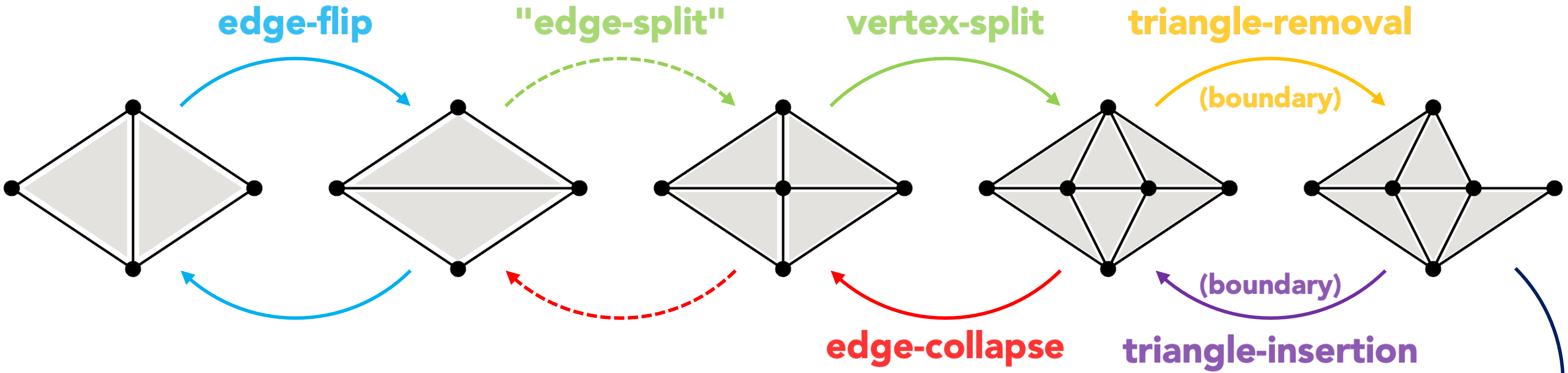

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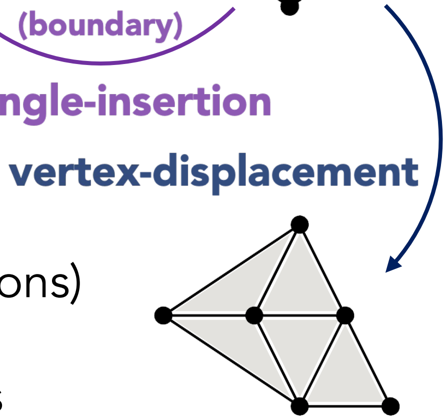

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  - Update embedding for vertex-inserting/removing operations

Simplicial complex edition with fixed vertices

- Optimization problem: given V and $\mathbf{p}$, and an initial (valid) simplicial complex K

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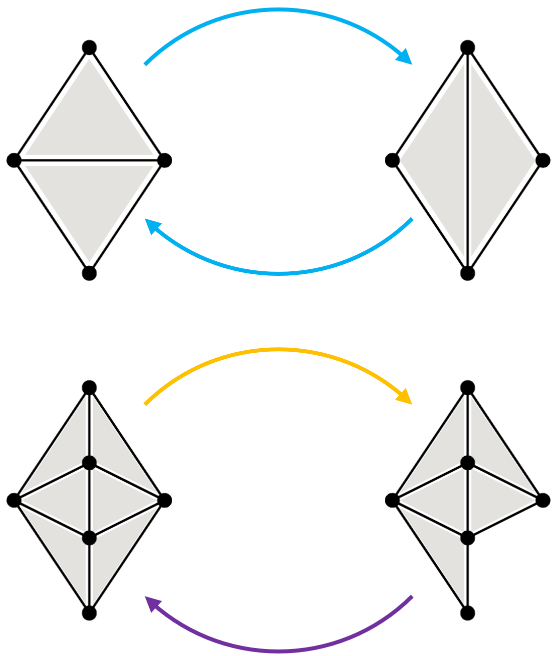

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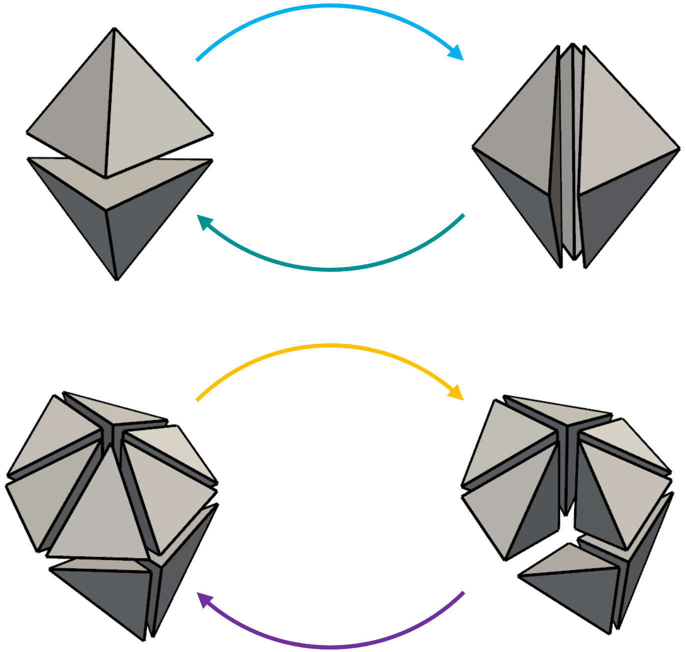

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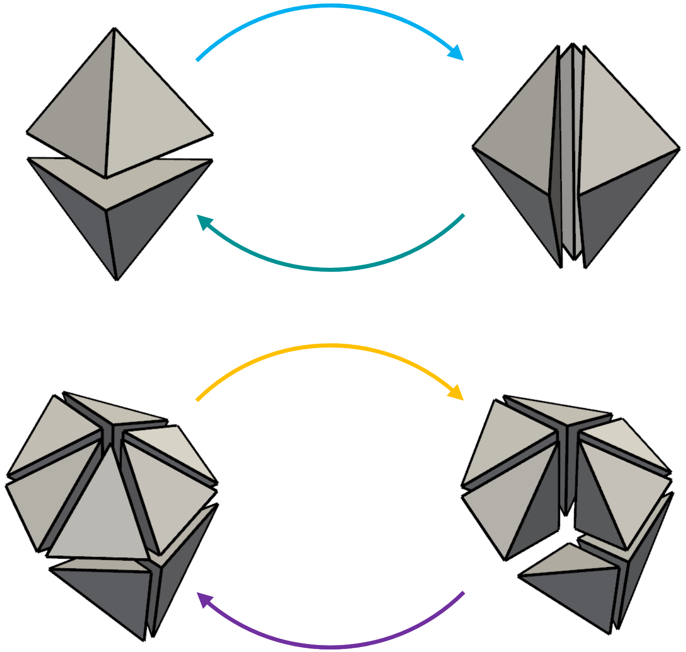

- Geometric constraints are imposed by $\mathbf{p}$
- Govern the feasibility of edit operations

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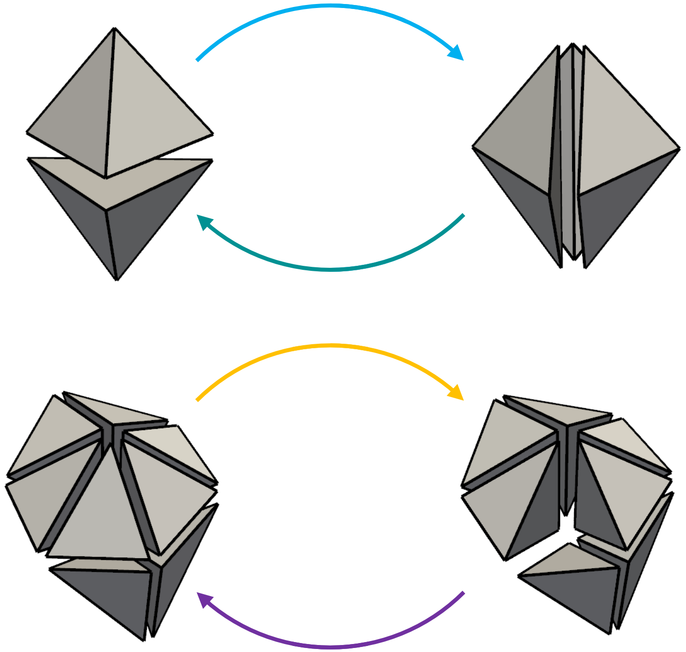
    - Maximize an objective function with only **vertex-preserving edit operations**



- Geometric constraints are imposed by $\mathbf{p}$
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Restricted "walkable" space

- Discrete SC space: combinatorial graph
- Valid edit operations define edges
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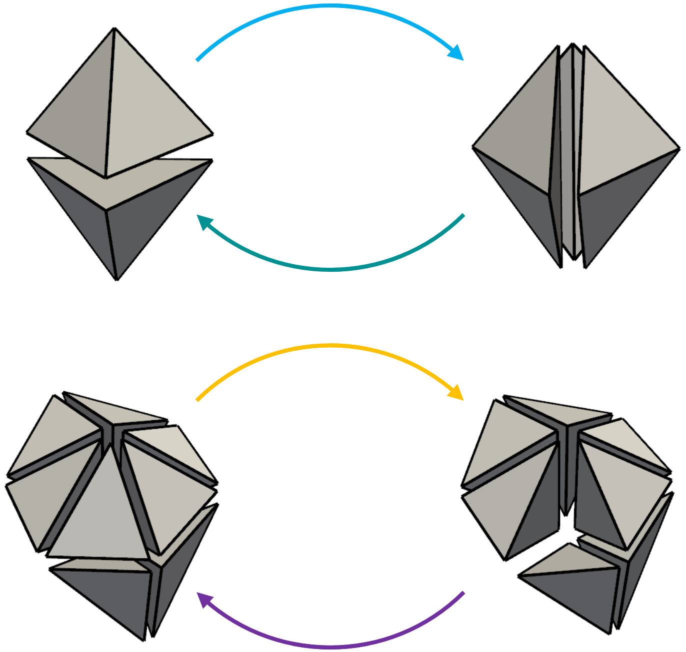

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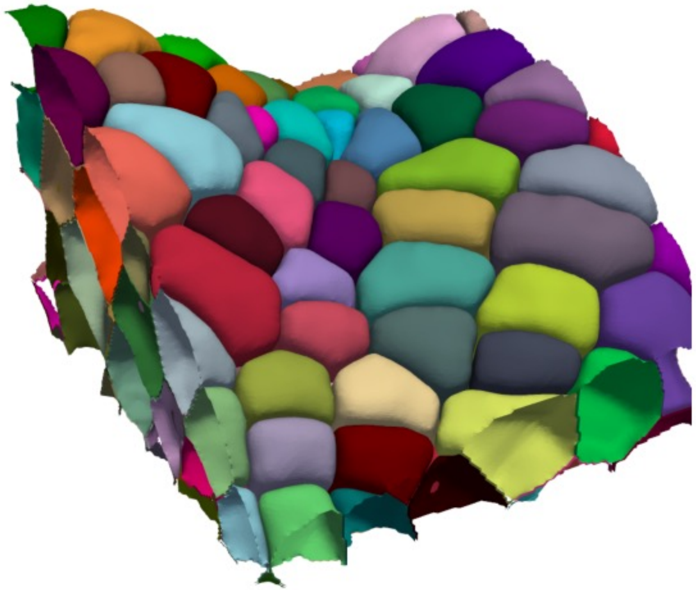
- Limits the accessibility of the optimal SC?

