Rosario y la teoría de control óptimo
Aplicaciones a la gestión de recursos naturales

Mabel Tidball

INRA: Institut National de Recherche en Agronomie, UMR LAMETA, Montpellier.

50 aniversario Licenciatura en matématicas, 2017

Who I am

Senior Researcher at INRA (National Institute of Research in Agronomy), Department of Economics, Montpellier, France.

- Mathematician:
  Licenciada en Matématicas, UNR, 1984: (# 56).
  Thesis UNR, 1991: "Numerical resolution of the Hamilton-Jacobi-Bellman equations". Supervisor: Roberto González. (#2).
- Economist:
  Habilitation à diriger des recherches, University of Montpellier (2005): "Environmental economics and strategic interactions".

Once upon a time: Robert González and optimal control theory

Emile Borel $\Longrightarrow$ Georges Valiron $\Longrightarrow$ Laurent Schwartz $\Longrightarrow$ Jacques Louis Lions $\Longrightarrow$ Alain Bensoussan $\Longrightarrow$

Roberto González Université Paris IX Dauphine. 1980

$\Downarrow$

Tidball, Aragone, Di Marco, Reyero, Mancinelli, Lotito, Katz

Once upon a time:
Robert González and optimal control theory

Mabel and UNR

Auxiliar... jefe de trabajos practicos (1985- 2000),
Investigadora CONICET (1995 - 2000).

1996-1997 et Dirección trabajos finales de la licenciatura: Ricardo Katz; M. Escalante, V. Leoni, H. Ponce de León;
1997 : Estadía M. Escalante INRIA Sophia Antipolis.
1998 : Estadía A. Lombardi. INRIA Sophia Antipolis.
2002 : Santiago Muro, INRA Montpellier.
2005 y 2009. Silvia Di Marco INRA Montpellier
2015 (?) María Evangelina Alvarez. INRA Montpellier.

Proyecto SticAmSud: DyGaMe, Dynamic Games Methods : theory, algorithms and applications. 2015-2017. Chile, Argentina, France.

Before

UMA ... ENIEF

Before

UMA ... ENIEF

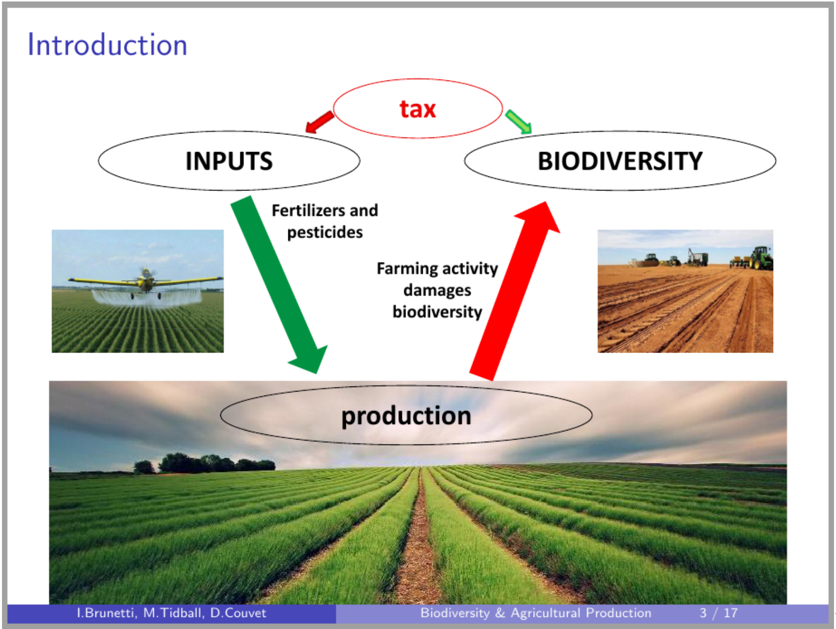
After

ISDG... EAERE

Some problems in environmental economics

- The relationship between biodiversity and agricultural production (with Ilaria Brunetti, post doc école Polytechnique, Denis Couvet, Muséum National d'Histoire Naturelle, Paris)
- Growth, environmental degradation and the creation of a new sector (with Pierre-André Jouvet, Univ. Nanterre, Julien Wolfersberger, AgroParisTech).
- Habitat and Fishery (with Georges Zaccour, HEC, Montréal, Ngo Van Long, Univ. McGill, Montréal).
- Cooperation with asymmetric environmental valuation and responsibility in a dynamic setting (with Francisco Cabo, Univ. de Valladolid, Spain).
- The impact of the pro-environmental behavior on the decision process in coordination and cooperation issues: theoretical modeling and experiments. Thesis...

