SEACS: Stochastic modEl-dAta-Coupled representationS for the analysis, simulation and reconstruction of upper ocean dynamics

A Joint CominLabs-Lebesgue-LabexMer Research Initiative

Co-PIs:

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Consortium

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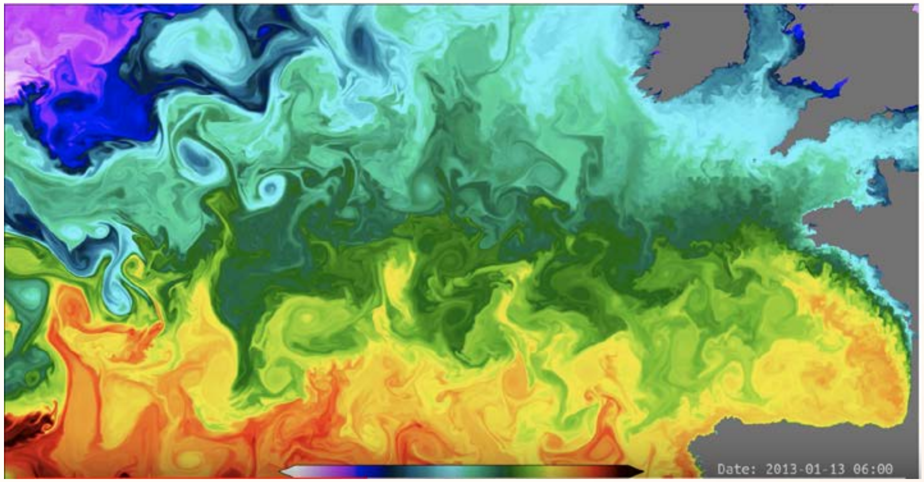
Context



NO observation/simulation system to resolve all scales and processes simultaneously
Requirement for stochastic representations to account for imperfectly resolved processes

Scientific background: model-driven vs. data-driven frameworks

Model-driven paradigm

Dynamical model

X_t → ODE solver $\partial_t X = F(X, \xi, t, \theta)$ → X_{t+1}

$+$

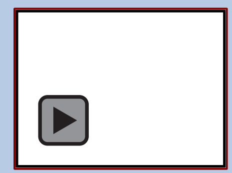
Observation model

$$Y_t = H(X, \zeta, t, \phi)$$

Data-driven paradigm

Dynamical model

X_t → **Training data** → X_{t+1}



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Observation model

$$Y_t = H(X, \zeta, t, \phi)$$

Scientific workplan

Methodological challenges

Stochastic representations of geophysical flows

Model-driven framework (Ch. I)

Data-driven framework (Ch. II)

Application challenge

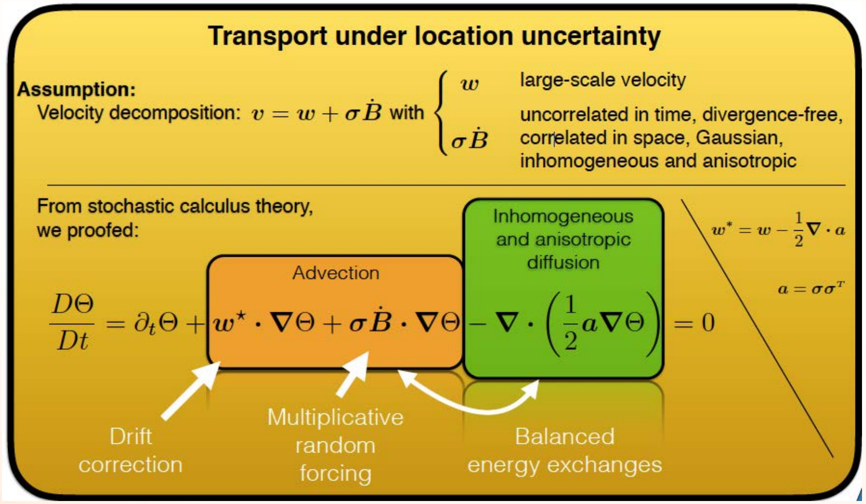
Simulation and reconstruction of upper ocean dynamics