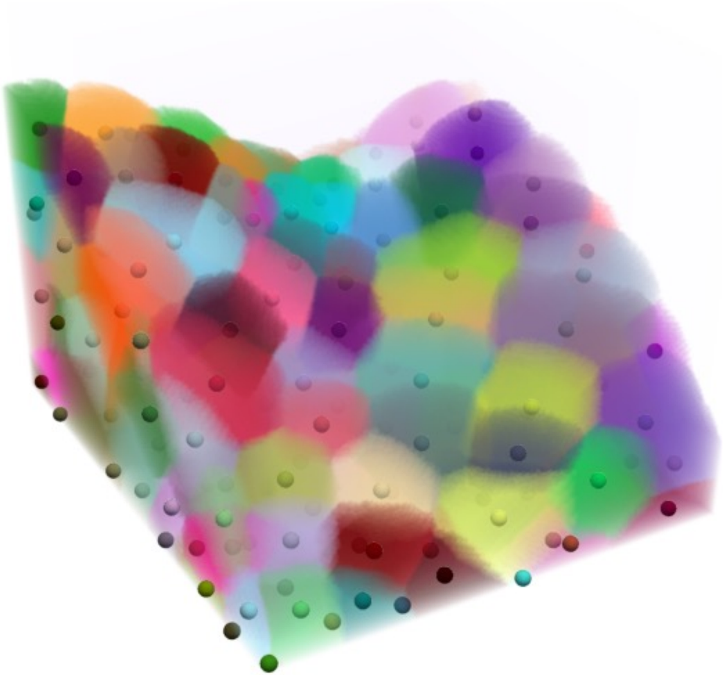
First method: Delaunay SC iterative optimization

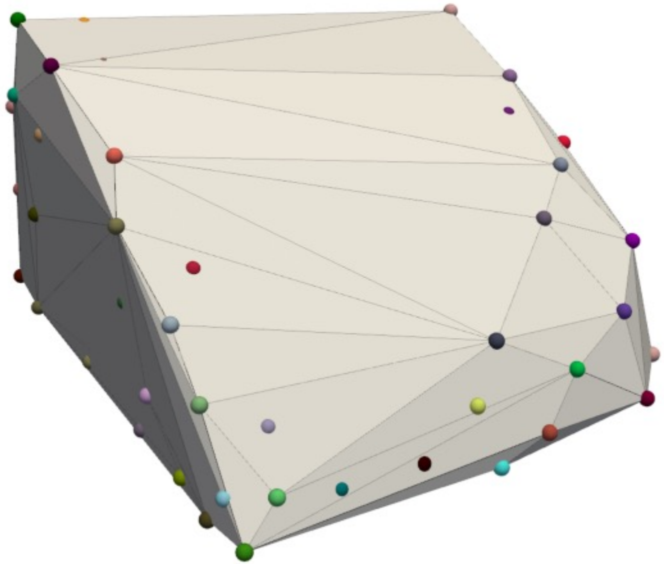


(G. Cerutti, O. Ali & C. Godin, Frontiers in Plant Science, 2017)

First method: Delaunay SC iterative optimization

- Compute the Delaunay tetrahedralization of cell centers





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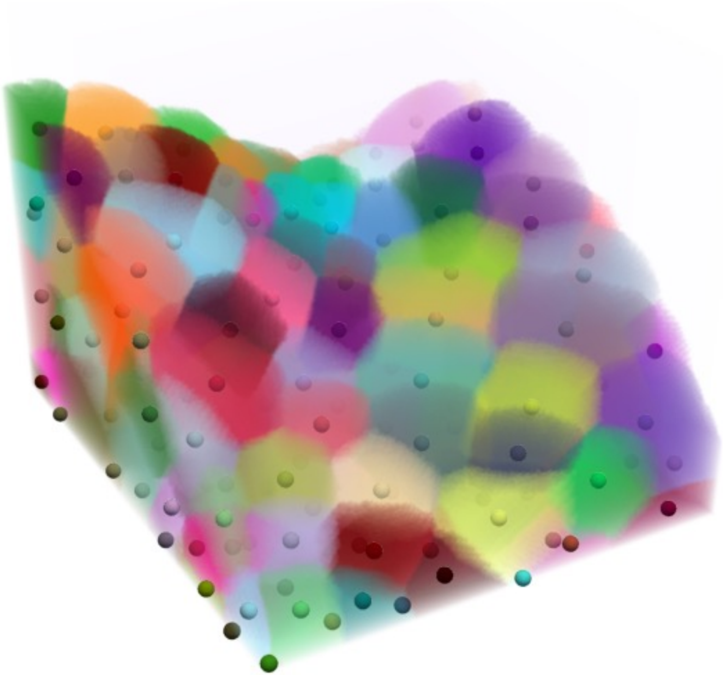
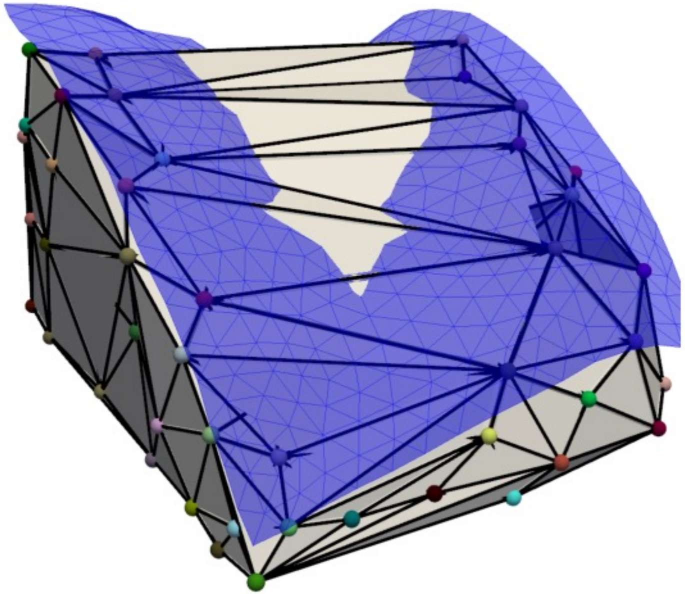
- Compute the Delaunay tetrahedralization of cell centers + removal w/r tissue surface





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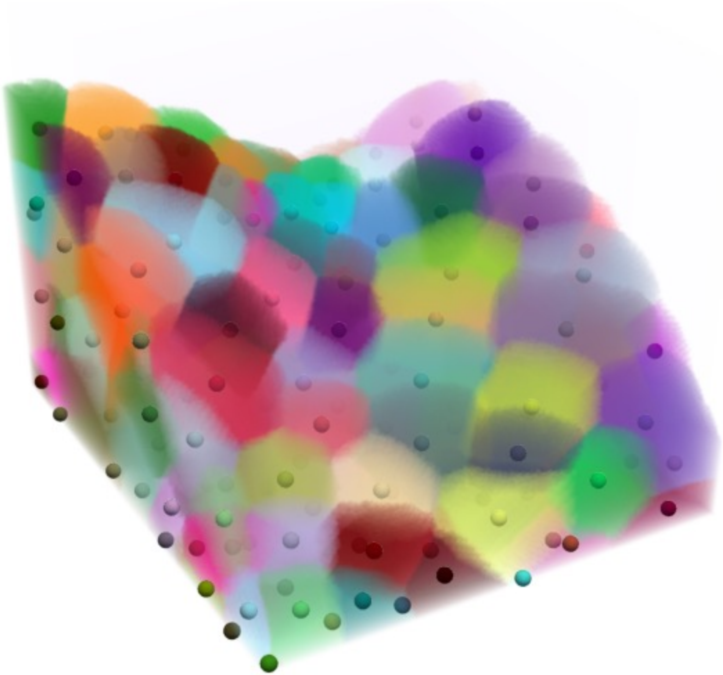

