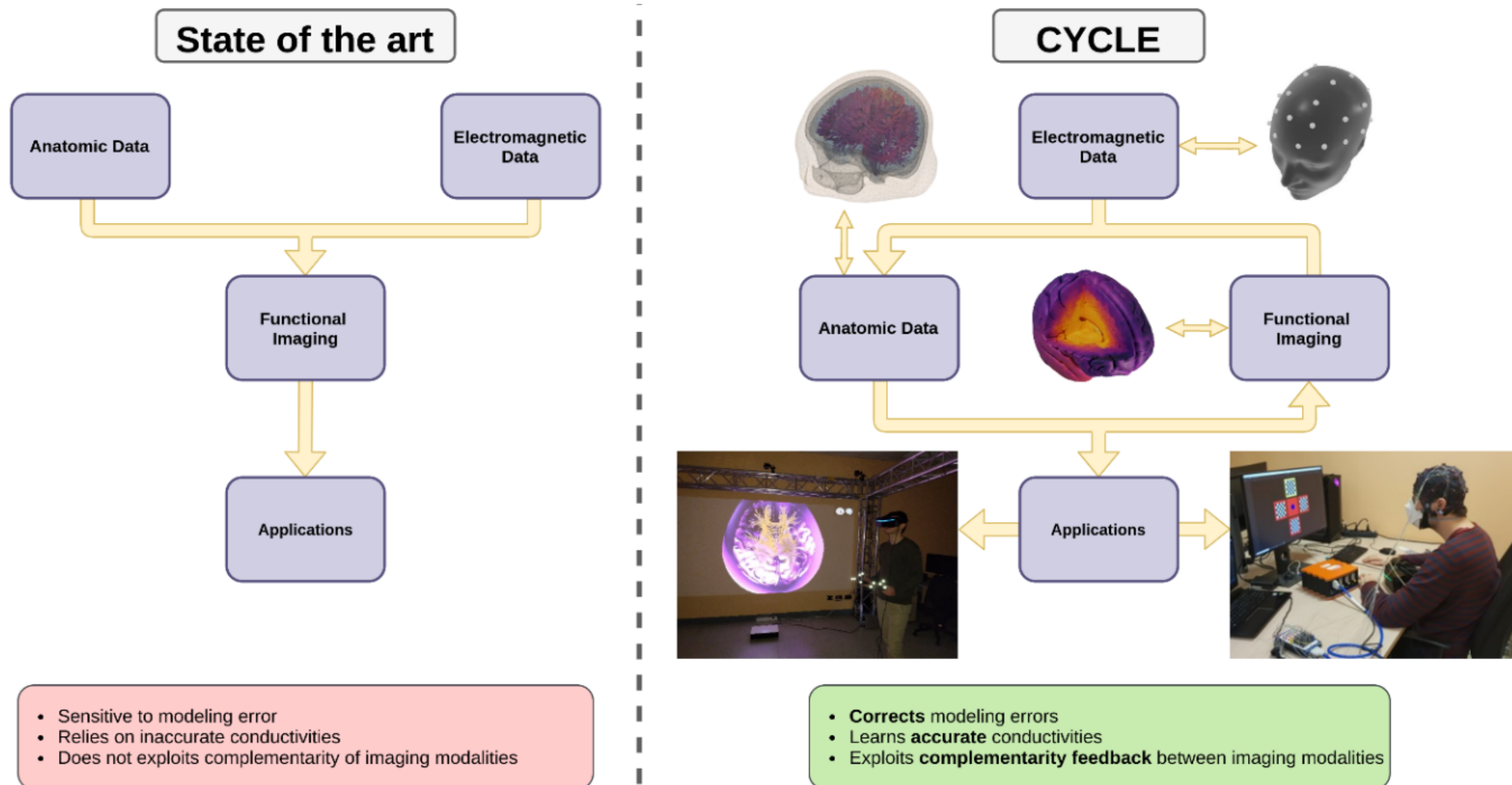


General Description



New High-Resolution and Full Wave Forward Models

Electric Flux Volume Integral Equation for Modeling Anisotropic and Inhomogeneous objects

$$\frac{D(\mathbf{r})}{\epsilon(\mathbf{r})} - (\mathcal{L}_\kappa^\Omega D)(\mathbf{r}) = E_i(\mathbf{r})$$

