CYCLE: Final Report

Functional to structural to functional imaging: a feedbaCked strategY for intraCraniaL widEband assessments

CominLabs 2021 call. Coordinator: Lab-STICC, IMT Atlantique. Partners: Lab-STICC and LaTIM (IMT Atlantique), IETR (Université de Rennes).

1. Objective and scope

Accurate knowledge of the broadband electromagnetic parameters of the head governs the performance of several distinct technologies. Localizing the seizure onset zone before epilepsy surgery through EEG source imaging depends on low-frequency tissue conductivities; transcranial magnetic stimulation and radiofrequency exposure assessment depend on the same tissues at much higher frequencies. Each of these fields has developed its own modelling tools, valid in its own band, and information does not pass between them.

CYCLE set out to reverse the usual direction of neuroimaging and, in doing so, to bridge those bands. Structural models normally feed functional imaging; the project investigated the opposite coupling, in which functional measurements are used to correct the structural and electromagnetic head model that produced them, and the corrected model is then used at higher frequencies. Four objectives followed from this: a fast and accurate broadband electromagnetic head model together with a learning-based inverse strategy able to exploit it; a framework for quantifying the sensitivity of imaging procedures to head-model errors at scale; an anatomic-functional feedback loop for broadband tissue-parameter imaging; and validation on realistic head phantoms.

Realizing this required three components that did not previously exist together: a forward solver valid across the whole frequency band rather than one solver per regime, an inverse stage that stays tied to the forward model when that model changes, and a physical object on which the resulting scheme can be measured. The project produced all three, together with the computational machinery that makes them usable at anatomical resolution. The work was carried out by three laboratories: the Lab-STICC on broadband electromagnetic modelling and numerical methods, the LaTIM on learning-based inverse methods, and the IETR on experimental validation and phantom development.

2. Scientific results

Broadband forward modelling

The project developed an electric-flux volume integral equation (D-VIE) for the head, regularized by a new family of *oblique* quasi-Helmholtz projectors. A single formulation is stable from the quasi-static regime of EEG up to the frequencies relevant to stimulation and exposure assessment, and remains well conditioned across the brain-to-skull conductivity contrast of about two orders of magnitude. The construction rests on three proven properties of the projectors: complementarity, annihilation of the low-frequency-unstable scalar-potential contribution on the solenoidal subspace, and a lower bound on the smallest singular value of the material Gram operator that rules out loss of rank as the contrast grows [1, 2]. The solver was validated against analytical solutions on layered spherical head models [3].

Fast direct solvers by operator filtering

Sensitivity studies and leadfield construction both require solving the same system hundreds to thousands of times. The project introduced a fast direct solver that filters the operator before compressing it: a quasi-Helmholtz Laplacian filter removes the part of the spectrum polluted by discretization error, which makes the compact part of a preconditioned equation compressible into a *single* skeleton, with no hierarchical structure [4, 5]. The system inverse is then obtained in closed form and stored, and applied to each new right-hand side at the cost of the rank rather than of the number of unknowns [6, 7].

Measured behaviour: storage follows an N trend against N^2 for the uncompressed operator, down to 0.6% of the uncompressed size at 8 032 unknowns at a solution error of 2.4×10^{-4}; under mesh refinement the error falls by three orders of magnitude at bounded skeleton rank [8, 9]. Applied to the EEG forward problem itself, on the Calderón-preconditioned symmetric formulation [10], the solver reproduces the standard solution on three-compartment spherical and anatomically realistic head models within the prescribed 5×10^{-3} tolerance, with the skeleton ranks of both diagonal blocks constant at 29 across the whole refinement sweep [11].

Learning-based EEG source imaging

The inverse strand progressed from direct supervised inversion to a bi-level, model-based formulation in which a learned prior and a learned solver are trained jointly around a variational cost that retains the forward operator. Graph neural networks are used for both components, so the inverse stage is invariant to renumbering of the

Figure 1: Left: scalp potential generated by an intracranial current source on an anatomically realistic three-compartment head model, computed with the fast direct solver. Right: realistic head phantom cast from an anatomical mold, with the corresponding mold hemisphere.

source space [12, 13, 14]. In-domain, learned inverters reach a localization error of 2.6 to 5.2 mm against 9.2 mm for sLORETA on the same data. Under change of subject and of electrode montage, direct inversion falls to near-random support overlap and cannot be applied at all when the electrode count changes, whereas the fully graph-based model-based variant retains an AUC of 92.4% with 3.9×10^5 parameters, an order of magnitude below its convolutional counterpart. This robustness under structural change is the property a feedbacked scheme requires. The same line has since been extended to majorization-minimization networks [15, 16].