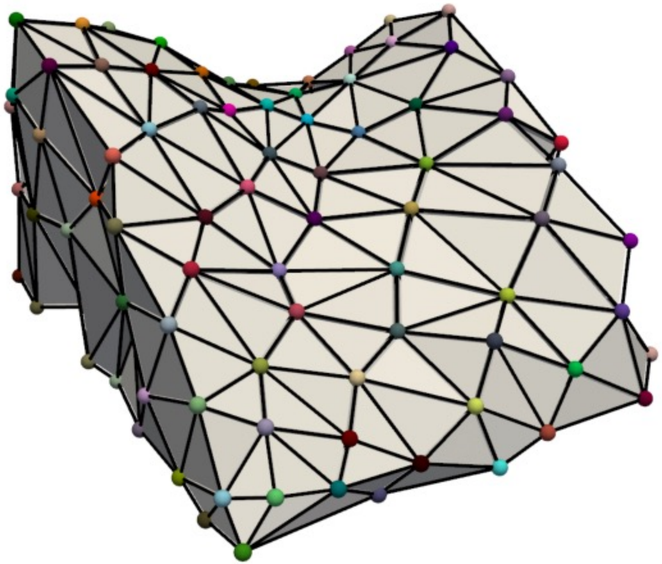

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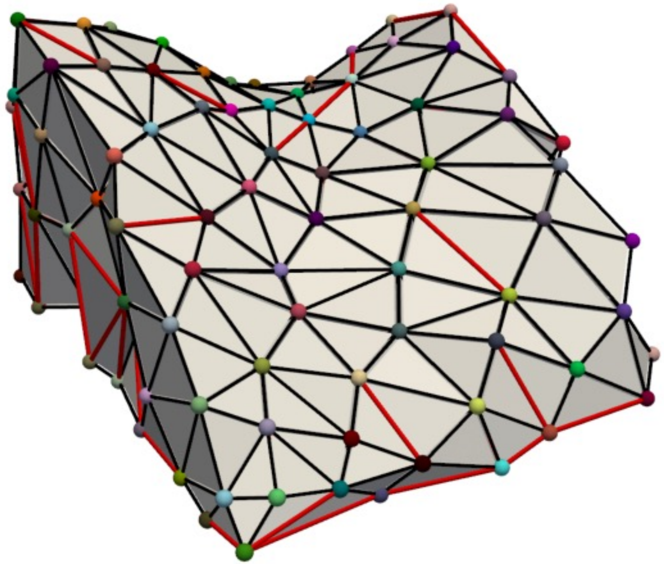

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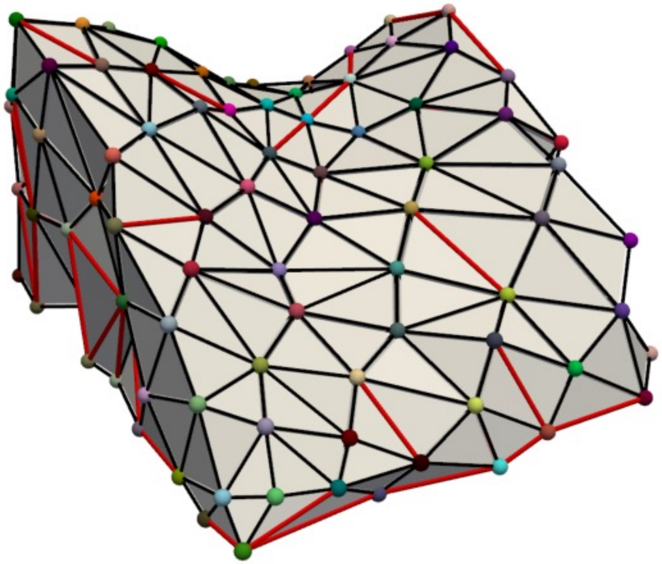

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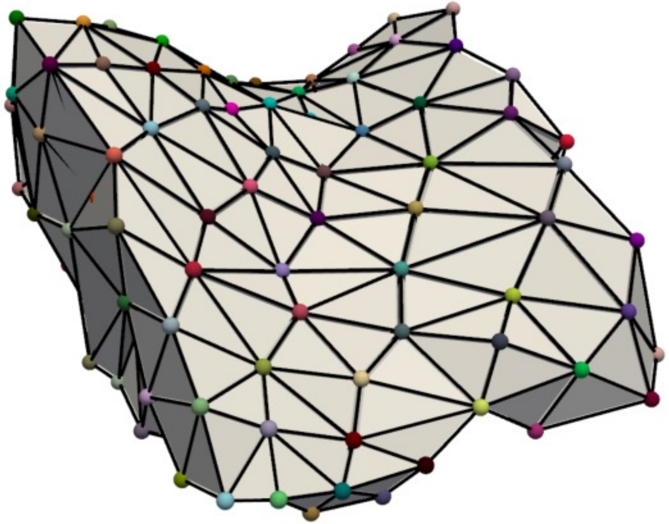

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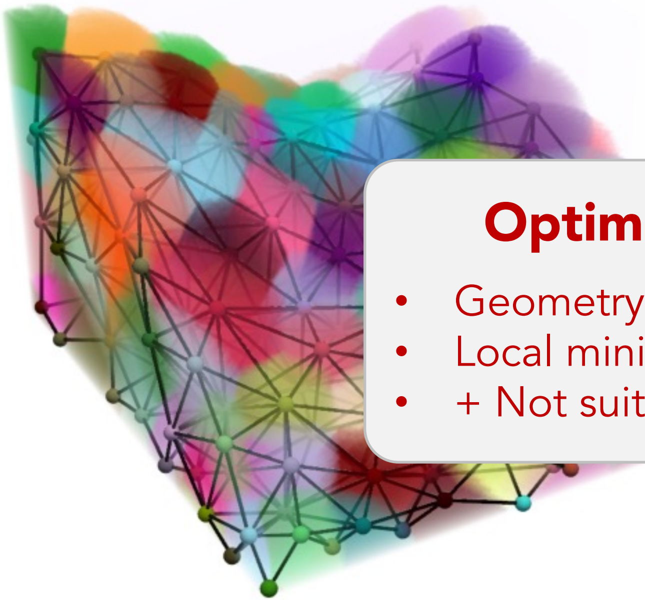
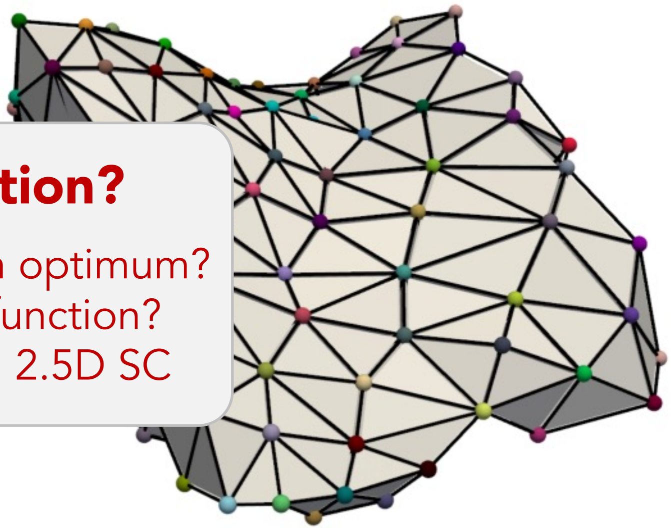




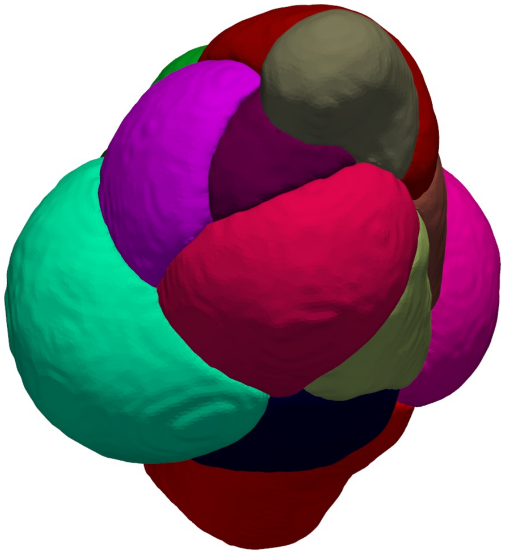
Optimality of the solution?

- Geometry impeding path to an optimum?
- Local minima of the objective function?
- \+ Not suited for reconstructing 2.5D SC

- Optimization: perform edit operations in a simulated-annealing iterative process

(G. Cerutti, O. Ali & C. Godin, Frontiers in Plant Science, 2017)

New favorite method: iteratively collapsing edges



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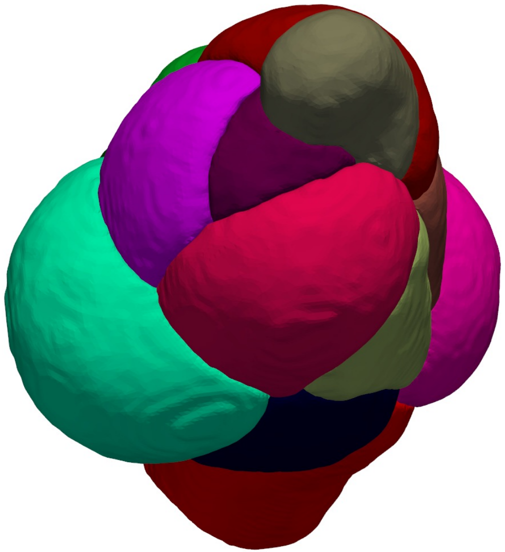
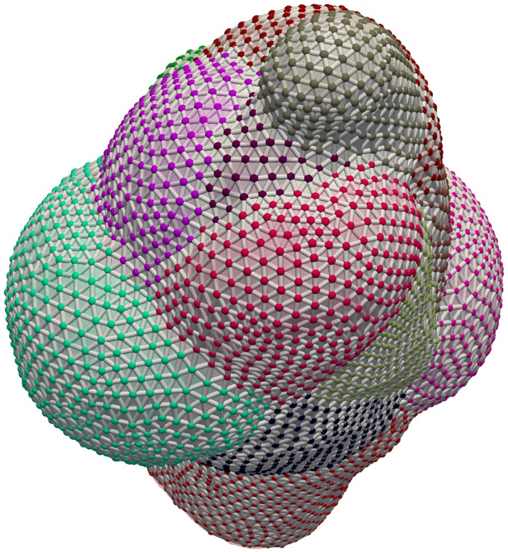
Mesh the image domain & project cell labels on vertices