Discretization



$$\mathbf{Z}\alpha = (\mathbf{G}_\epsilon + \mathbf{Z}_\Phi + \mathbf{Z}_A)\alpha = \mathbf{v}$$

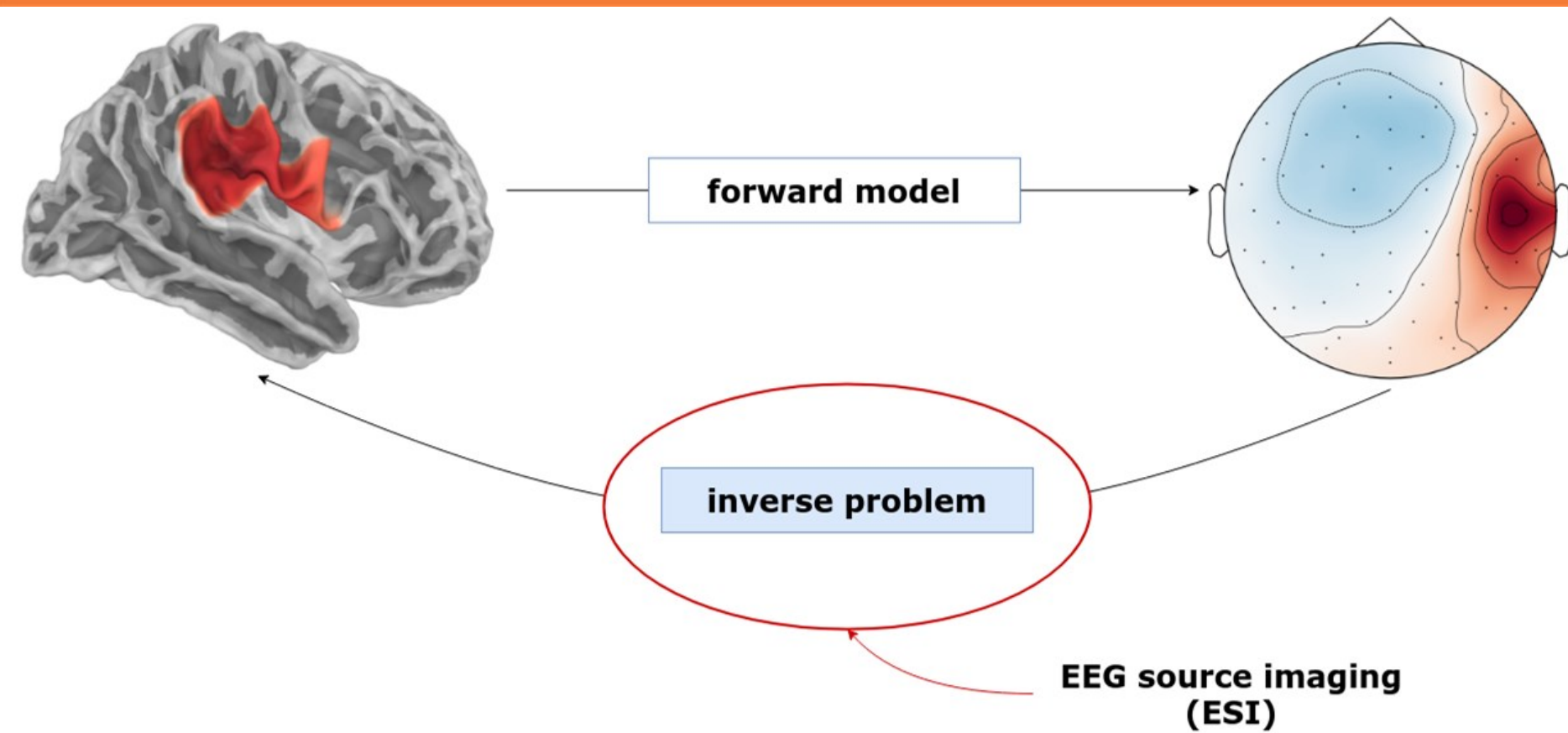


Ill conditioned Gram matrix

→ Low-Frequency (LF) breakdown

→ High-Contrast (HC) breakdown

Inverse Problem in EEG



Forward model: $\mathbf{M} = \mathbf{G}\mathbf{D} + \epsilon$ (ϵ = additive noise)

