



ω -regular Energy Problems

GT DAAL 2024

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Energy Büchi Problem

- Timed automata
- Büchi condition
- weighted over integers
 - ▶ negative weight: **consumption** of energy
 - ▶ positive weight: **collection** of energy
- energy bounded
 - ▶ from below (battery must not become empty)
 - ▶ weakly from above (maximal battery capacity)



Energy Büchi Problem

Does a Büchi accepted **feasible** run exist?

energy always within bound $[0, b]$

weak upper bound b

Remember, Remember, the 15 September

- ... 2008
- Bouyer, F., Larsen, Markey, Srba: *Infinite Runs in Weighted Timed Automata with Energy Constraints*, FORMATS 2008
- Dziadek, Fahrenb., Schlehuber: *Energy Büchi Problems*, FM 2023:
 - ▶ extend to Büchi conditions
 - ▶ fix problems
 - ▶ **implement everything**: TChecker + Spot
- Dziadek, Fahrenb., Schlehuber: *ω -regular Energy Problems*, submitted
 - ▶ extend to Parity condition
 - ▶ fix more problems
 - ▶ add trace extraction
 - ▶ update implementation



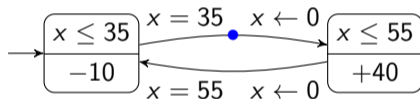
Weighted Timed Büchi Automata



Weighted Timed Büchi Automata

Weighted Timed Büchi Automata

- generalized Büchi acceptance on transitions
- (only) locations are weighted



Note: we only handle **one clock**

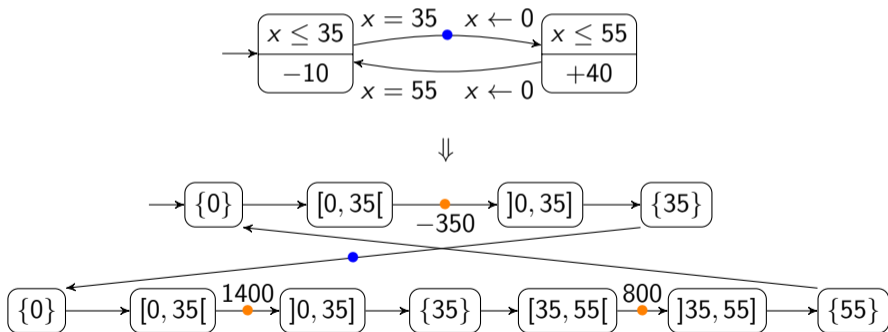
Energy problems **undecidable** for **four** clocks (Bouyer, Larsen, Markey 2014)

open for **two** or **three** clocks

Corner-Point Abstraction

One-clock timed automaton \rightarrow untimed automaton

- TChecker computes the zone graph
- compute corner-point abstraction (Behrmann, Fehnker, Hune et al. 2001)
- Zeno-exclusion



Weighted Büchi Automata



Weighted Büchi Automata

Weights

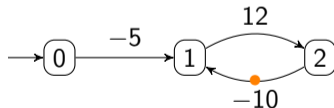
Given values: c : initial credit
 b : weak upper bound

Weights: $e_0 = \min(b, c)$
 $e_{i+1} = \min(b, e_i + w_i)$ for transition weight w_i

Feasible Run

Always: $e_i \geq 0$

Example



Feasible with $c \geq 5$ and $b \geq 10$.

Bellman-Ford (BF)

Recall: BF finds **shortest** paths \Rightarrow Invert to find **maximal** energy

BF relaxes a distance approximation until solution is found

BF asserts that no “negative loops” exist \Leftrightarrow here, positive cycles are desired

BF not aware of Büchi acceptance

Our solution:

- Decompose **strongly connected components**
- Treat accepting **back-edges** one-by-one
- **modify BF** for “energy positive” loops

Our Algorithm

Take a weighted Büchi automaton:

- find strongly connected components (SCC) (we use Couvreur)
- degeneralize SCCs (produces Büchi accepting **back edges**)
- with modified Bellman-Ford search for feasible lassos:
 - ▶ on original graph for maximal **prefix** energy
 - ▶ in SCCs for **non-negative cycles** including a **Back-edge**