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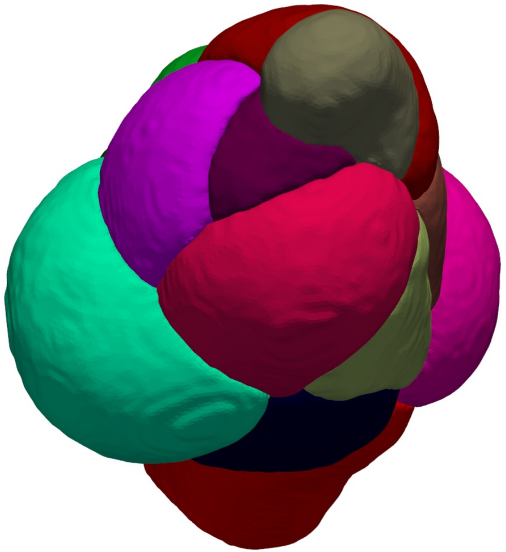

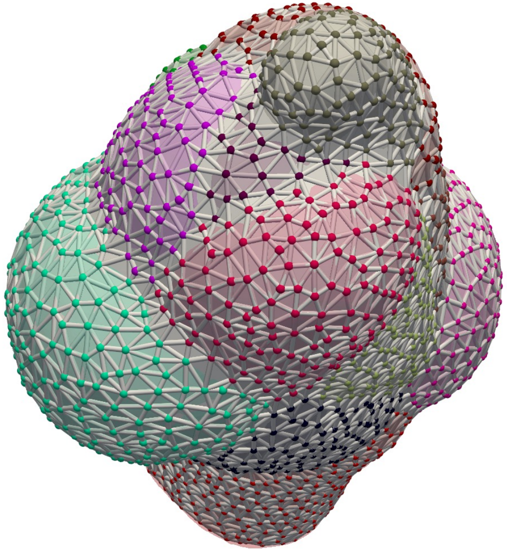

Apply **edge-collapses** on 1-simplices including **identically labelled** vertices

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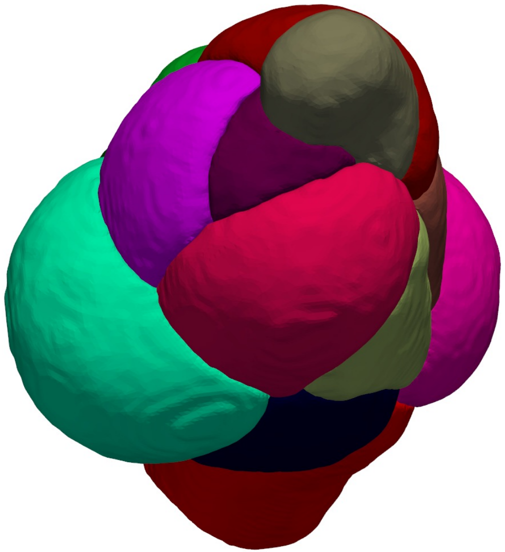

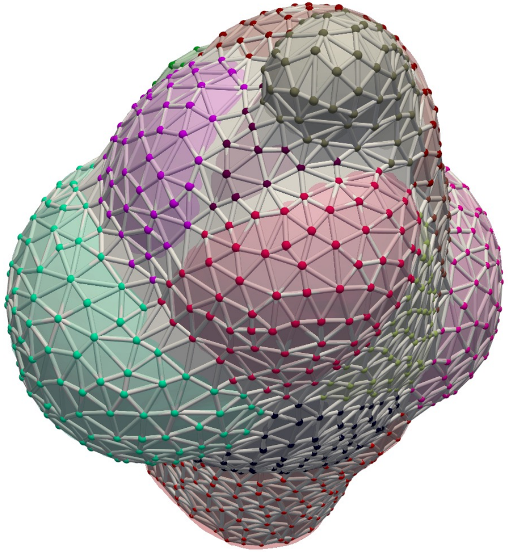

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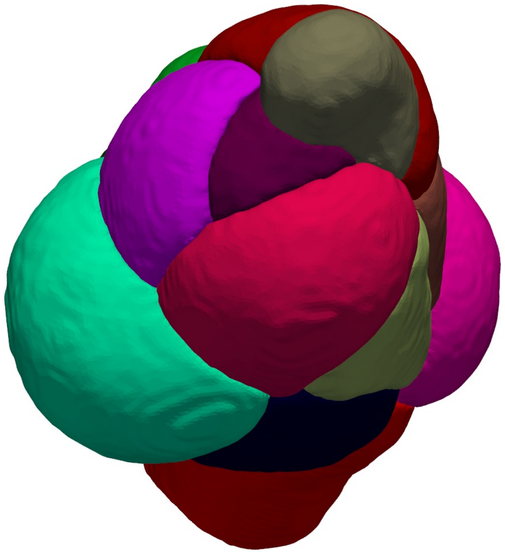

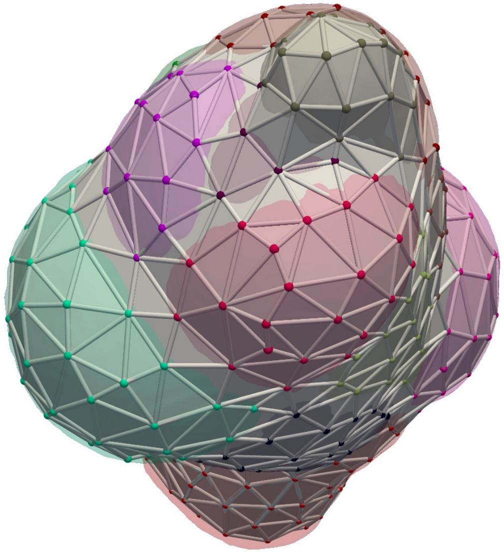

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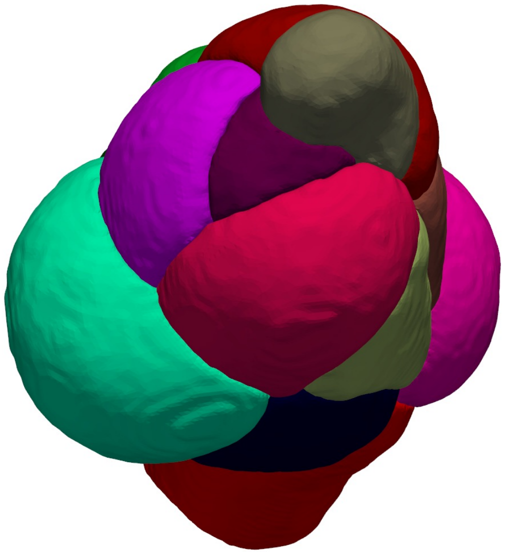

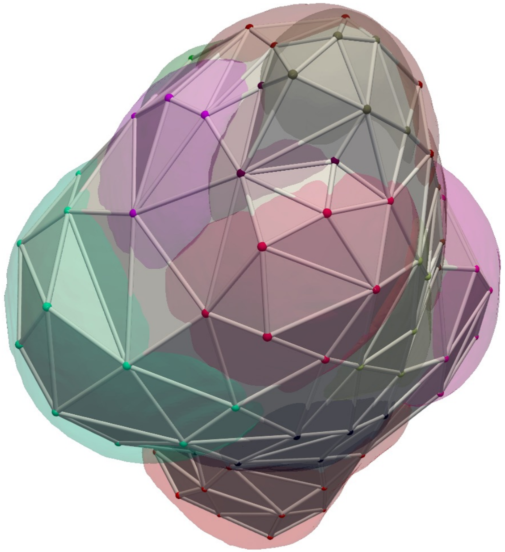

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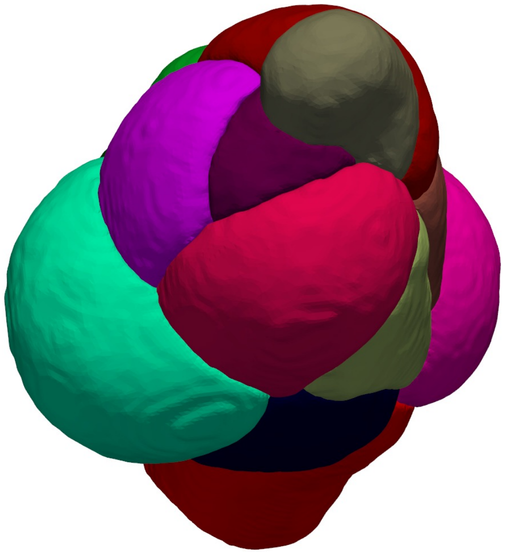

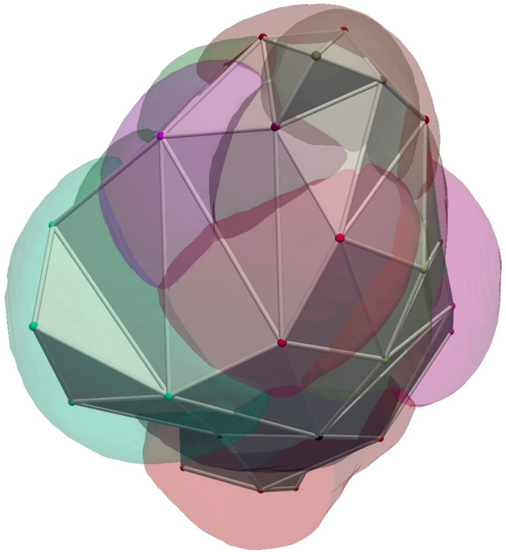

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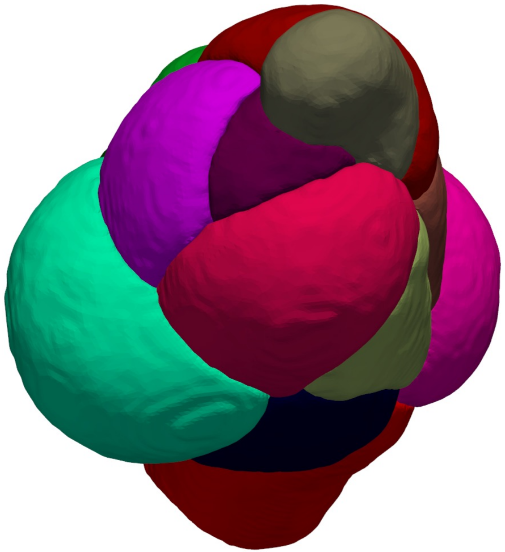