SEACS workplan & outreach

- **Non-permanent resources:** 7 PhD, 4 Eng. & postdocs
- **Animation & visiting scientists:**
  - 4 summer schools, 3 workshops, 1 national conference, 2 doctoral courses
  - More than 40 short incoming visits: eg, Prof. D. Giannakis (NYU), Prof. S. Gotwald (Univ. Sydney),, Prof. S. Brunton (Univ. Washington)
- **Publications & awards:**
  - > 20 journal papers
  - > 40 communications in int. conferences
  - Best PhD SMAI/GAMNI 2018 (V. Resseguier)

Key scientific results

Advances on model-driven approaches



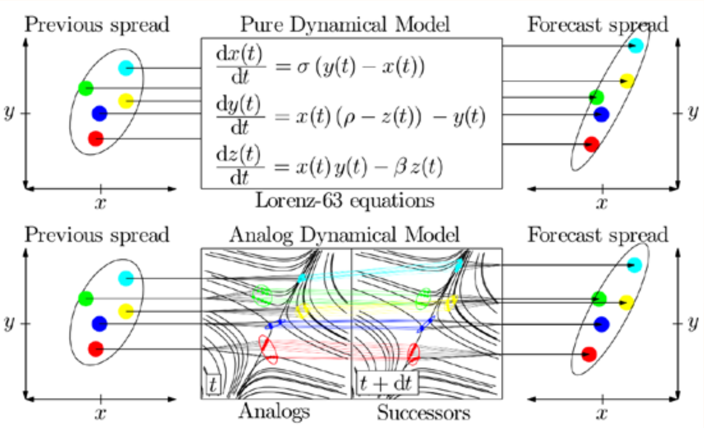
Improved LES (wake flows, Green-Taylor, TBL, …)

Derivation of stochastic geophysical flows dynamics (eg, QG)

