Dynalearn

Physics for Deep Learning and Deep Learning for Physics

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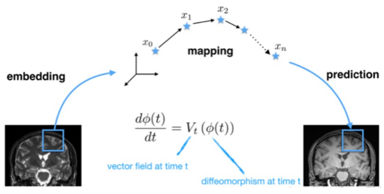
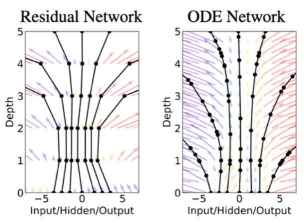
Overview

- Though extremely popular and efficient, deep neural networks require well-defined mathematical frameworks and often suffer from lack of physical justifications.
- Some connections exist between neural nets and dynamical systems (ex : ResNets and ODE)





Goals of the project

Interactions between deep learning and physical models

- Can dynamical formulations of learning process help in understanding deep neural architectures ?
- Can neural representations help new data-driven dynamical simulation models ?

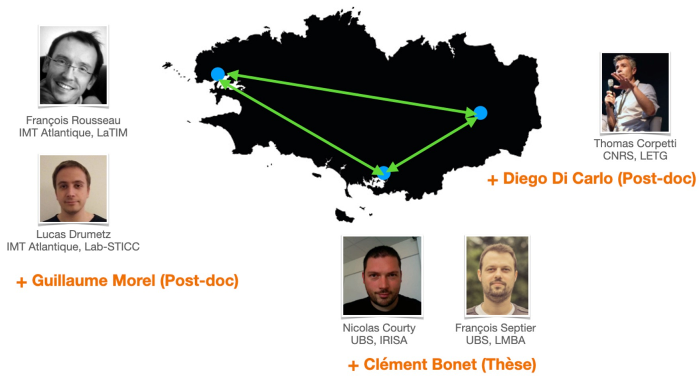
Questions :

1) How can a learning system be expressed as a dynamical system?

2) How can learning-based methods help in simulating complex dynamic systems?

3) How to handle the probabilistic nature of data ?

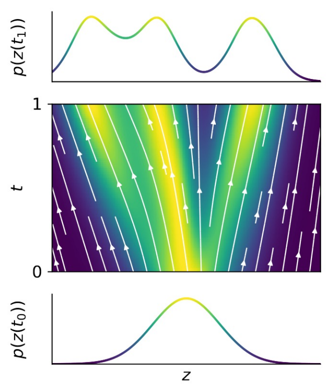
Consortium



1) Optimal Transport and incompressible flows

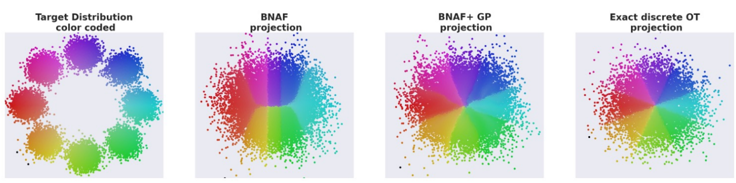
Normalizing Flows (NF) are powerwul likelihood based generative models.

Optimal Transport (OT) is used to find models with minimal effort between the source and target distributions.



We proposed a method called GP-flow based on Brenier's polar factorization theorem to transform any trained NF into a more OT-efficient version without changing the final density.

We do so by learning a rearrangement of the source (Gaussian) distribution that minimizes the OT cost between the source and the final density.



Target distribution : mixture of 8 Gaussians.

2) Physical super-resolution

Goal : Improve spatial and temporal resolution of physical images

Model-driven

- Interpolation, deconvolution, sparse coding, . . .
- Scale interaction physical laws

→ Limited performances (computational complexity and numerical instability) & difficulties to embed new priors

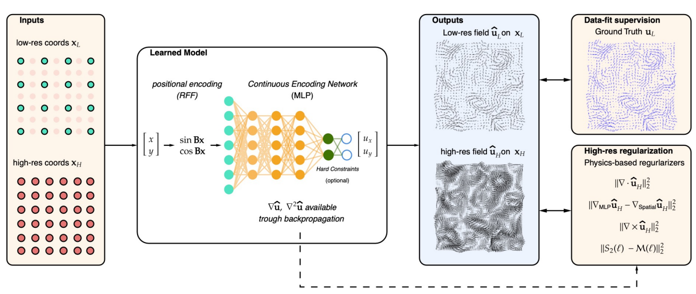
Data-driven

- Kernel interpolation (Gaussian Process)
- Deep Learning (CNNs, UNets, GANs, etc

→ impressive visual results but no evaluation on single points, so physical consistency

This project : PINNs for Super-resolution

- Implicit representation (Multiplicative Filters), unsupervised, meshless, data and model driven
- Regularize with physical models (divergence free, Kolmogorov subgrid), possible use of CNN for spatial features



3) Sliced Wasserstein Flows

Minimizing functionals with respect to probability measures is a ubiquitous problem in machine learning.