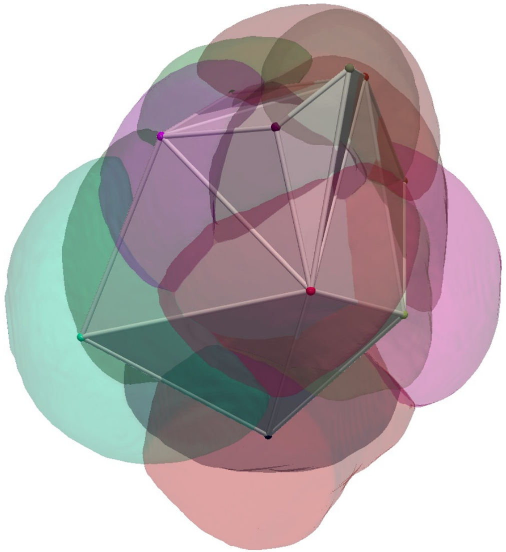

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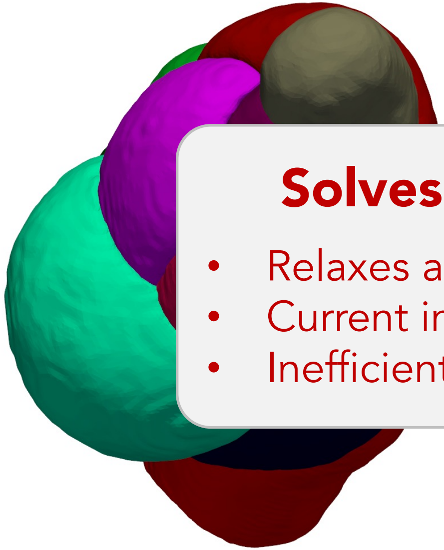
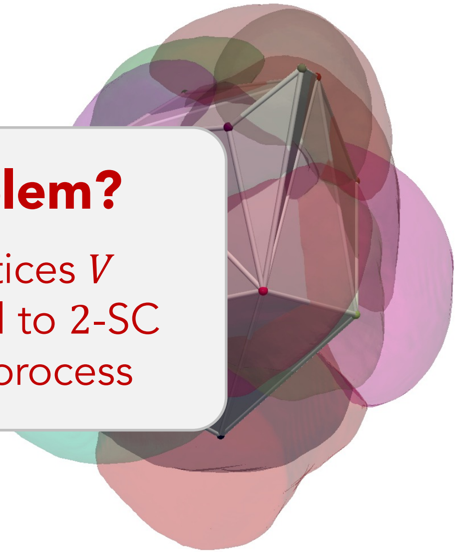


Solves a different problem?

- Relaxes a constraint on the vertices V
- Current implementation limited to 2-SC
- Inefficient data structure: slow process



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New combinatorial challenge: inter-layer filling

E. Gascon: Ph.D. Work (Mosaic/RDP)

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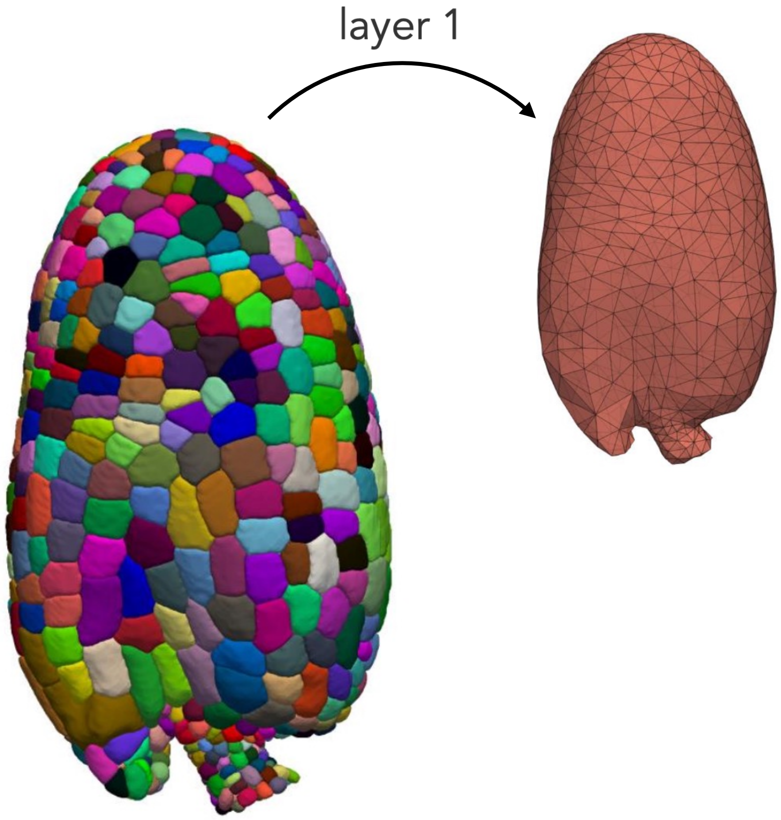

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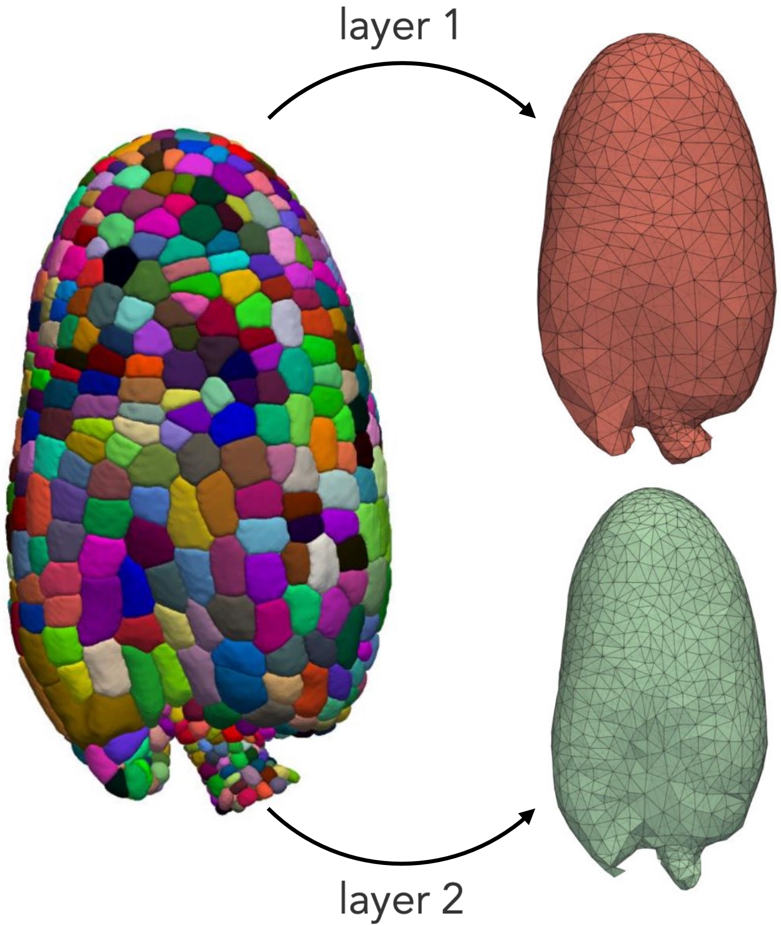

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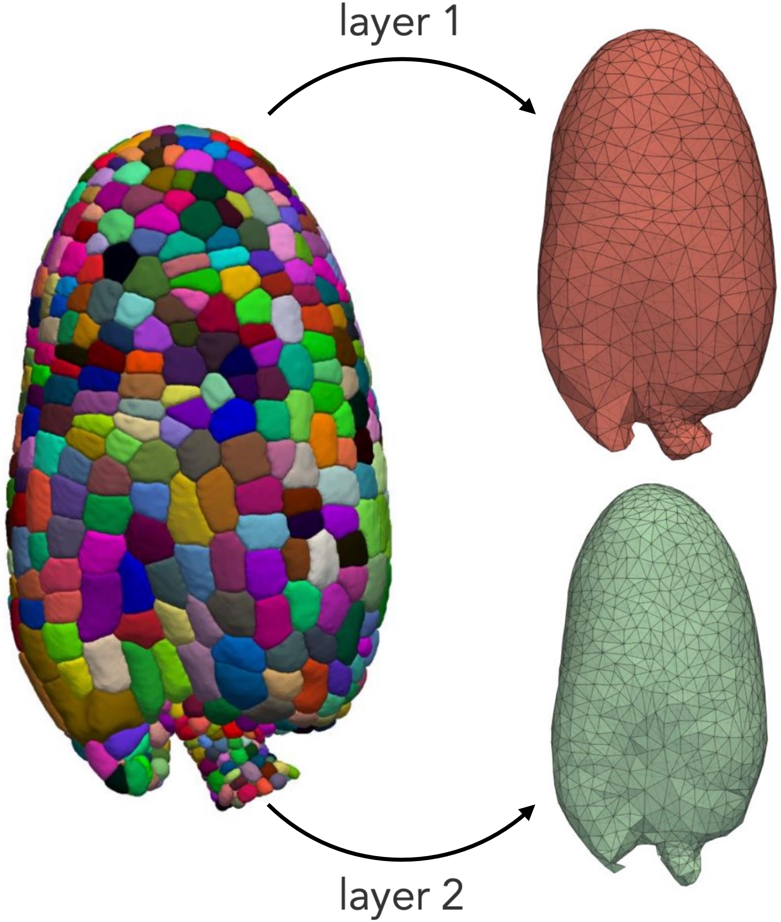

