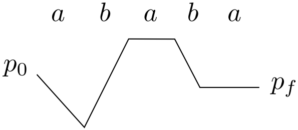
Explorable Automata

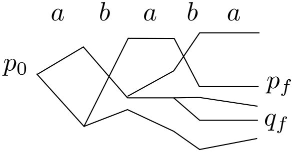
Emile Hazard, Olivier Idir, Denis Kuperberg

GT DAAL, Rennes, April 26th 2024

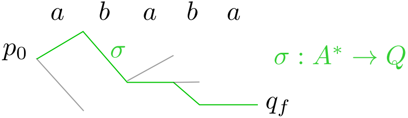
History-Deterministic Automata



Deterministic



Non-deterministic



History-Deterministic
(a.k.a. Good-for-Games)

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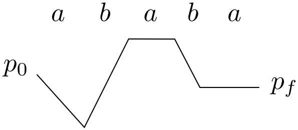

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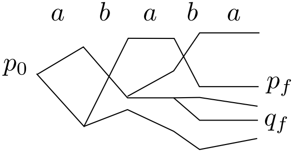

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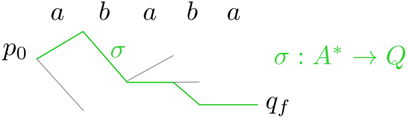

History-Deterministic
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Motivations

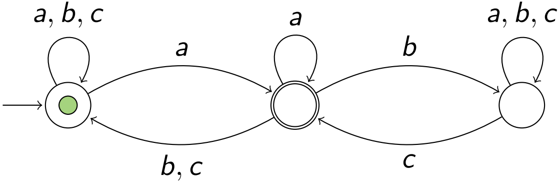
- Solve Church Synthesis more efficiently
- Intermediate model between Det. and Nondet.
- Exponential Succinctness wrt Det. [K., Skrzycpzak '15]

Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters:

Eve: resolves non-deterministic choices for transitions



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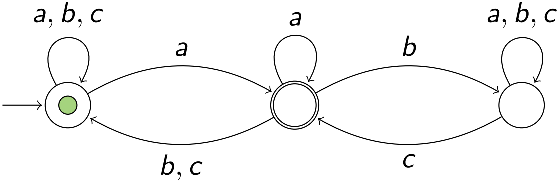

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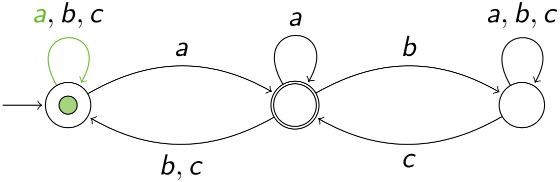

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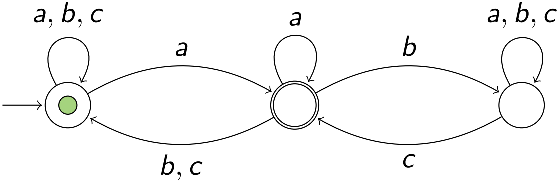

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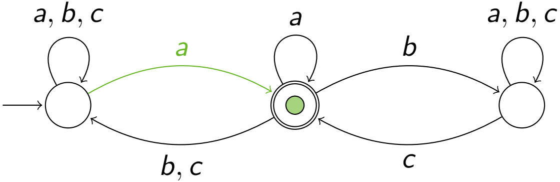

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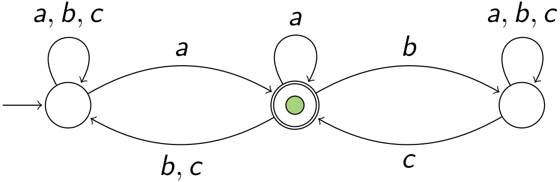

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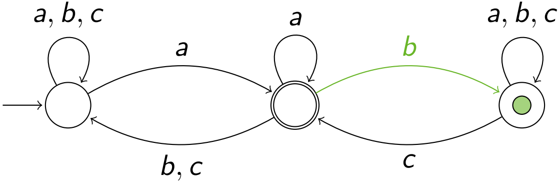