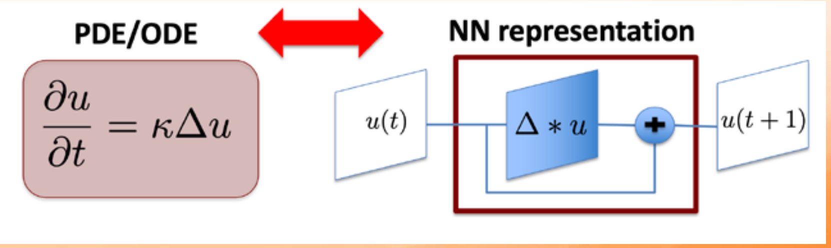
Accurate representation of errors

New analysis tool of the small-scales

Advances on data-driven approaches



Stochastic analog forcasting strategies



NN representation for (S)ODE/PDE

Data-driven schemes for geophysical flows

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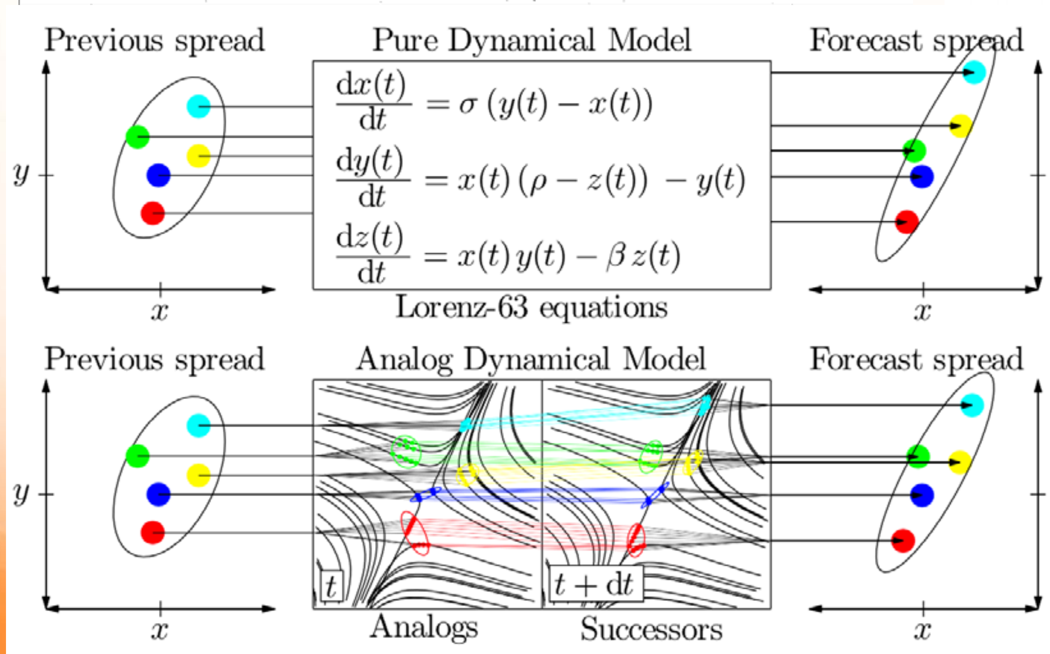
Empirical Orthogonal Functions and Statistical Weather Prediction



Deterministic Nonperiodic Flow[1]

Edward N. Lorenz

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form January 1963)

Abstract

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

[illegible]oduction

[illegible] in hydrodynamical systems exhibit steady-state [illegible]terns, while others oscillate in a regular periodic [illegible] Still others vary in an irregular, seemingly [illegible]rd manner, and, even when observed for long [illegible] of time, do not appear to repeat their previous [illegible]

[illegible] modes of behavior may all be observed in the [illegible] rotating-basin experiments, described by Fultz, [illegible]959) and Hide (1958). In these experiments, a [illegible]cal vessel containing water is rotated about its [illegible]d is heated near its rim and cooled near its center [illegible]ady symmetrical fashion. Under certain condi[illegible]e resulting flow is as symmetric and steady as the [illegible] which gives rise to it. Under different conditions [illegible]n of regularly spaced waves develops, and pro[illegible]at a uniform speed without changing its shape. [illegible]till different conditions an irregular flow pattern [illegible]nd moves and changes its shape in an irregular [illegible]odic manner.

[illegible] of periodicity is very common in natural sys[illegible]d is one of the distinguishing features of turbu[illegible]v. Because [illegible]ntaneous turbulent flow patterns [illegible]rregular, [illegible]on is often confined to the sta[illegible] [illegible]f turbul[illegible]ch, [illegible] of [illegible]nce, of [illegible] in [illegible] well [illegible]ed [illegible]. The [illegible] r fo[illegible] [illegible]d wil[illegible]ly to [illegible] [illegible]rbulent eddies—the cyclones and anticyclones—[illegible]ontinually arrange themselves into new patterns.

[illegible]esearch reported in this work has been sponsored by the [illegible]ics Research Directorate of the Air Force Cambridge [illegible] Center, under Contract No. AF 19(604)-4969.

Thus there are occasions when more than the statistics of irregular flow are of very real concern.

In this study we shall work with systems of deterministic equations which are idealizations of hydrodynamical systems. We shall be interested principally in nonperiodic solutions, i.e., solutions which never repeat their past history exactly, and where all approximate repetitions are of finite duration. Thus we shall be involved with the ultimate behavior of the solutions, as opposed to the transient behavior associated with arbitrary initial conditions.