Given 2 "concentric" 2-simplicial complexes

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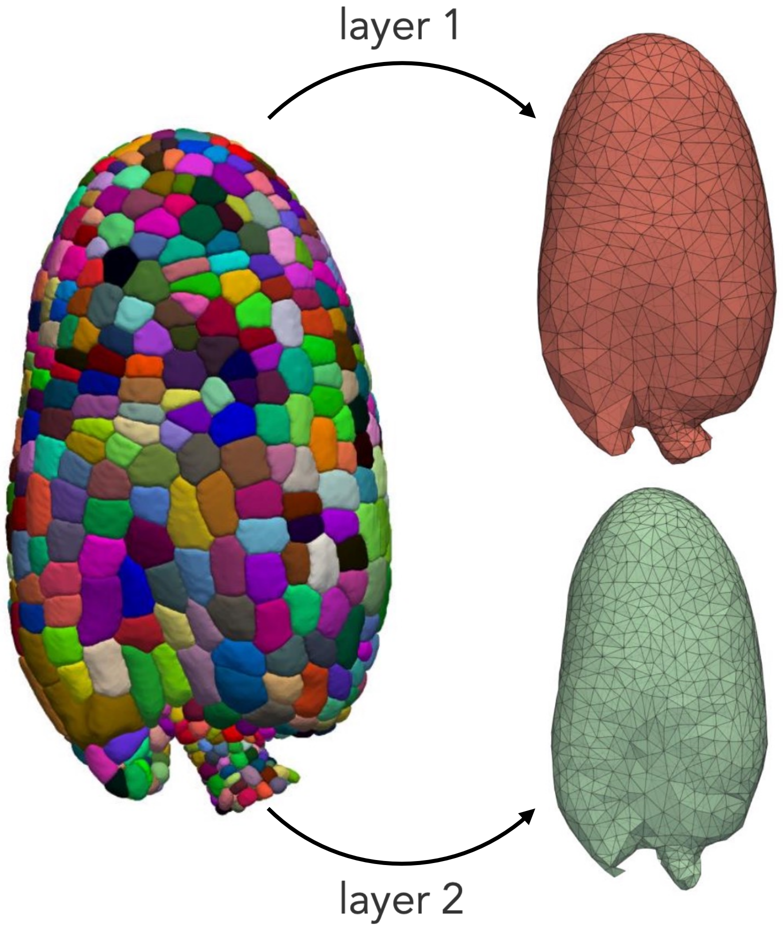

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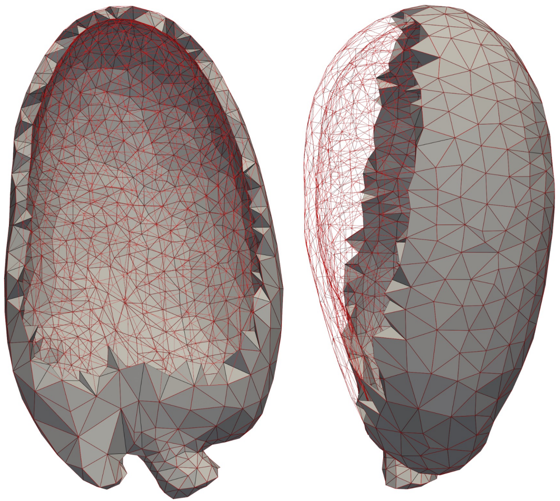

Reconstruct a 3-SC including all 2-simplices

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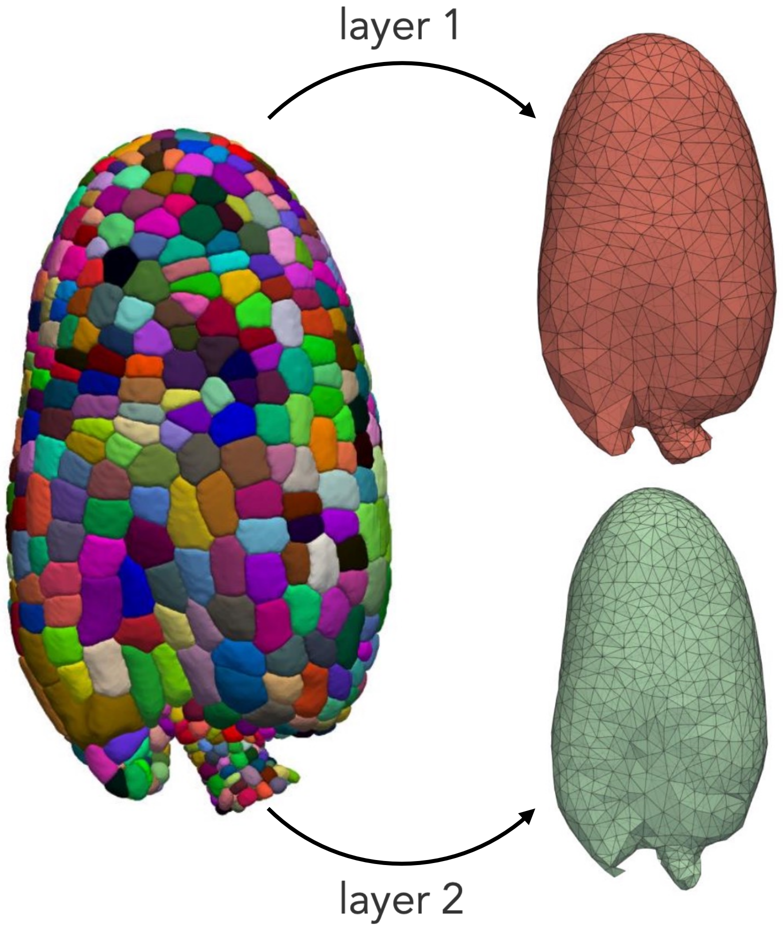

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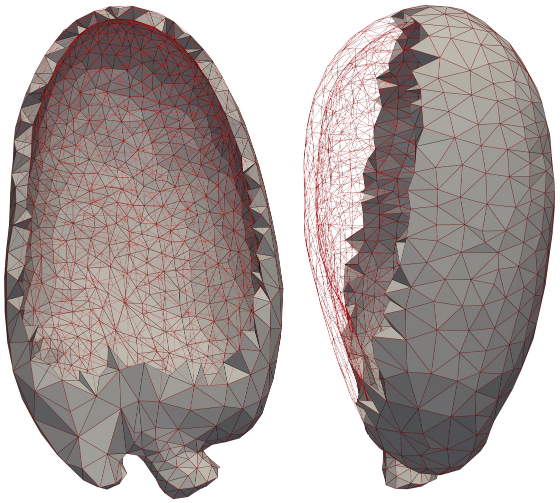

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Match image adjacencies and ensure quality

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Beyond reconstruction:

Beyond reconstruction: lineaging & SC edit paths

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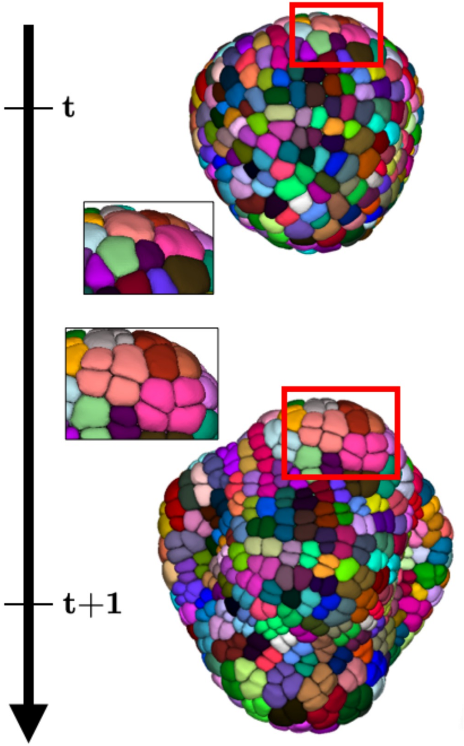
- Lineaging problem: find a **mapping function** between cells at **consecutive times**



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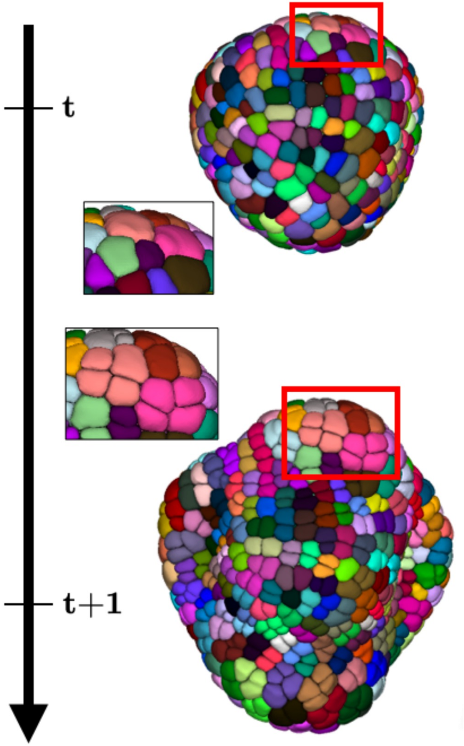

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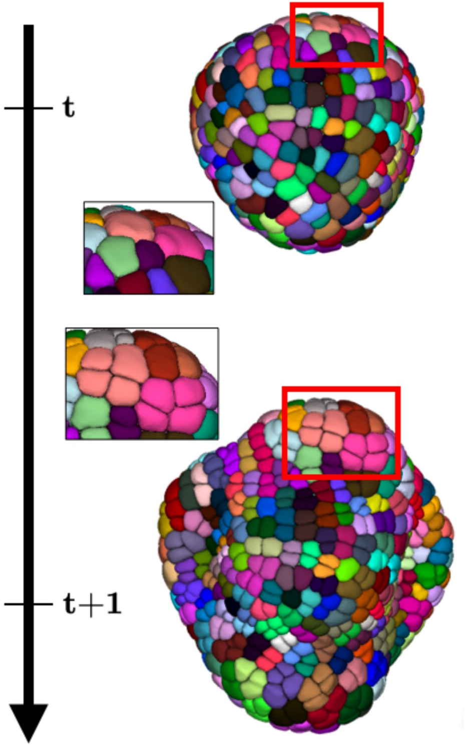

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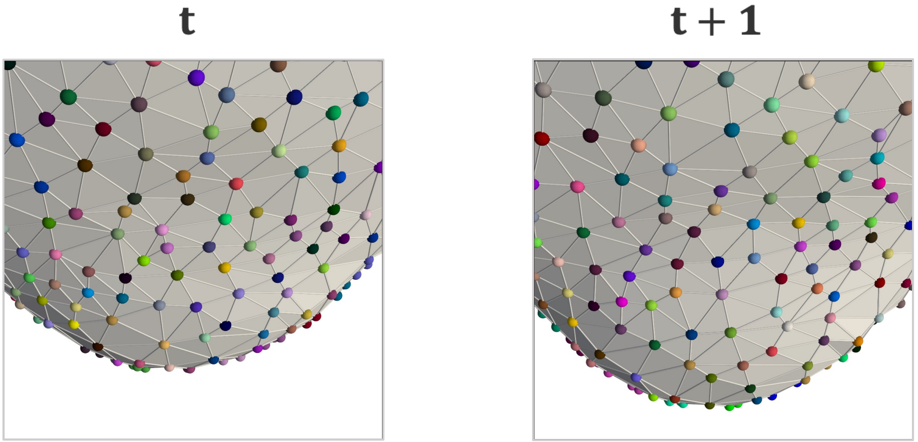
Reformulated as a SC edition problem?