Simulation and evaluation framework

Supervised inverse imaging has no ground truth on real recordings, so the methods above required a simulation chain built for the purpose. It combines realistic source spaces, forward operators derived from head models, parameterized source supports, and temporal waveform families ranging from signal-oriented generators to neural-mass dynamics, and it varies source extent, the number of simultaneously active regions, and the spatial organization of the sources. Evaluation is multi-criteria rather than reduced to a single score: localization error, support overlap, amplitude at the peak and over the whole time series, and temporal alignment, four of them evaluated at the instant of maximum activity of the ground-truth seed. This framework is what allows the comparison between classical estimators, direct inversion, and model-based methods to be made on equal terms [13, 17].

Dispersion-constrained imaging of tissue parameters

For the material-imaging side, the project constrained the reconstruction of complex conductivity by causality across the frequency band. Reconstructing a set of Debye parameters common to all frequencies, rather than a complex conductivity independently at each frequency, yields a spectrum that is Kramers-Kronig consistent by construction, with fewer unknowns and information shared across the band. On skull complex conductivity the overall reconstruction error falls from 11.37% to 1.75% at 30 dB SNR, and from 17.69% to 14.39% under forward-to-inverse mesh mismatch.

Tissue-equivalent head phantoms

A three-compartment head phantom was designed from the ICBM-NY model, characterized, and fabricated. Conductivities were tuned to the literature reference values and verified by three independent methods: 0.409 S/m for the scalp mimic and 0.469 S/m for gray matter against targets of 0.41 and 0.47 S/m, and 0.023 ± 0.00114 S/m for a conductive-PLA skull printed at 25% infill against a whole-skull target of 0.02 S/m, the latter after correction of probe contact resistance by a linear fit across sample lengths. Material recipes, observed lifetimes, and the five printed components of the assembly are documented, and the phantoms were instrumented with EEG headsets and dense-electrode acquisition.

3. Originality and contribution to the state of the art

- **Oblique quasi-Helmholtz projectors.** Existing quasi-Helmholtz regularization projects orthogonally and stabilizes the low-frequency breakdown alone. Projecting along a direction set by the material operator lets one pair of projectors follow both the frequency-induced and the contrast-induced rotation of the relevant subspaces, so low-frequency and high-contrast breakdowns are treated by a single mechanism.
- **Fast direct solvers without hierarchy.** Classical fast direct solvers rely on hierarchical matrix structures because the kernel singularity makes the spectrum decay too slowly for a single skeleton. Filtering the operator first removes that obstacle, giving a single-skeleton solver and merging the acceleration and preconditioning

steps that are usually treated separately.

- **Bi-level model-based learning for source imaging.** Variational source imaging requires a hand-selected penalty that suits some source configurations and not others. Learning the prior inside a variational cost that keeps the forward operator explicit removes that choice while retaining the physics, and is, to the consortium's knowledge, the first bi-level model-based formulation applied to EEG source imaging.
- **Geometry-aware inverse operators.** Direct-inversion networks implicitly learn a correspondence between source position and output index, so they break when the source space is re-meshed or the montage changes. Graph operators are invariant to that relabelling by construction, which is what allows an inverse stage to survive the structural updates a feedback loop produces.
- **Causality as an imaging constraint.** Reconstructing a causal dispersion model rather than per-frequency values makes the recovered spectrum physically admissible by construction and couples the low- and high-frequency regimes, since a causal parameterization fitted over the measured band determines the response outside it.
- **Refinement-free preconditioning of the EEG forward problem.** A Calderón preconditioner for the symmetric formulation that avoids dual elements and barycentric mesh refinement, yielding a symmetric positive-definite system [10]. The conditioning and preconditioning strategies underlying this family of formulations were surveyed by the consortium in [18].