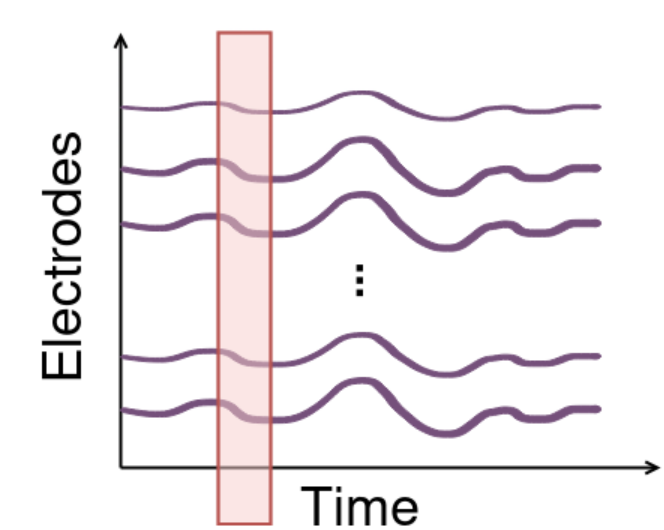


- $p \gg N$: \mathbf{G} non-invertible
- Volume mixing/conduction

Ill-posed inverse problem: add prior on \mathbf{D} to solve.

$$\hat{\mathbf{D}} = \arg \min_{\mathbf{D}} (\underbrace{\|\mathbf{M} - \mathbf{G}\mathbf{D}\|^2}_{\text{data fitting}} + \underbrace{\lambda \mathcal{R}(\mathbf{D})}_{\text{regularization}})$$