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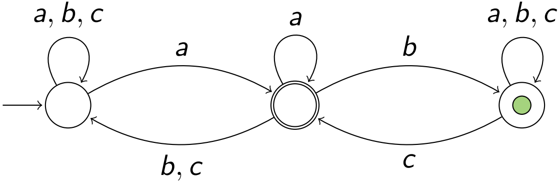

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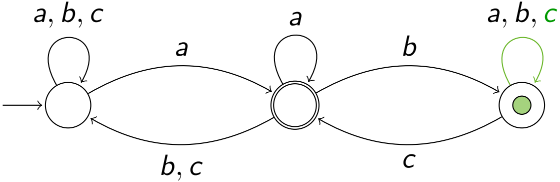

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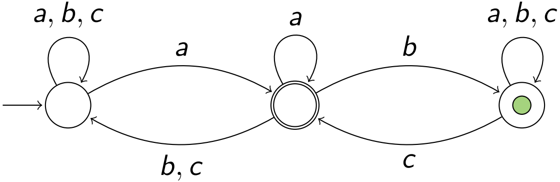

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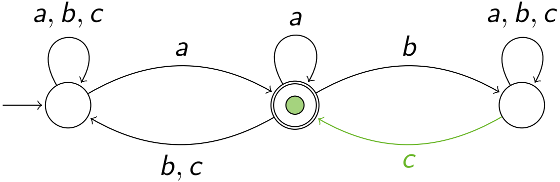

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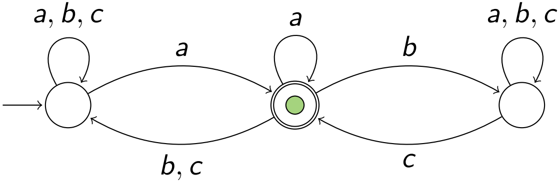

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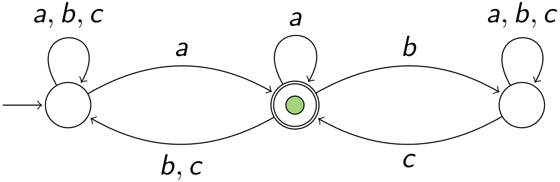

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$\mathcal{A}$ HD $\Leftrightarrow$ Eve wins the Letter game on $\mathcal{A}$

$\Leftrightarrow$ there is a strategy $\sigma_{\mathrm{HD}} : A^* \to Q$ accepting all words of $L(\mathcal{A})$.

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
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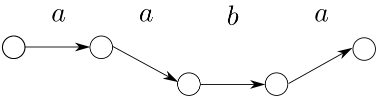
To tackle these questions, we generalize the notion of HD . . .

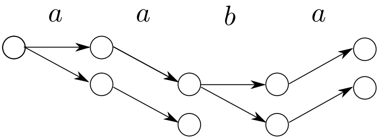
Allowing more runs

Idea: Allow to build several runs, at least one accepting.



1 state - 1 run

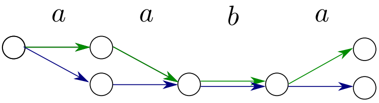
HD



2 states

Width 2

[K. + Majumdar '18 '19]



2 runs

2**-Explorable**

[This work]

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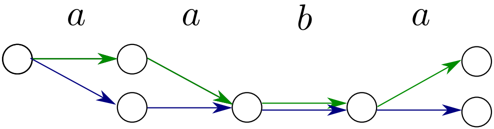
k-**explorability game**:

Adam plays letters, Eve moves k tokens



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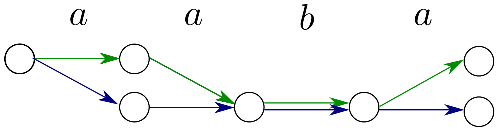