4. Applied results

- **Physical validation platform.** Spherical and anatomically realistic phantoms with tuned conductivities, a documented fabrication and measurement chain, and EEG-headset and dense-electrode instrumentation, usable for electrophysiological measurement campaigns.
- **Brain-computer interface demonstration.** A three-class imagined-flicker visual-imagery protocol over 20 subjects with subject-specific frequency selection and a compact ten-electrode montage, reaching 60.4% average offline and 51.2% average online accuracy against a 33.3% chance level. It provides the real-data functional task on which head models and inverse operators can be evaluated end to end.
- **Dosimetry-oriented computation.** Linear-in-complexity computational strategies for modelling and dosimetry at terahertz frequencies, aimed at exposure assessment for fifth-generation and later wireless systems [19].
- **Reusable software.** The integral-equation machinery and the supervised-learning source-imaging code are publicly released, so the numerical core and the reported benchmarks are available beyond the consortium.

5. Indicators

Indicator	Value
Doctoral theses defended	1. Sarah Reynaud, *Machine learning for inverse problem resolution to study brain electrophysiological activity*, IMT Atlantique, doctoral school SPIN no. 648, defended 29 September 2025 at Brest (thesis no. 2025IMTA0495, HAL tel-05379638). Jury chaired by R. Fablet, with J. Cologner (CNRS) and T. Papadopoulo (INRIA) as reviewers and F. Andriulli (Politecnico di Torino) and S. Bourguignon (Centrale Nantes) as examiners.
Journal articles	5, in *IEEE Transactions on Antennas and Propagation* (2), the *Journal of Computational Physics*, the *IEEE Open Journal of Antennas and Propagation*, and *Frontiers in Neuroscience*.
International conference papers	11, at EUSIPCO (2), URSI EMTS, URSI GASS, ICEAA (2), IEEE CAMA (3), EuCAP, and AP-S/URSI.
Preprints and articles under review	1 preprint and 1 submitted article, on model-based and majorization-minimization networks for EEG imaging.
National scientific communications	2 oral presentations, at ORASIS 2023 and GDR IASIS 2025.
CominLabs Days	Posters/presentations at all CominLabs Days from 2022 to 2024.
Open-source software releases	2 public repositories: the integral-equation technology at `https://github.com/CERL-PoliTo/open321`, and the supervised-learning source-imaging code at `https://github.com/SarahReynaud/stESI_pub`.

6. International visibility

The project's scientific output was produced in sustained collaboration with two foreign institutions. Politecnico di Torino co-authored the majority of the electromagnetic modelling results, and Universität Rostock contributed to the work on conditioning and preconditioning of integral equations and on refinement-free preconditioning of the EEG forward problem. This collaboration also reached into the doctoral examination, with F. Andriulli (Politecnico di Torino) sitting on the thesis jury.

Results were presented at the principal international venues of the field: EuCAP, ICEAA, IEEE CAMA, AP-S/URSI, the URSI General Assembly and Scientific Symposium, the URSI International Symposium on Electromagnetic Theory, and EUSIPCO, held across Europe, Asia, Africa, and North America. Publication venues include the two reference journals of computational electromagnetics and antennas, together with a journal of computational physics and one of neuroscience, which reflects the position of the project between numerical electromagnetics, inverse problems, and neuroimaging.

The open-source release of the integral-equation technology makes the numerical core available beyond the consortium, and the source-imaging code release allows the reported benchmarks to be reproduced. The inverse-imaging line continues to publish beyond the funded period, with a preprint and a EUSIPCO 2026 paper on majorization-minimization networks for inverse problems and an article under review on model-based and graph-based source imaging.

Project publications

[1] C. Henry, A. Merlini, and F. P. Andriulli, "Oblique quasi-Helmholtz projectors for electric flux volume integral equations: Differential forms and conditioning analysis," in *IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (AP-S/URSI)*, 2022.

[2] C. Henry, A. Merlini, and F. P. Andriulli, "On the solution of the brain-to-skull contrast breakdown with a new integral scheme: Mathematical framework and medical application scenarios," in *XXXIVth General Assembly and Scientific Symposium of the International Union of Radio Science (URSI GASS)*, 2021.

[3] C. Henry, A. Merlini, L. Rahmouni, and F. P. Andriulli, "On a low-frequency and contrast-stabilized full-wave volume integral equation solver for lossy media," *IEEE Transactions on Antennas and Propagation*, vol. 71, no. 3, pp. 2571–2586, 2023.