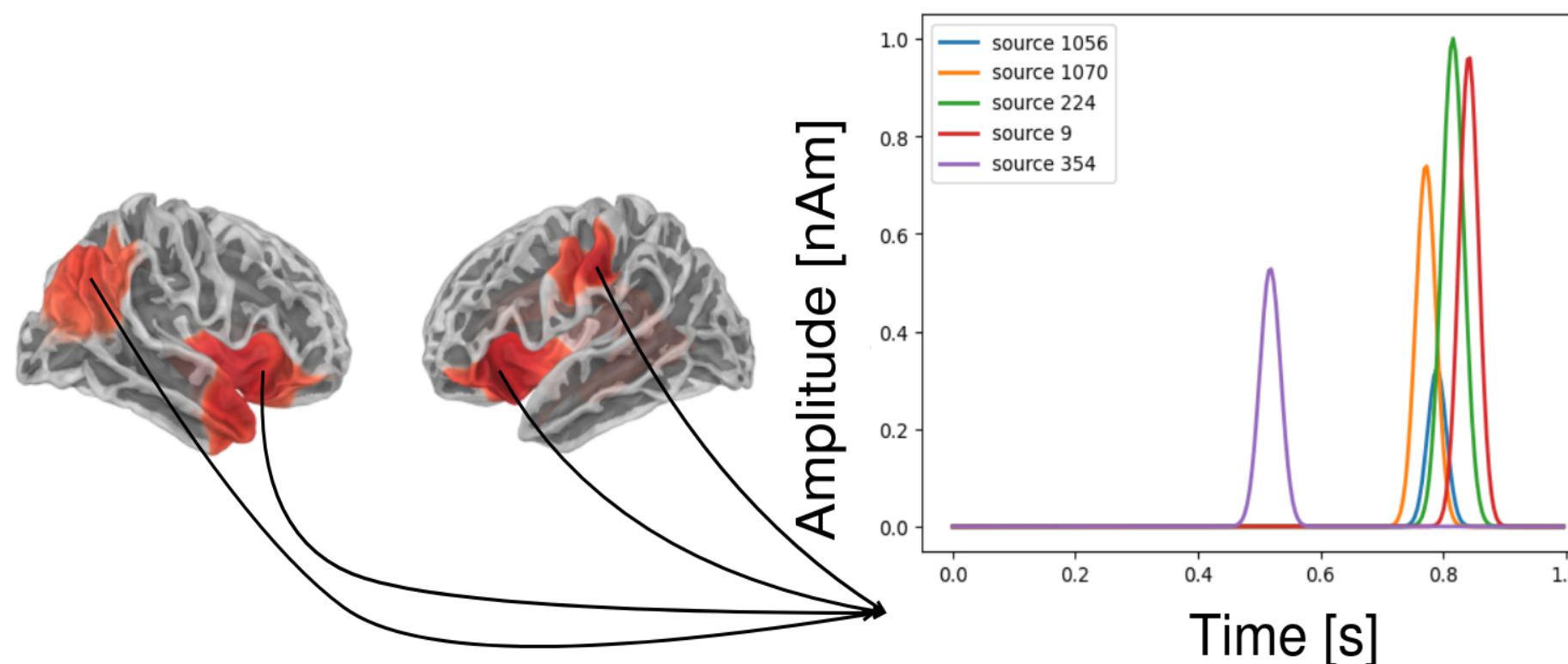
Deep Learning for ESI



Network parameters:

- 1D-CNN: 7 102 464 params
- LSTM: 513 400 params
- Ground truth source activity (\mathbf{D}) with SEREEGA

$$\text{loss}(\mathbf{D}, \hat{\mathbf{D}}) = -\frac{1}{T} \sum_{k=1}^T \left(\frac{\sum_{i=1}^{N_s} \mathbf{D}_{i,k} \hat{\mathbf{D}}_{i,k}}{\|\mathbf{D}_{:,k}\| \|\hat{\mathbf{D}}_{:,k}\|} \right)$$



Regularization of HC and LF Breakdowns with Generalized Volume Quasi-Helmholtz Projectors [9]