A closed hydrodynamical system of finite mass may ostensibly be treated mathematically as a finite collection of molecules—usually a very large finite collection —in which case the governing laws are expressible as a finite set of ordinary differential equations. These equations are generally highly intractable, and the set of molecules is usually approximated by a continuous distribution of mass. The governing laws are then expressed as a set of partial differential equations, containing such quantities as velocity, density, and pressure as dependent variables.

It is sometimes possible to obtain particular solutions [illegible]f these equations analytically, especially when the [illegible]lutions [illegible] periodi[illegible] [illegible]ith time, and, in[illegible]ee[illegible] [illegible]ork[illegible] [illegible]de[illegible] [illegible]btaining such [illegible] [illegible]ordinarily, how[illegible]ve[illegible] [illegible]onp[illegible] [illegible]luti[illegible] [illegible]dily be deter[illegible]mine[illegible] [illegible]by n[illegible] [illegible]such procedures involve replacing [illegible]nuous variables by a new finite set of functi[illegible]e, which may perhaps be the values of the continuous variables at a chosen grid of points, or the coefficients in the expansions of these variables in series of orthogonal functions. The governing laws then become a finite set of ordinary differential

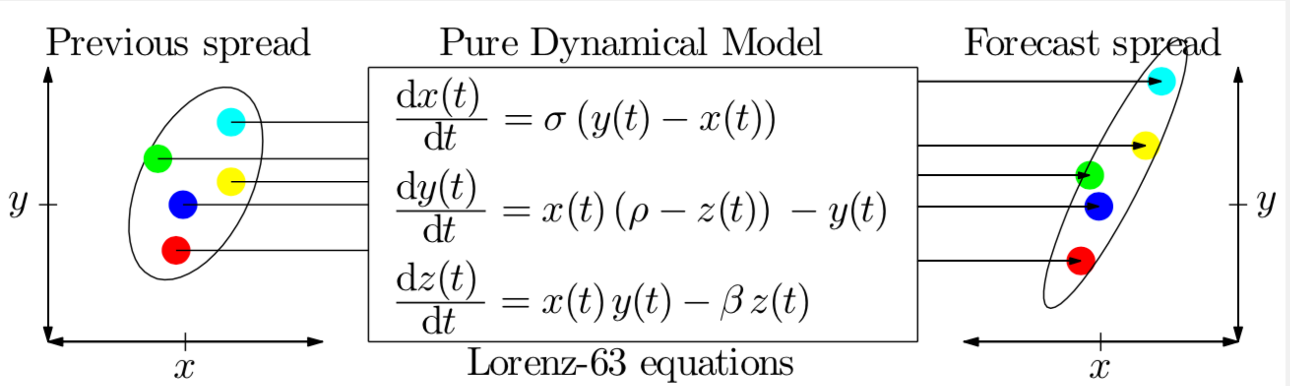
Analogs

Analog forecasting operator & Data Assimilation

Key idea

Replacing the explicit dynamic model by an analog forecasting operator

***Plug-and-play application to stochastic filters** (e.g., EnKF, PF)*



$$\boxed{X_{t+1} | X_t = \mathcal{M}_{X_t} X_t + \epsilon(X_t)}$$

« Local » Gaussian/Linear state-dependent model fitted using analogs

Lguensat et al., 2017

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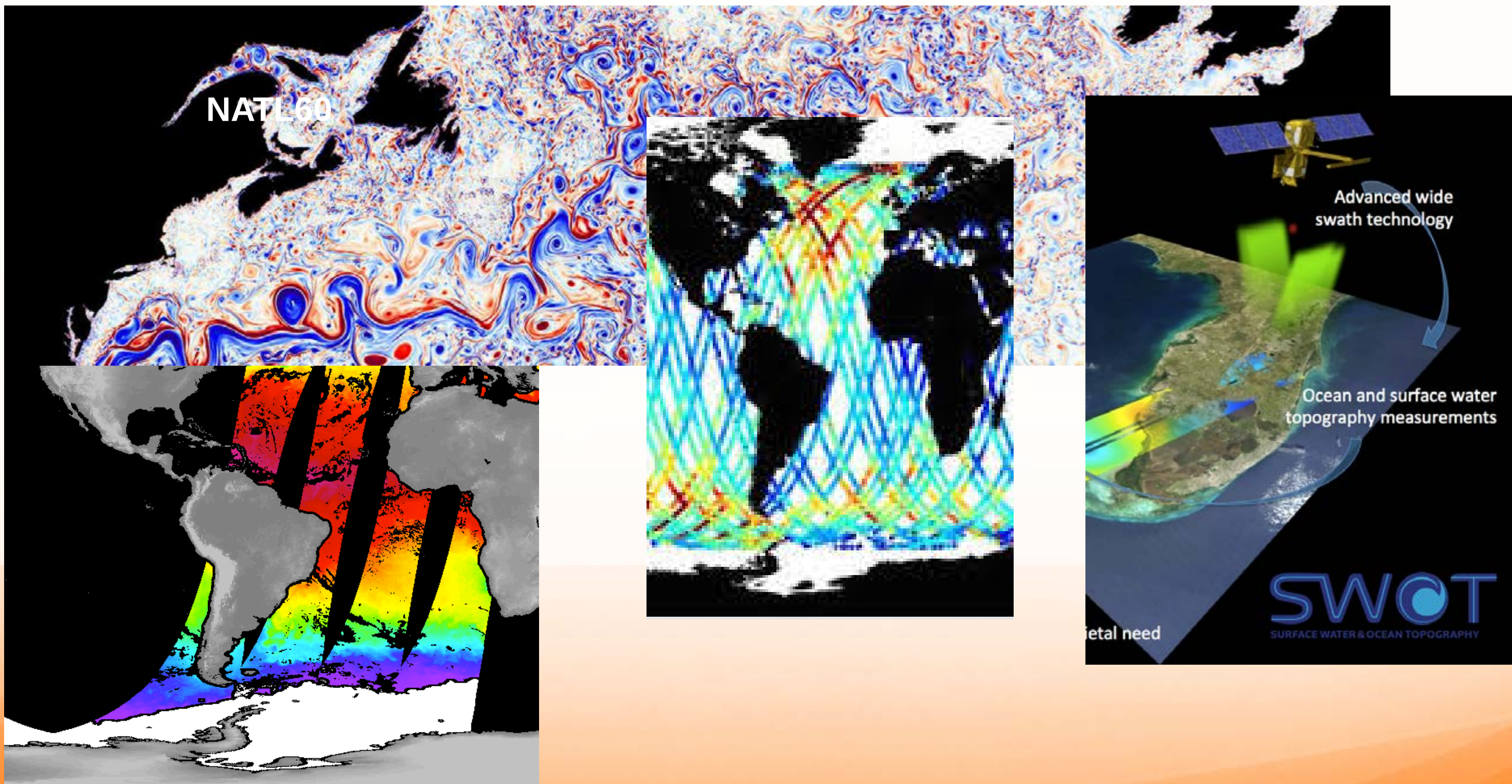
Analog forecasting operator & Data Assimilation

Extension to 2D+t geophysical fields:



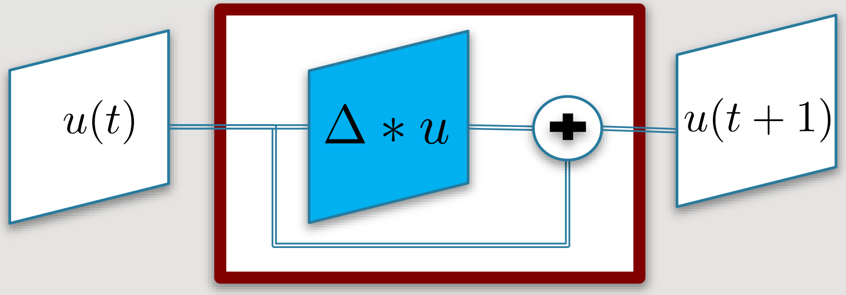
e.g., (Fablet et al., 2017, Lopez Radcenco et al., 2019)