Biodiversity and Agricultural Production



A farmed land is equally divided among N farmers. We denote by

- $B_i^t \in [0,1]$ local **biodiversity** on farmer's i land at time $t = 0, 1, \ldots$;
- $\tilde{B}_i^t \in [0,1]$, **after migration biodiversity**:

$$\tilde{B}_i^t := \sum_{j=1}^{N} m_{ij} B_j^t, \qquad \sum_{j=1}^{N} m_{ij} = 1,$$

- where the coefficient m_{ii} represents the **migration effect** within the same land, while m_{ij} from land j to i, $j \neq i$.

Biodiversity and Yield

We suppose that agricultural production depends on **inputs**, **labor** and on after migration **biodiversity**. We define yield through a

Cobb-Douglas function:

$$Y_i^t := \beta_i v_i(\tilde{B}_i^t)(\ell_i^t)^{\nu_i}(A_i^t)^{\eta_i}, \qquad \eta_i + \nu_i < 1$$

where A_i^t and ℓ_i^t are the production factors, with decreasing return to scale, $\beta_i > 0 \forall i$. Function v represents the **productivity**, which depends on $\tilde{B}_i^t$:

$$v_i(x) := x^{\gamma_i},$$

which leads to:

$$Y_i^t := \beta_i(\tilde{B}_i^t)^{\gamma_i}(\ell_i^t)^{\nu_i}(A_i^t)^{\eta_i}$$

Biodiversity Dynamics

Biodiversity evolves on time according to a **continuous-time dynamic equation**:

$$\frac{dB_i^t}{dt} = R\tilde{B}_i^t\left(1 - \frac{\tilde{B}_i^t}{M_i^t(A_i, Y_i)}\right)$$

where $R > 0$ is the **intrinsic growth rate** of biodiversity and M_i^t represents the **carrying capacity** of farmer's i land to host biodiversity at time t:

$$M_i^t(A_i, Y_i) := a_i - b_i(\alpha A_i^t + Y_i^t), \qquad \alpha \geq 0.$$

The evolution of **biodiversity** is thus **damaged** by **inputs and production**: the higher are A_i^t and Y_i^t, the lower is the growth rate of B_i^t.

Farmers Profit Function

Farmer *i* **instantaneous profit** is given by:

$$\pi_i^t = pY_i^t - w_A A_i^t - w_\ell \ell_i^t - \tau A_i^t, \qquad w_A > 0, w_\ell > 0$$

where:

- p is the **fixed price**;
- τ is a **fixed tax** on inputs;
- w_A and w_ℓ represent the **cost** per unit of inputs and labor respectively.

Farmers are supposed to be **myopic**, i.e. farmer i maximizes π_i^t considering $\tilde{B}_i^t$ as fixed and given.

Biodiversity Dynamics

We look for the **biodiversity dynamics** when farmers adopt their **optimal strategies** (system of differential equations).

$$\frac{dB_i^t}{dt} = R\tilde{B}_i\left(1 - \frac{\tilde{B}_i}{a_i - b_i(\alpha A_i^* + Y_i^*)}\right) = R\tilde{B}_i\left(1 - \frac{\tilde{B}_i}{a_i - b_iK_i(\tilde{B}_i)^\theta}\right)$$

Growth, environmental degradation and the creation of a new sector

- Neoclassical growth model with capital accumulation, and we use an approach related to Stokey's paper to generate pollution.
- Actual and potential output are related by the techniques of production chosen by the planner. Dirtier techniques provide a higher level of output but also more pollution.
- Define long-term unlimited growth by the existence of a balanced growth path where capital accumulation and consumption grow at a positive rate.

The model

$$\max_{\{C,L,z\in[0,1]\}} \int_0^{\infty} U(C)e^{-\rho t}dt,$$

subject to

$$\begin{aligned} \dot{K} &= zA(P,K)F(K,BL) - wBL - C, \\ \dot{P} &= g(E) - \delta P, \\ \dot{B} &= hB, \\ E &= \psi(z)A(P,K)F(K,BL). \end{aligned}$$

$A(P,K) = A,\quad A(P,K) = A(P/K)$, implies optimal growth

$A(P,K) = A(P)$, implies SS

The model with the new sector

$$\max_{\{C,K^X,L,L^X,z\in[0,1]\}} \int_0^\infty U(C)e^{-\rho t}dt, \quad (1)$$

subject to

$$\begin{aligned} \dot{K} &= zA(P)F(K-K^X, B(L-L^X)) - wBL - C, \\ \dot{P} &= g(E) - \delta P - X(K^X, BL^X), \\ \dot{B} &= hB, \\ E &= \psi(z)A(P)F(K-K^X, B(L-L^X)). \end{aligned} \quad (2)$$