New Projectors

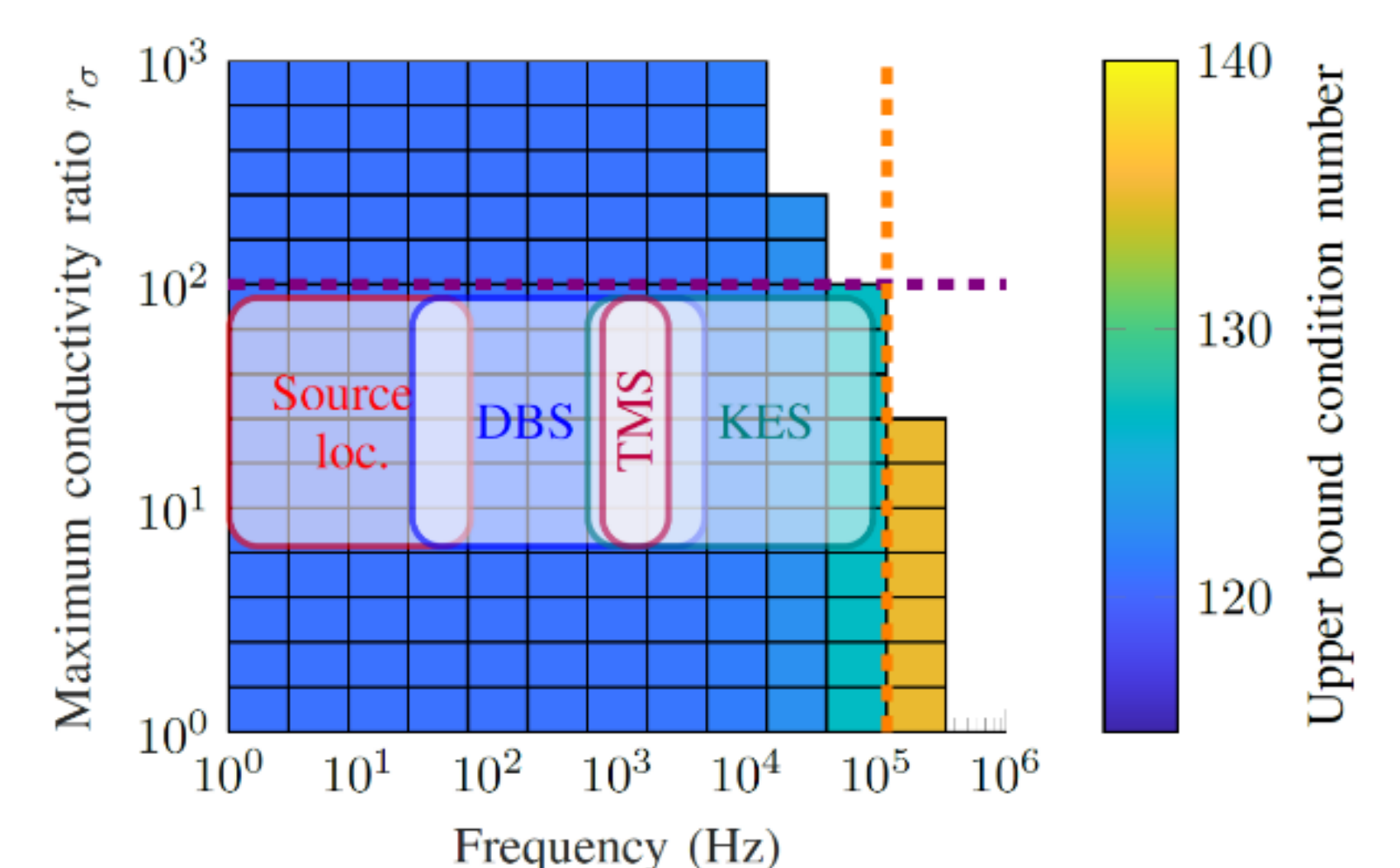
$$\mathbf{P}_{\mathbf{G}_\epsilon}^\Sigma = \mathbf{G}_\epsilon^{-1} \Sigma (\Sigma^T \mathbf{G}_\epsilon^{-1} \Sigma)^+ \Sigma^T$$

$$\mathbf{P}_{\mathbf{G}_\epsilon}^\Lambda = \mathbf{I} - \mathbf{P}_{\mathbf{G}_\epsilon}^\Sigma = \Lambda (\Lambda^T \mathbf{G}_\epsilon \Lambda)^+ \Lambda^T \mathbf{G}_\epsilon$$