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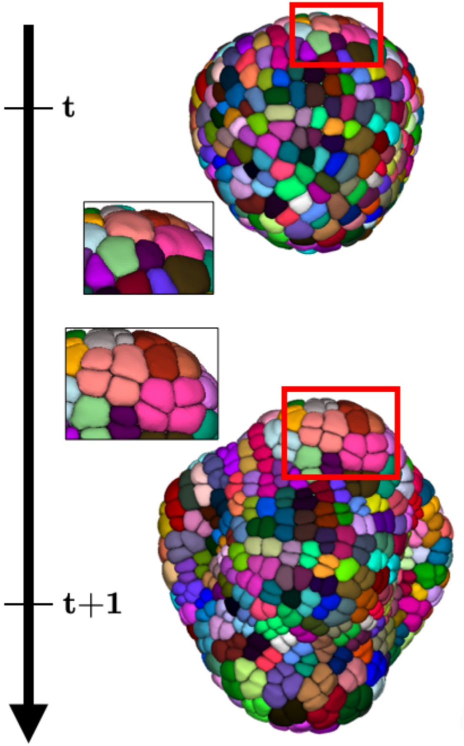

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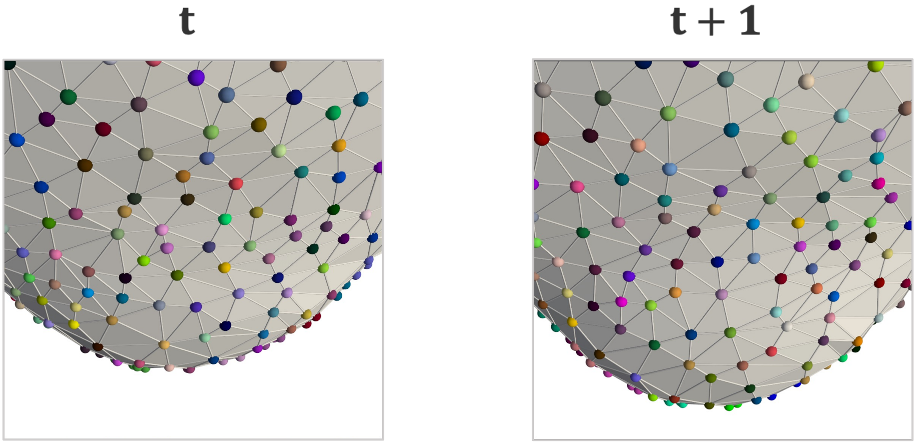


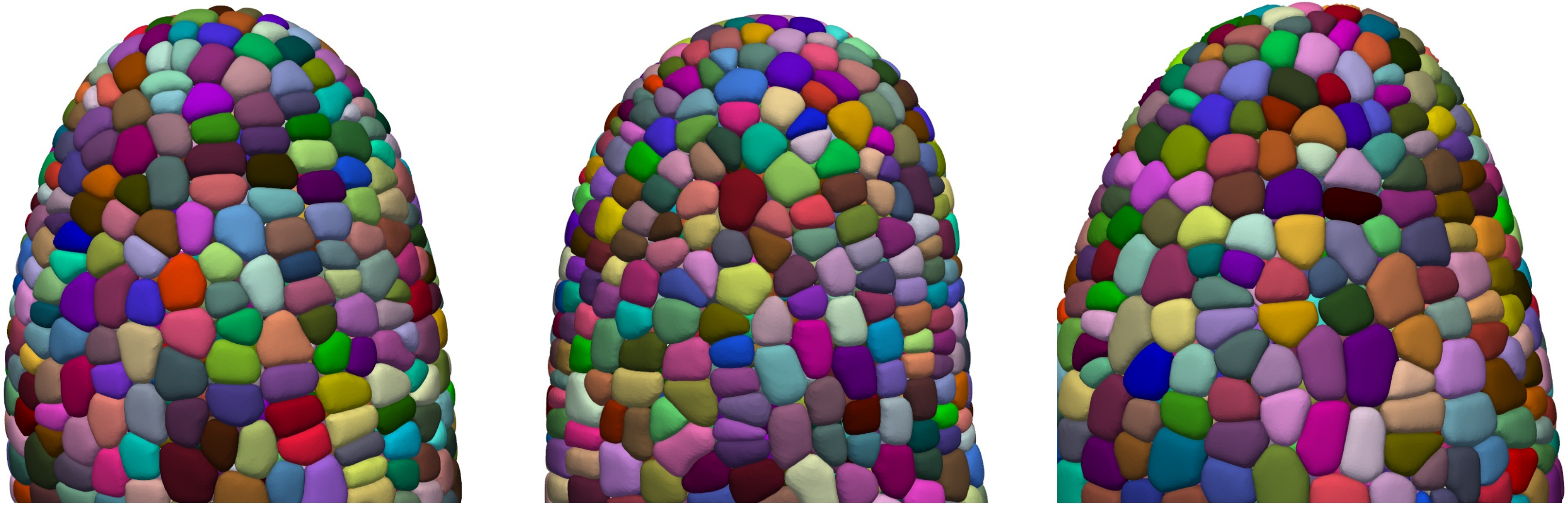
- Min. cost path of SC edit operations: mapping $V_{\mathbf{t+1}}$ to $V_{\mathbf{t}}$

Beyond reconstruction: inter-individual comparison

Data courtesy: J. Braat (RDP)

Beyond reconstruction: inter-individual comparison

Different acquisitions (individuals) of the same biological tissue



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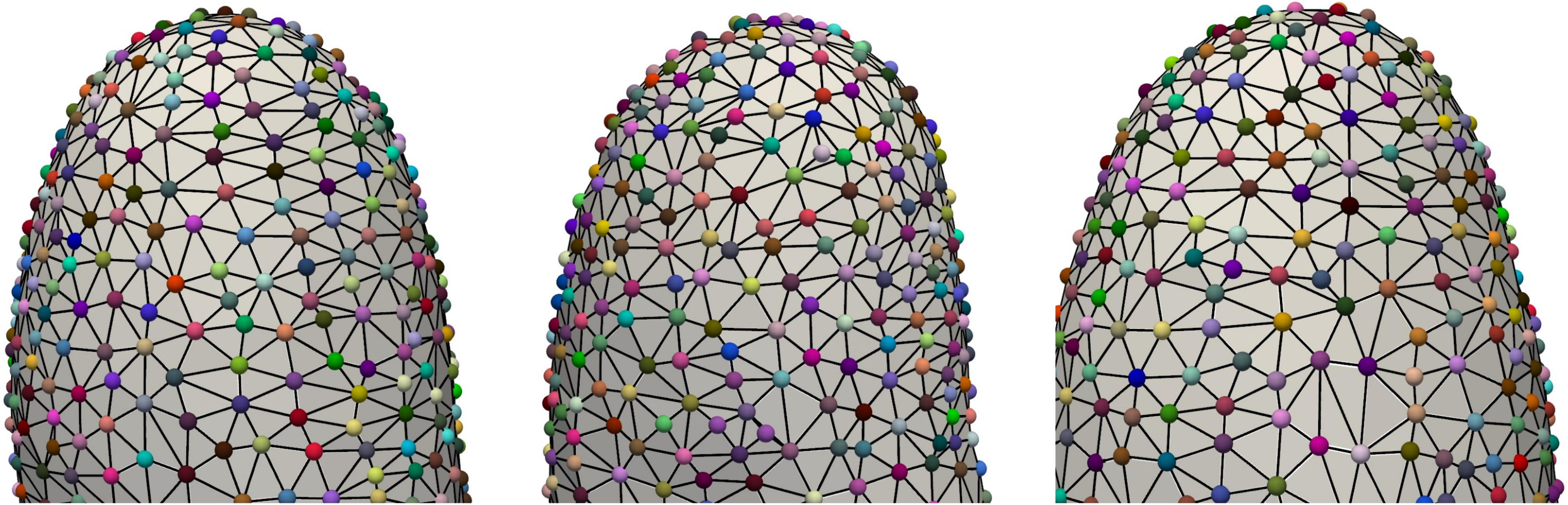

Could we compute the pairwise **edit paths** between their **simplicial complexes**?