Towards learning white-boxes

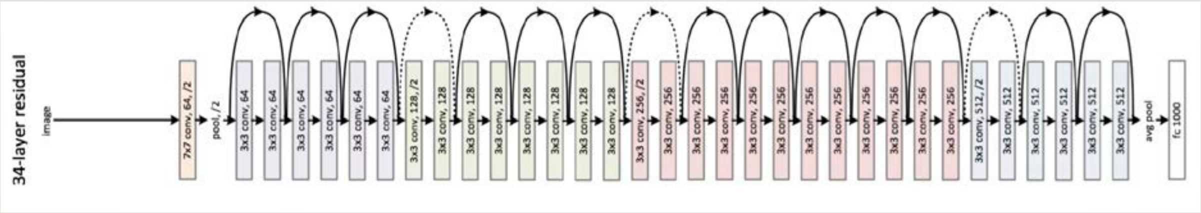
PDE/ODE

$$\frac{\partial u}{\partial t} = \kappa \Delta u$$

NN representation



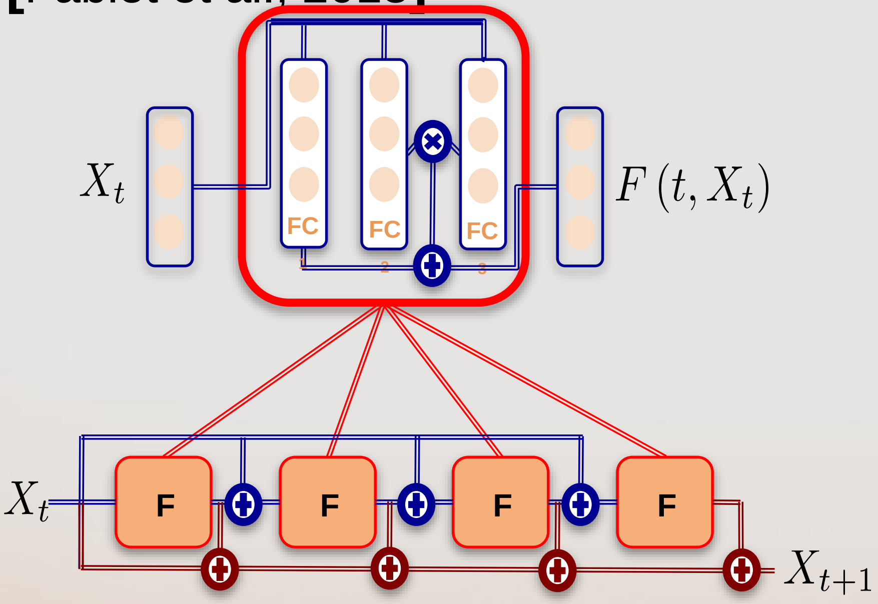
Numerical integration schemes as ResNets [He etal. 2015]



NN representations for ODEs/PDEs

An example: Residual RK4 Bilinear Network

[Fablet et al., 2018]



$$\frac{dx(t)}{dt} = \sigma\,(y(t) - x(t))$$
$$\frac{dy(t)}{dt} = x(t)\,(\rho - z(t)) - y(t)$$
$$\frac{dz(t)}{dt} = x(t)\,y(t) - \beta\,z(t)$$