Left Preconditioner

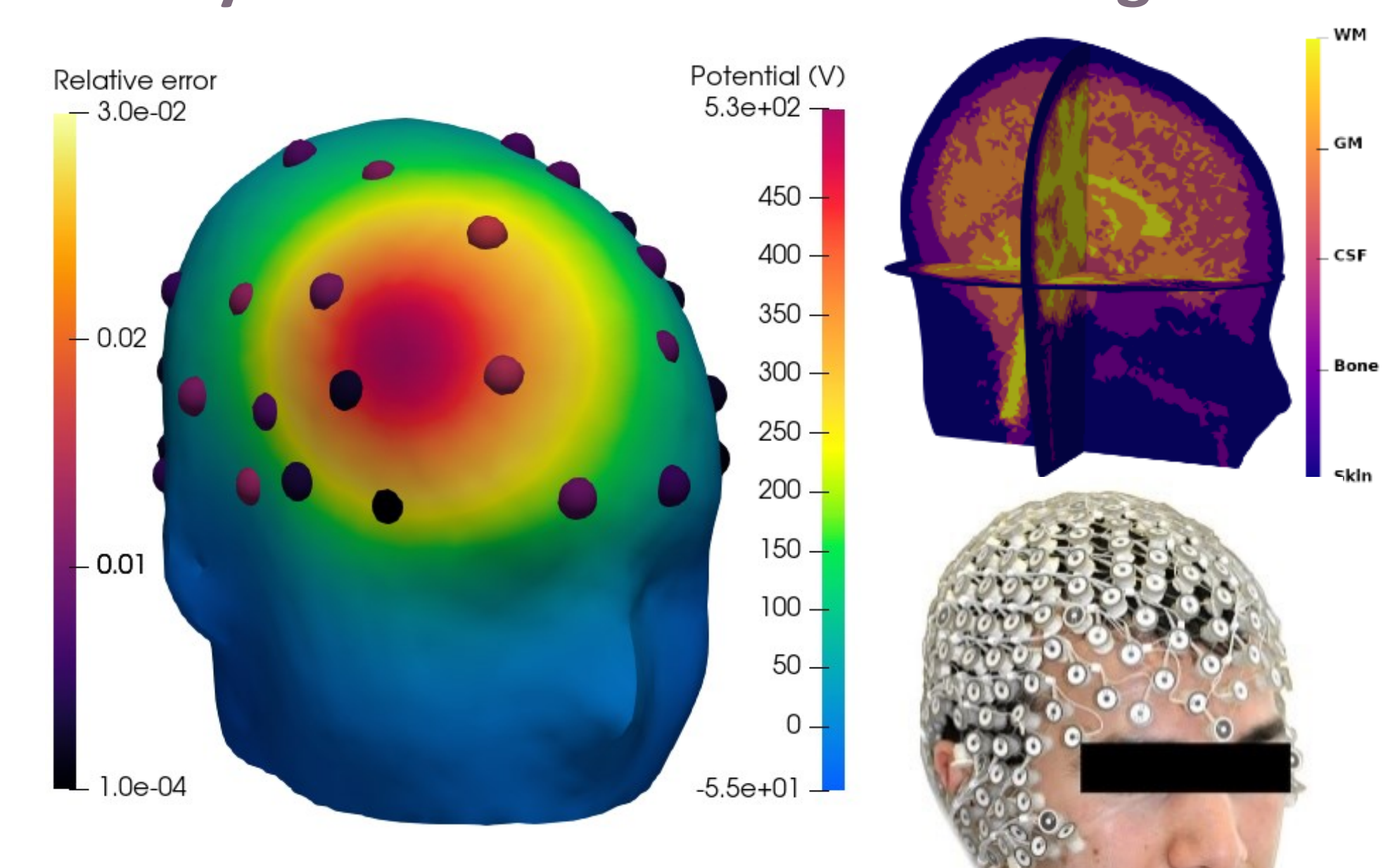
$$\mathbf{L}_{\mathbf{G}_\epsilon} = \mathbf{P}_{\mathbf{G}_\epsilon}^\Lambda \mathbf{G}_\epsilon^{-1} + \mathbf{P}_{\mathbf{G}_\epsilon}^\Sigma \mathbf{G}^{-1}$$

Range of operation of the formulation



DBS: Deep brain stimulation, TMS: Transcranial Magnetic stimulation, KES: Kilohertz electrical stimulation

Validity of the EEG forward modeling



1d-CNN for ESI

SNR	Methods Metrics	Single extended source				Multiple extended sources			
		1D-CNN	LSTM	MNE	sLORETA	1D-CNN	LSTM	MNE	sLORETA
30dB	AUC ↑	0.9860	0.9878	0.7862	0.7844	0.7818	0.7782	0.6600	0.6558
	LE [mm] ↓	2.94	2.45	9.87	6.25	4.42	5.22	11.05	7.87
	nMSE ↓	0.0039	0.0024	0.0231	0.0328	0.0053	0.0045	0.0232	0.0375
	PSNR ↑	39.5436	41.8593	32.0799	29.7051	37.6688	38.3320	30.3987	27.6743
	time error [ms] ↓	0.05	0.45	1.25	1.07	1.91	3.23	7.42	6.61
20dB	AUC ↑	0.9859	0.9874	0.7847	0.7847	0.7807	0.7763	0.6528	0.6510
	LE [mm] ↓	3.31	2.48	10.07	6.26	4.76	5.48	11.87	8.63
	nMSE ↓	0.0046	0.0024	0.0233	0.0354	0.0062	0.0046	0.0257	0.0440
	PSNR ↑	38.7850	41.6129	29.4784	27.5368	36.8713	37.6045	27.6764	25.3797
	time error [ms] ↓	0.11	0.46	2.62	2.00	2.65	5.76	15.14	10.89