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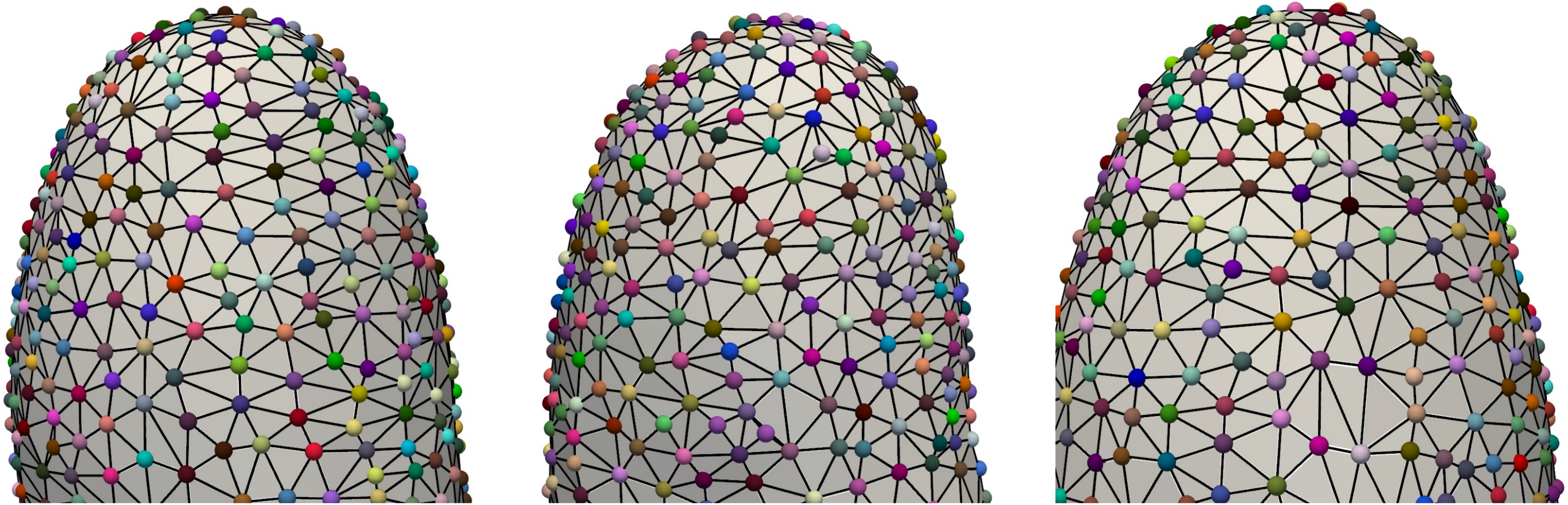
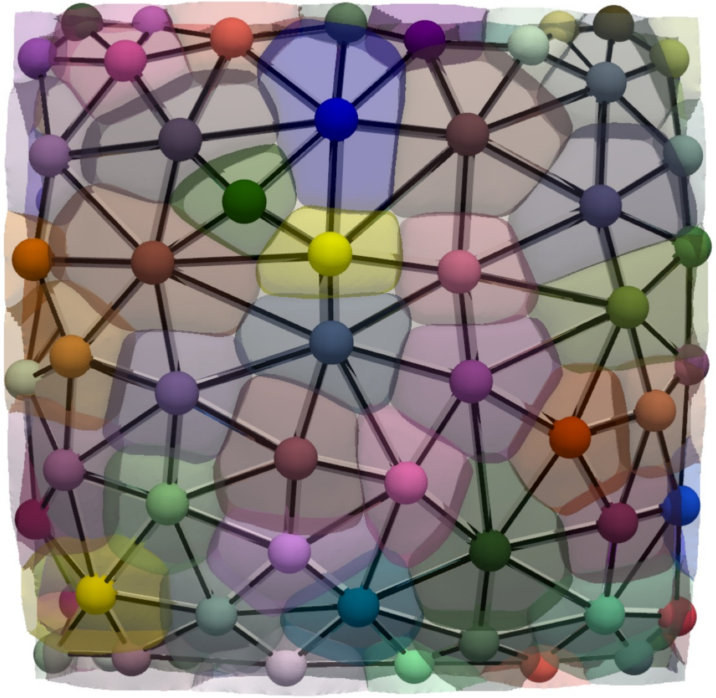


Could we compute the pairwise **edit paths** between their **simplicial complexes**?

Use mappings and edit costs to quantify local **variability of cellular organization**

Data courtesy: J. Braat (RDP)

Simplicial complexes of tissues: final thoughts



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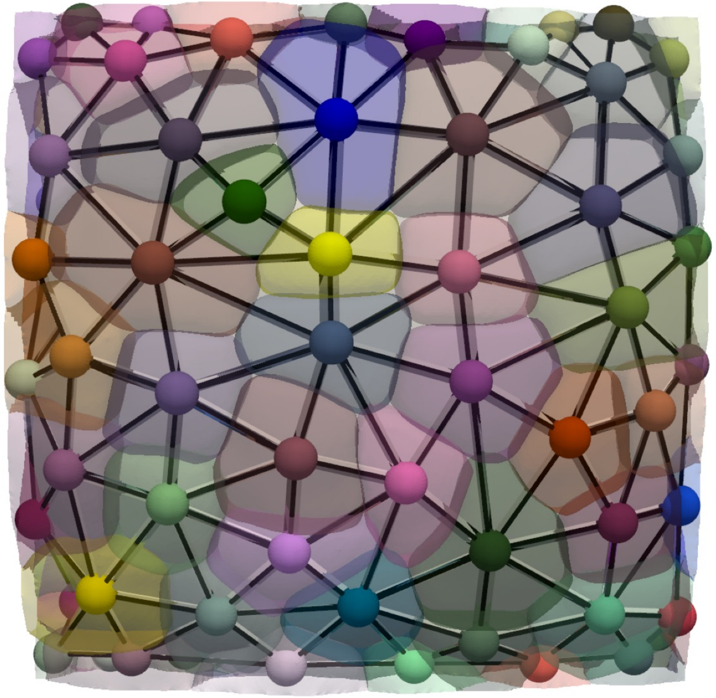

Versatile representations of multicellular tissues, suited for many applications

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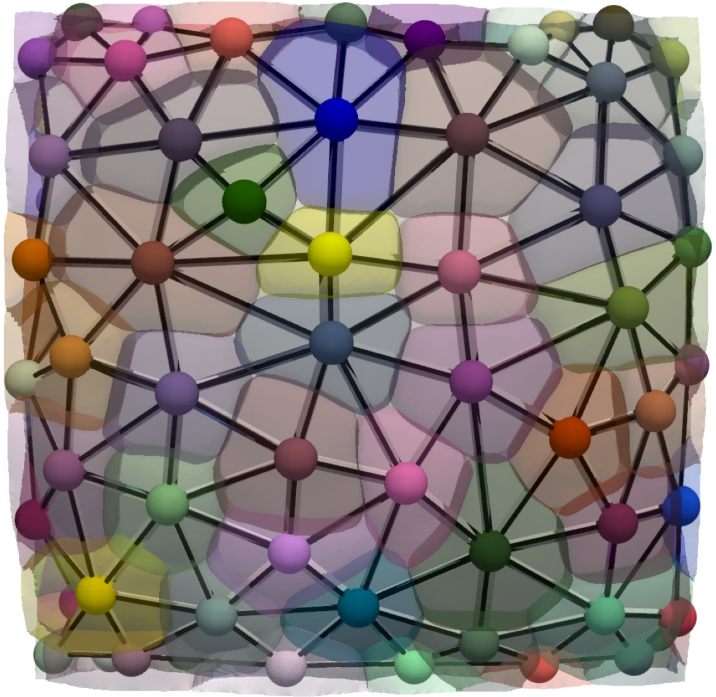
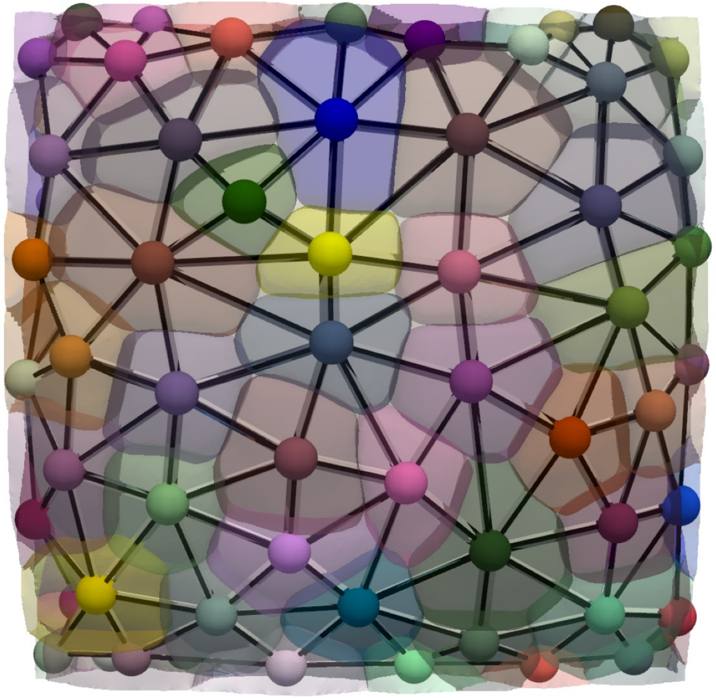


Versatile representations of multicellular tissues, suited for many applications

Higher order combinatorial space …with an extra layer of geometric constraints

Simplicial complexes of tissues: final thoughts



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Lack of (expertise on) efficient algorithmic tools to build & compare

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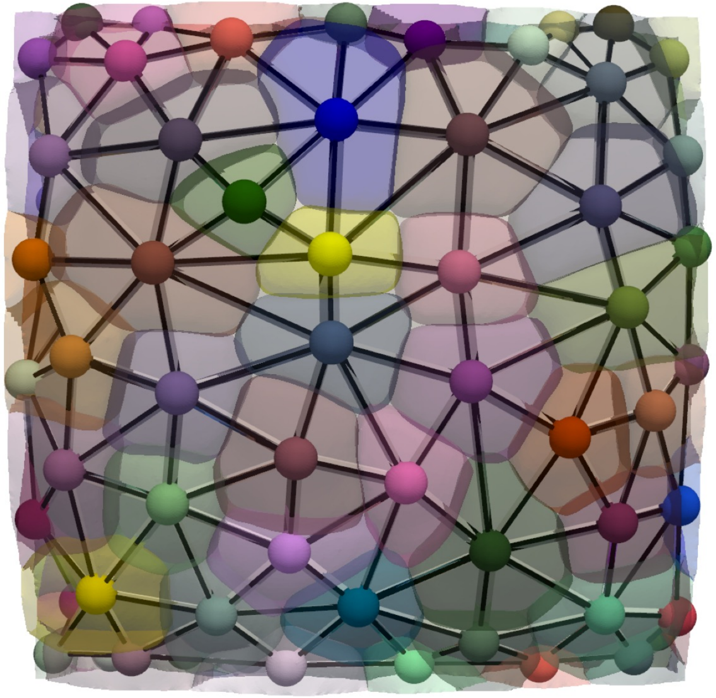

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Open exciting challenges on both biological and combinatorial sides

Thank you for your attention!