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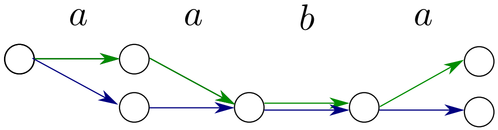

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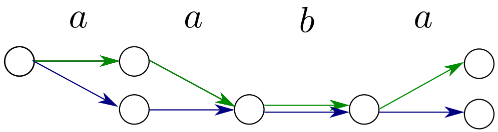

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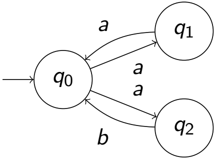

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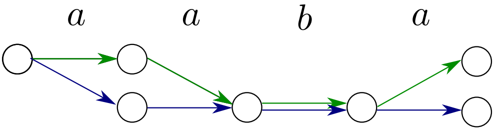

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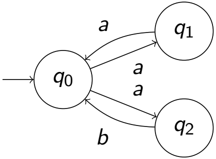

A non-explorable safety NFA

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is EXPTIME-complete.

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How many tokens might be needed in explorable automata ?

A related paper

Similar questions in [Betrand et al 2019: Controlling a population]

k-**population game**: Arena like *k*-explorability game on NFA,
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Our goal: Generalize to Explorability, but

- Game harder to solve: the input word has to be in $L(\mathcal{A})$
- Must deal with acceptance conditions on infinite words.

Results

Theorems [Hazard, K. 2023]

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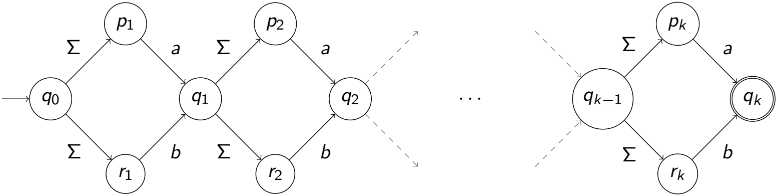

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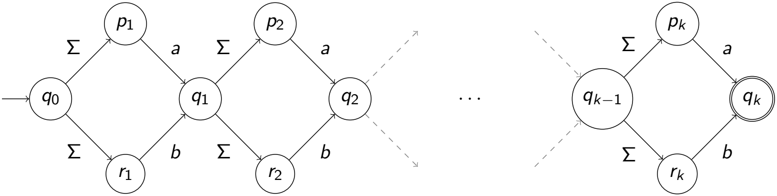

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Theorems [Idir, K.]

Explorability is EXPTIME for coBüchi, $[0, 2]$-Parity.

ω-explorability

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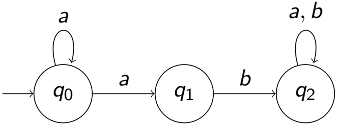

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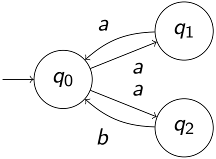

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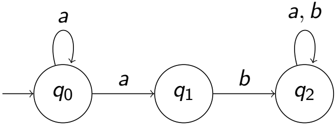

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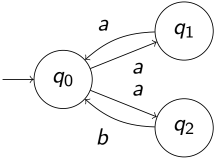

not ω-explorable

Intuition:

Non-ω-explorable: Adam can always kill any run

Results on ω-explorability

Facts:

- any NFA is ω-explorable,
- any automaton $\mathcal{A}$ with $L(\mathcal{A})$ countable is ω-explorable.
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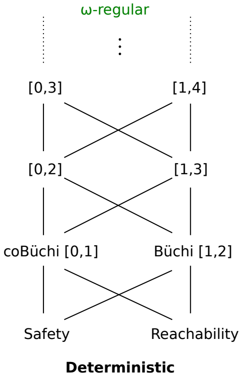
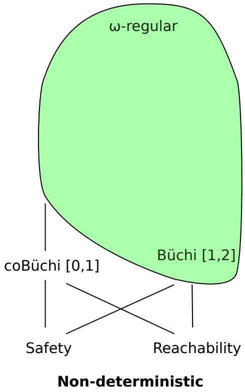
Decidability open for Büchi.