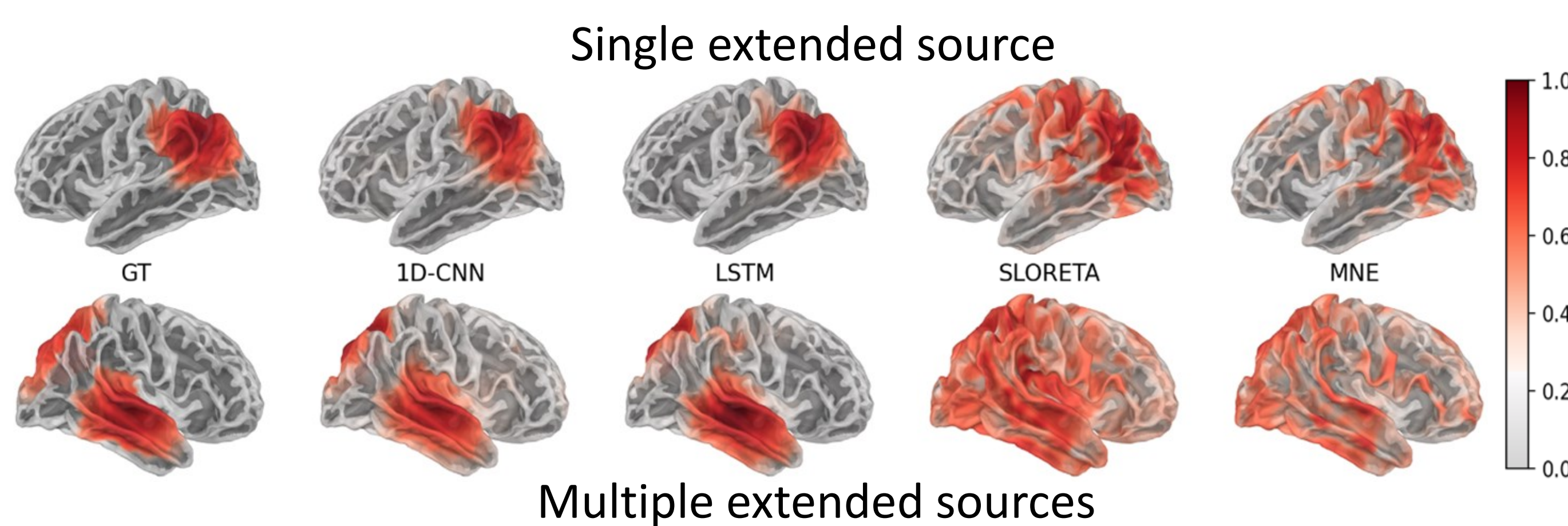
- AUC**: Area under the ROC curve (extension)
- LE**: localisation error
- nMSE**: normalized MSE

Training details:

- 10000 samples / dataset (80% training, 20% evaluation)
- ADAM, 100 epochs, gradient clipping for LSTM

Baseline:

MNE[4], sLORETA: non learning based methods



Ongoing Work and Next Steps

- Acceleration of the forward model using via fast direct solver
- Other brain activity models
- BCI feedbacked strategy
- Experimental Validation



Published Results

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- [4] A. Merlini *et al.*, ICEAA 2023.
- [5] C. Henry *et al.*, IEEE CAMA 2023.
- [6] C. Henry *et al.*, EuCAP 2023.
- [7] V. Giunzioni *et al.*, *Journal of Computational Physics*, 491, 2023.
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