[4] A. Merlini, C. Henry, D. Consoli, L. Rahmouni, A. Dély, and F. P. Andriulli, "Laplacian filtered loop-star decompositions and quasi-Helmholtz filters: Definitions, analysis, and efficient algorithms," *IEEE Transactions on Antennas and Propagation*, vol. 71, no. 12, pp. 9289–9302, 2023.

[5] C. Henry, D. Consoli, A. Dély, L. Rahmouni, A. Merlini, and F. P. Andriulli, "Fast direct solvers based on the quasi-Helmholtz laplacian filters," in *IEEE Conference on Antenna Measurements and Applications (CAMA)*, 2022.

[6] D. Consoli, C. Henry, A. Dély, L. Rahmouni, J. E. Ortiz Guzmán, T. L. Chhim, S. B. Adrian, A. Merlini, and F. P. Andriulli, "On the fast direct solution of a preconditioned electromagnetic integral equation," in *International Conference on Electromagnetics in Advanced Applications (ICEAA)*, pp. 193–195, 2022.

[7] C. Henry, D. Consoli, A. Dély, L. Rahmouni, A. Merlini, and F. P. Andriulli, "Fast direct solvers for integral equations at low-frequency based on operator filtering," in *17th European Conference on Antennas and Propagation (EuCAP)*, pp. 1–4, 2023.

[8] A. Merlini, C. Henry, D. Consoli, A. Dély, L. Rahmouni, and F. P. Andriulli, "On the fast direct solution of low-frequency integral formulations via operator filtering," in *URSI International Symposium on Electromagnetic Theory (EMTS)*, pp. 237–240, 2023.

[9] C. Henry, A. Merlini, and F. P. Andriulli, "A fast direct solver for volume integral equations based on quasi-Helmholtz laplacian filters," in *IEEE Conference on Antenna Measurements and Applications (CAMA)*, pp. 954–955, 2023.

[10] V. Giunzioni, J. E. Ortiz Guzmán, A. Merlini, S. B. Adrian, and F. P. Andriulli, "On a Calderón preconditioner for the symmetric formulation of the electroencephalography forward problem without barycentric refinements," *Journal of Computational Physics*, vol. 491, p. 112374, 2023.

[11] A. Merlini, V. Giunzioni, C. Henry, S. B. Adrian, and F. P. Andriulli, "On well conditioned integral formulations for fast electrophysiological source imaging of the brain," in *International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 2023.

[12] S. Reynaud, A. Merlini, D. Ben Salem, and F. Rousseau, "EEG source imaging by supervised learning," in *31st European Signal Processing Conference (EUSIPCO)*, pp. 1170–1174, 2023.

[13] S. Reynaud, A. Merlini, D. Ben Salem, and F. Rousseau, "Comprehensive analysis of supervised learning methods for electrical source imaging," *Frontiers in Neuroscience*, vol. 18, p. 1444935, 2024.

[14] S. Reynaud, M. Q. Pham, R. Fablet, A. Merlini, D. Ben Salem, and F. Rousseau, "Model based deep learning and graph neural networks for EEG source imaging." Submitted, 2026.

[15] L. M. T. Tran, S. Reynaud, R. Fablet, A. Merlini, F. Rousseau, and M. Q. Pham, "Majorization-minimization networks for inverse problems: An application to EEG imaging." arXiv preprint arXiv:2602.03855, 2026.

[16] S. Reynaud, R. Fablet, A. Merlini, F. Rousseau, and M. Q. Pham, "Sparse MM-Net for inverse problems: Applied to EEG imaging," in *34th European Signal Processing Conference (EUSIPCO)*, Aug. 2026.

[17] S. Reynaud, A. Merlini, D. Ben Salem, and F. Rousseau, "Imagerie de sources électrophysiologiques par apprentissage supervisé," in *ORASIS 2023, 19e édition du colloque d'ORASIS, journées francophones des jeunes chercheurs en vision par ordinateur*, May 2023.

[18] S. B. Adrian, A. Dély, D. Consoli, A. Merlini, and F. P. Andriulli, "Electromagnetic integral equations: Insights in conditioning and preconditioning," *IEEE Open Journal of Antennas and Propagation*, vol. 2, pp. 1143–1174, 2021.

[19] V. Giunzioni, G. Ciacco, C. Henry, A. Merlini, and F. P. Andriulli, "Linear-in-complexity computational strategies for modeling and dosimetry at TeraHertz," in *IEEE Conference on Antenna Measurements and Applications (CAMA)*, pp. 950–953, 2023.