Expressivity of explorable automata





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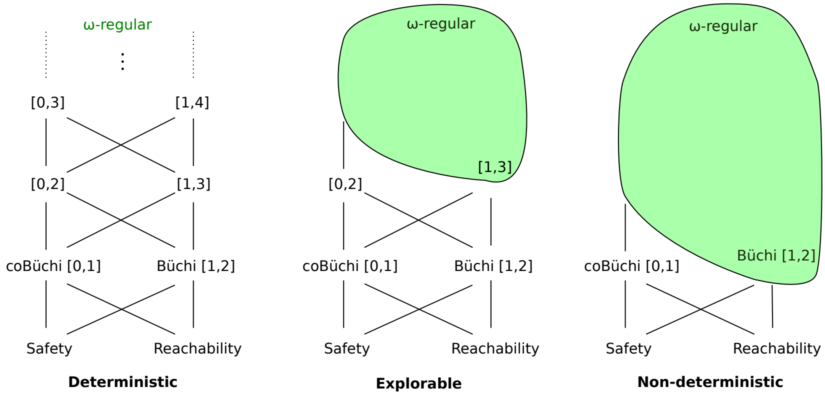

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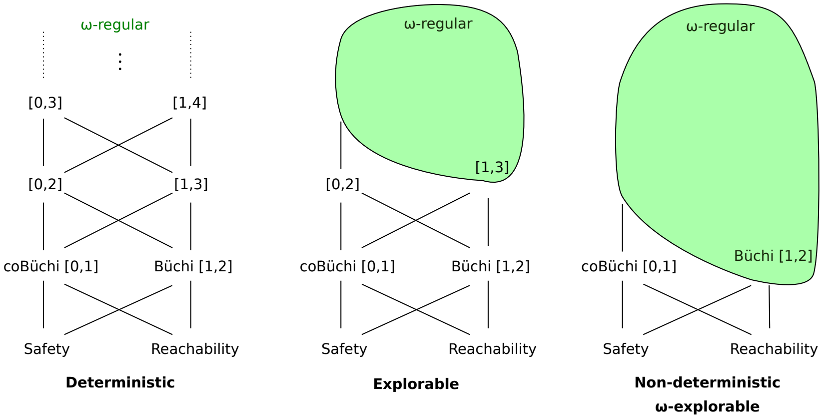

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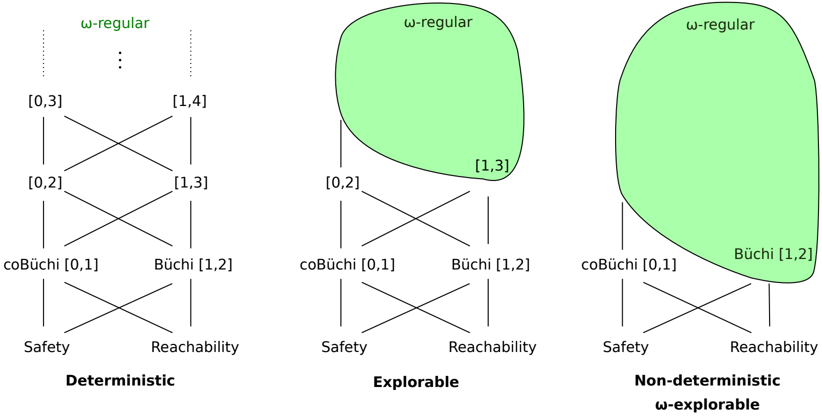

Theorem (Idir, K.)

$[1,3]$-*explorability decidable* $\Leftrightarrow$ *Parity explorability decidable*
Büchi ω-*explorability decidable* $\Leftrightarrow$ *Parity* ω-*explorability decidable*

Future work

- Open decidability: $[1,3]$-expl., Büchi ω-expl.
- Complexity of k-expl. with k in binary?
- Studying HD and expl. models in other frameworks.
- Practical applications, experimental evaluations.
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Thanks for your attention!