Note: Energy and Büchi condition cannot be fully separated

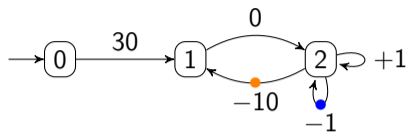


Figure: Original WBA

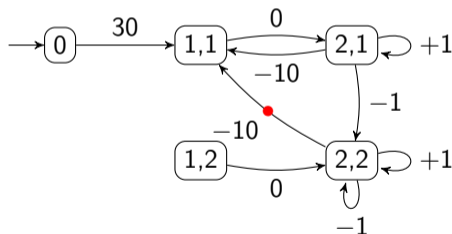
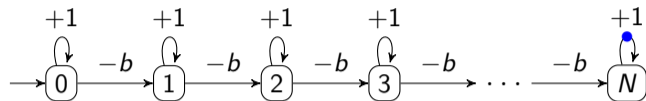


Figure: Degeneralizing SCC {1, 2} with level 1 rooted in

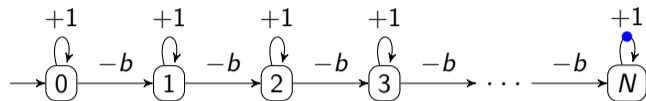
Modified BF: Challenges

Example



Modified BF: Challenges

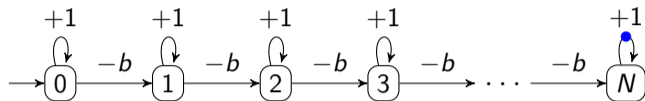
Example



Get weak upper bound b out of complexity

Modified BF: Challenges

Example

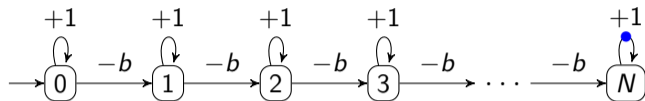


Get weak upper bound b out of complexity

\Rightarrow After each iteration, positive loops are *pumped*

Modified BF: Challenges

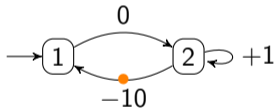
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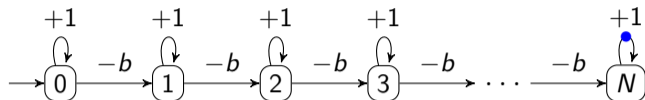
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Example (for $c = 30, b = 30$)



Modified BF: Challenges

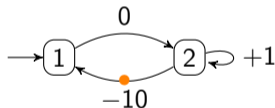
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Get weak upper bound b out of complexity

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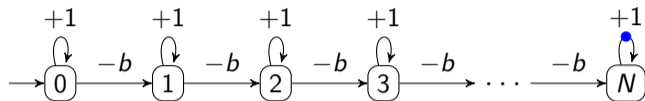
Example (for $c = 30, b = 30$)



Max energy: 30

Modified BF: Challenges

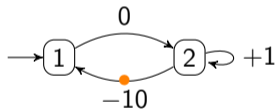
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Get weak upper bound b out of complexity

\Rightarrow After each iteration, positive loops are *pumped*

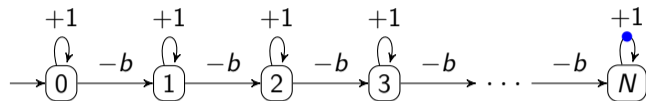
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Max energy: 30 30

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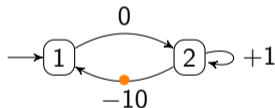
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Get weak upper bound b out of complexity

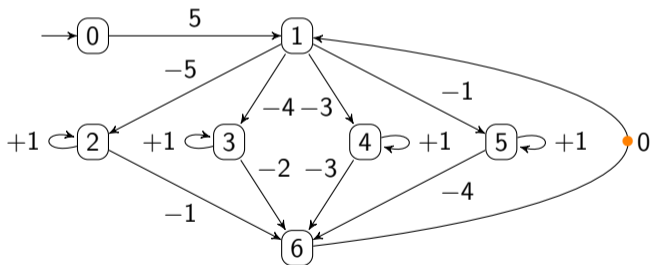
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Example (for $c = 30, b = 30$)



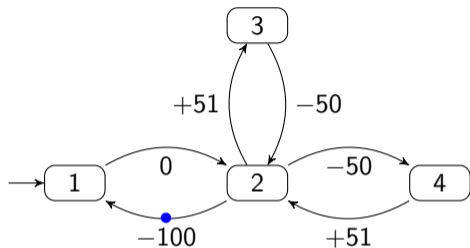
Max energy:

30
20
30

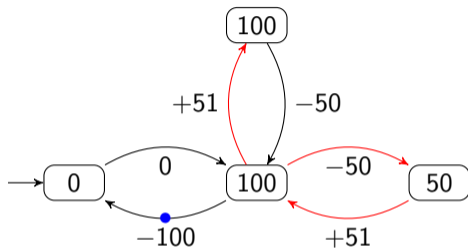


WBA where two iterations do not suffice

Challenges trace extraction

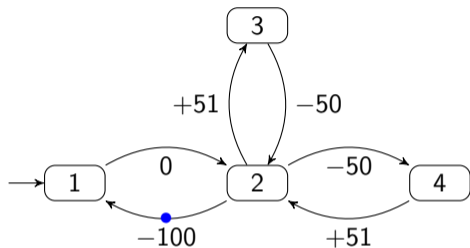


Nontrivial example of trace extraction
(for $b = 100$)



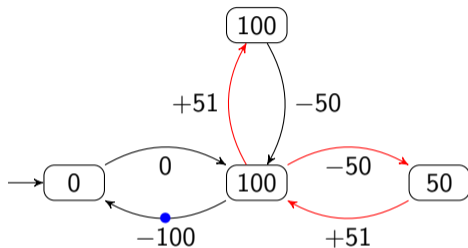
Results obtained from the Algorithm.

Challenges trace extraction



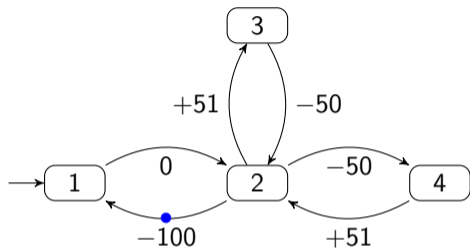
Nontrivial example of trace extraction
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Important information is lost during the iterations.



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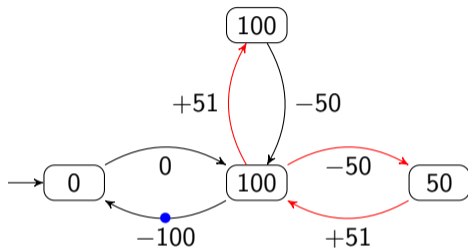
Challenges trace extraction



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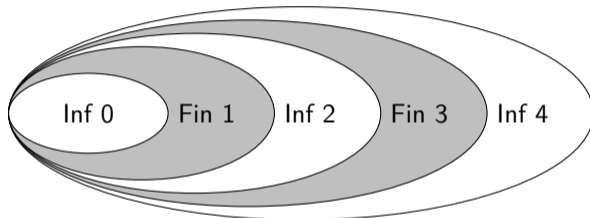
Important information is lost during the iterations.

⇒ Storing all predecessors and launch an adapted backwards-forward search.



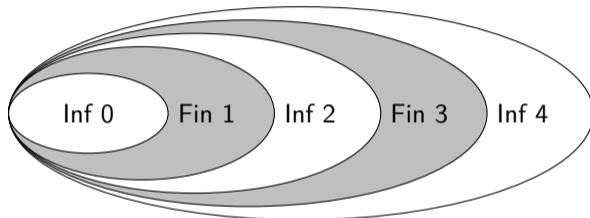
Results obtained from the Algorithm.

Max-Even 5



Parity condition for priorities up to 4:
 $\text{Inf}(4) \mid (\text{Fin}(3) \& (\text{Inf}(2) \mid (\text{Fin}(1) \& \text{Inf}(0))))$

Max-Even 5



Parity condition for priorities up to 4:

$\text{Inf}(4) \mid (\text{Fin}(3) \ \& \ (\text{Inf}(2) \mid (\text{Fin}(1) \ \& \ \text{Inf}(0))))$

⇒ Reduce to Büchi case from most to least important color

Conclusion



Results on Energy Büchi problems



1. *Weighted ω -regular automata*
 - Modified Bellman-Ford with Couvreur's algorithm
2. *One-clock weighted timed ω -regular automata*
 - Reduce to **1.** using corner-point abstraction
3. *Solved the trace extraction problem*

All algorithms are implemented using TChecker and Spot

Future Work

- edge weights
 - ▶ Bouyer, F., Larsen, Markey: *Timed automata with observers under energy constraints*, HSCC 2010
- Avoid iteration over all maximal states.
- **parametric problem**: synthesize b and/or c
 - ▶ F., Juhl, Larsen, Srba: *Energy Games in Multiweighted Automata*, ICTAC 2011
 - ▶ (in some cases that's easier!)
- **implement everything!**