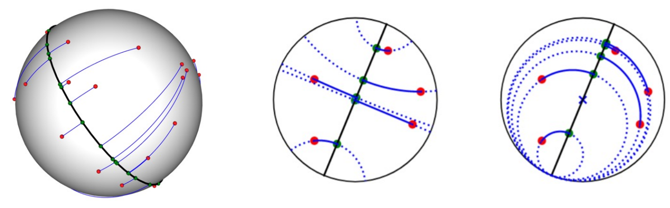
We derived a new class of gradient flows in the space of prabability measures endowed with the sliced-Wasserstein metric, and the corresponding algorithms.
→less computationally intensive and more versatile than the state-of-the-art methods.

	JKO-ICNN		SWGF+RealNVP	
Dataset	Acc	t	Acc	t
covtype	$0.755 \pm 5 \cdot 10^{-4}$	33702s	$0.755 \pm 3 \cdot 10^{-3}$	103s
german	$0.679 \pm 5 \cdot 10^{-3}$	2123s	$\mathbf{0.68} \pm 5 \cdot 10^{-3}$	82s
diabetis	$0.777 \pm 7 \cdot 10^{-3}$	4913s	$\mathbf{0.778} \pm 2 \cdot 10^{-3}$	122s
twonorm	$0.981 \pm 2 \cdot 10^{-4}$	6551s	$0.981 \pm 6 \cdot 10^{-4}$	301s
ringnorm	0.736 ± 10^{-3}	1228s	$\mathbf{0.741} \pm 6 \cdot 10^{-4}$	82s
banana	0.55 ± 10^{-2}	1229s	$\mathbf{0.559} \pm 10^{-2}$	66s
splice	$0.847 \pm 2 \cdot 10^{-3}$	2290s	$\mathbf{0.85} \pm 2 \cdot 10^{-3}$	113s
waveform	$\mathbf{0.782} \pm 8 \cdot 10^{-4}$	856s	$0.776 \pm 8 \cdot 10^{-4}$	120s
image	$\mathbf{0.822} \pm 10^{-3}$	1947s	$0.821 \pm 3 \cdot 10^{-3}$	72s

4) Manifold Sliced Wasserstein

Sliced Wasserstein distance has received a lot interest since it leverages one-dimensional projections for which a closed-form solution of the Wasserstein distance is available.

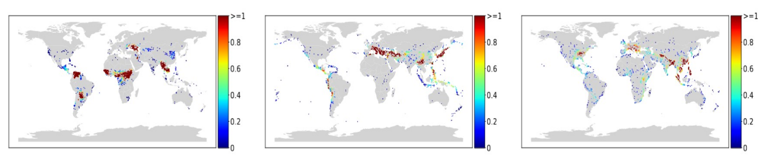
We derived a new class of sliced-Wasserstein discrepancies on non-trivial manifolds.



(a) Along geodesics. (b) Along horospheres.

Hyperspheres **Hyperbolic embeddings**

This discrepancy has been successfully applied on different learning tasks: **density estimation, classification, self-supervised learning, …**



(a) Fire (b) Earthquake (c) Flood

Density estimation on Earth of real world phenomena

Publications

• Bonet, C., Courty, N., Septier, F., Drumetz, L. Sliced-Wasserstein Gradient Flows. arXiv:2110.10972, 2021
• Bonet, C., Vayer, T., Courty, N., Septier, F., Drumetz, L. Subspace Detours Meet Gromov-Wasserstein. Algorithms, MDPI. 14. 2021
• Bonet, C., Berg, P., Courty, N., Septier, F., Drumetz, L., Pham, M.-T. Spherical Sliced-Wasserstein. arXiv:2206.08780, 2022
• Di Carlo, D, Heitz, D., Corpetti, T. and Courty, N. Post Processing Sparse And Instantaneous 2D Velocity Fields Using Physics-Informed Neural Networks, 20th *Int. Symp. on applications of laser techniques to fluid mechanic*, Lisbon, 2022
• Morel, G., Drumetz, L., Courty, N., Rousseau, F. and Benaïchouche, S. Turning Normalizing Flows into Monge Maps with Geodesic Gaussian Preserving Flows. arXiv preprint arXiv:2209.10873, 2022.