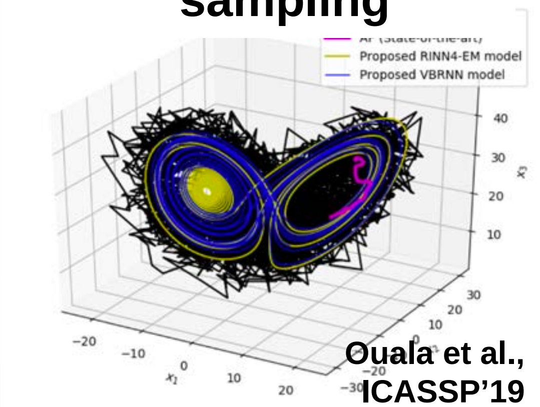
Lorenz-63 equations

Noise-free training data

Forecasting time step	**t_0+h**	**t_0+4h**	**t_0+8h**
Analog forecasting	$<10^{-6}$	0.002	0.005
Sparse regression	$<10^{-6}$	0.002	0.006
MLP	$<10^{-6}$	0.018	0.044
Bi-NN(4)	***$<10^{-6}$***	***$<10^{-6}$***	***$<10^{-6}$***

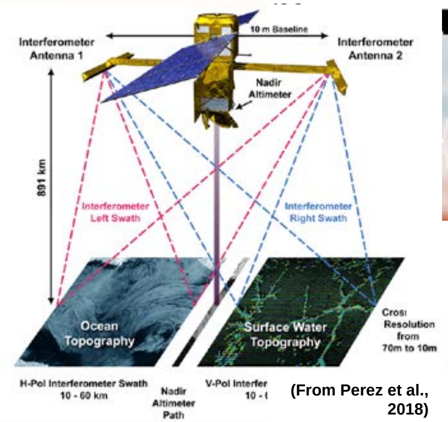
Learning from real observation data ?

Scarce time sampling

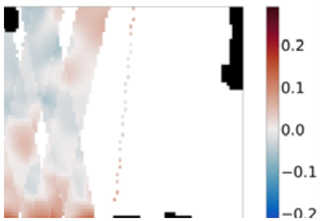


Ouala et al., ICASSP'19

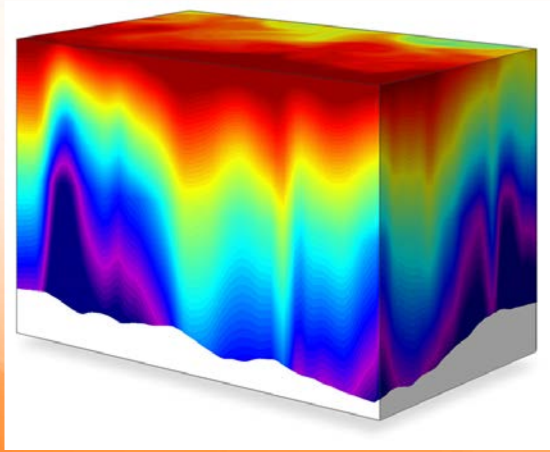
Noisy and irregular sampling



(From Perez et al., 2018)



Nguyen et al., preprint, 2019



Partially-observed system

Ouala et al., preprint 2019

Leveraging effects

- **Supporting grants and fellowships:**
  - ESA postdoc fellowship (C. Gonzalez, 2016-2017)
  - SAD Region SeaStorm (W. Bauer)
  - Teralab grant (2016-2018)
  - Microsoft AI4Ocean grant (2018, GPU resources)
  - OSTST MANATEE (co-PI, R. Fablet, CNES, 2017-2020)
- **New initiatives and partnerships:**
  - Industrial partnerships: eg, ITGA, CSTB, OceanNext, e-odyn,…
  - Isblue theme « Observing Systems » (co-PIs B. Chapron, R. Fablet)
  - H2020 Eurosea (2020-) (PI: A. Franke, 2019-2023)
  - LEFE/MANU IA-OAC (PI. R. Fablet, 2019-2021)
  - ANR Melody (PI: R. Fablet, 2019-2023)
  - ERC Synergy STUOD (co-Pis: B. Chapron, E. Mémin, 2020-2025)
  - Joint INRIA team between INRIA Rennes, IMT Atlantique and Ifremer under discussion