optimal growth

Habitat and Fishery

We consider a fishery described at any instant of time $t \in [0, \infty)$ by the available stock of fish $x(t)$ and an index measuring its (marine) environmental quality $(M(t))$.
Denote by $e(t)$ the fishing effort at time t, and let the harvest be given by $h(t) = e(t)x(t)$. We assume that the state variables evolve according to the following differential equations:

$$\dot{x}(t) = (a + bM(t))x(t)\left(1 - \frac{x(t)}{f + gM(t)}\right) - h(t), \quad x(0) = x_0,$$

$$\dot{M}(t) = M(t)(1 - M(t)) - \beta h(t), \quad M(0) = M_0.$$

Agents myopic behavior

A representative fisherperson behaves myopically when maximizing her profit. Denote by p the exogenous price of fish and by $C(e) = \frac{c}{2}e^2$ the convex increasing fishing effort cost. The fisherperson's maximization problem is given by

$$\max_e \pi(e) = (p - \tau(x))\, ex - \frac{c}{2}e^2,$$

where $\tau(x)$ is the tax collected by the government (or regulator), satisfying $\tau'(x) \leq 0$. In the sequel, we shall consider the following specification for the tax function:

$$\tau(x) = \tau_0 + \frac{\tau_1}{x},$$

where $\tau_i, i = 1, 2$ are non negative constants.

Agent's Optimal Harvest

$$e = \max\{0, \alpha x - \gamma\}, \quad \alpha = (p - \tau_0)/c, \quad \gamma = \frac{\tau_1}{c}.$$

The agent's optimal profit is

$$\pi^{op}(x) = \begin{cases} \frac{c}{2}[x\alpha - \gamma]^2 & \text{if } x > \tilde{x}, \\ 0 & \text{otherwise.} \end{cases}$$

The profit is decreasing in the tax.
Inserting the harvesting rule into the dynamics, we obtain

$$\dot{x} = (a + bM)\, x \left(1 - \frac{x}{f + gM}\right) - \max\left\{0, \alpha x^2 - \gamma x\right\}, \quad x(0) = x_0,$$

$$\dot{M} = M(1 - M) - \beta \max\left\{0, \alpha x^2 - \gamma x\right\}, \quad M(0) = M_0.$$

The role of a regulator

Suppose the government must choose a tax τ to maximize steady state "welfare", where welfare is here defined as profit (i.e. producer's surplus) plus tax revenue. The idea is to find an optimal tax τ, and the resulting steady state x^∞, M^∞, and steady-state welfare, W^∞.

$$\max_{\alpha,\gamma}\left[ph^\infty(\alpha,\gamma) - \left(\frac{c}{2}\right)\left(\frac{h^\infty(\alpha,\gamma)}{x^\infty(\alpha,\gamma)}\right)^2\right]$$

subject to

$$M^\infty(\alpha,\gamma)(1 - M^\infty(\alpha,\gamma)) - \beta h^\infty(\alpha,\gamma) = 0$$

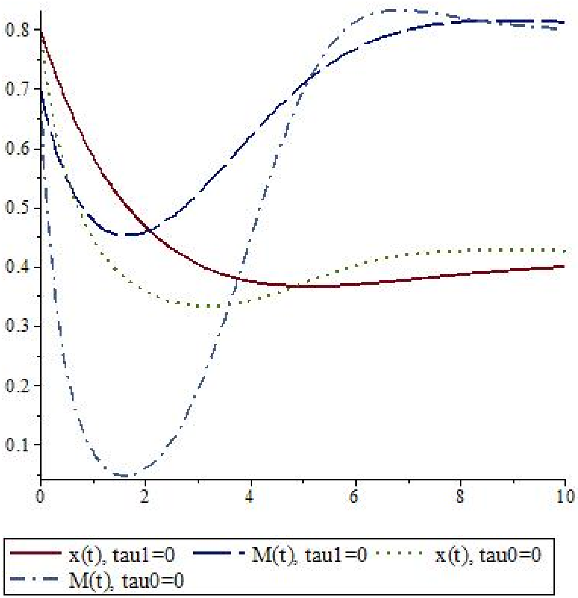
and

$$bM^\infty(\alpha,\gamma)x^\infty(\alpha,\gamma) - \frac{b}{g}\left(x^\infty(\alpha,\gamma)\right)^2 - h^\infty(\alpha,\gamma) = 0.$$

with the constraints

$$\alpha \le p, \quad \alpha g Z_2^* \ge \gamma, \quad \gamma \ge 0.$$

Evolution of dynamics



MCV... La mas vieja?

Silvia

Silvia

Silvia

Il y a

Il y a des détails invisibles aux yeux
Des rêves impossibles à réaliser
Des sensations difficiles à retrouver.

Il y a des accords brisés
De chansons oubliées
Des couleurs compliquées à composer.

Il y a des mystères insondables
Des chemins interminables
Des douleurs à surmonter.

Et il y a des vides
Qui seront vides à jamais.