Incomplete Knowledge and Collective Decision Making

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Plan

Social Choice

Designing and analyzing methods for collective decision making

- *Voting*
  - single winner: elect a president; find a date for a meeting
  - multi-winner: choose a set of lecturers for EKAW 2020
  - multi-issue: decide where to hold EKAW 2020 and who should be the program chair
- *Fair Division* (divisible or indivisible goods)
  - allocate classes to teach or time slots in a high school
  - allocate items among pirats after a successful attack
  - divorce settlement: how to divide the bank account, who will have the chidren's custody, who keeps the stereo and who keeps the cat.
- *Matching*
  - assign students to universities
- *Group Formation*
  - find a partition of a set of employees into work teams
- *Belief/Opinion/Judgment Aggregation*
  - jury agreeing on a verdict

A very rough history of social choice

1. from ancient Greece to ∼1800 (Condorcet's and Borda's talks at EKAW 1789, Versailles)
2. 1951: birth of modern social choice
   - results are mainly *axiomatic* (economics/mathematics)
   - impossibility theorems: *incompatibility of a small set of seemingly innocuous conditions*, such as in Arrow's theorem (1951):

     With at least 3 alternatives, an aggregation function satisfies *unanimity* and *independence of irrelevant alternatives* if and only if it is a *dictatorship*.

   - computational issues are neglected
3. early 90's: computer scientists come into play

   Computational social choice Using computational notions and techniques (mainly from Artificial Intelligence, Operations Research, Theoretical Computer Science) for solving complex collective decision making problems.

Plan

Voting

- $X = \{x_1, \ldots, x_m\}$ set of candidates

$$X = \{a, b, c, d\}$$

- $N = \{1, \ldots, n\}$ set of voters

$$N = \{1, 2, .., 9\}$$

- each voter reports a ranking $\succ_i$ over candidates;
- voting profile: $P = \langle \succ_1, \ldots, \succ_n \rangle$

	voters 1, 2, 3, 4 :	$c \succ b \succ d \succ a$
P:	voters 5, 6, 7, 8 :	$a \succ b \succ d \succ c$
	voter 9 :	$c \succ a \succ b \succ d$

- **plurality rule**: the winner is the candidate ranked first by the largest number of voters

$$plurality(P) = c$$

Voting

- $X = \{x_1, \ldots, x_m\}$ set of candidates

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P:	voters 5, 6, 7, 8 :	$a \succ b \succ d \succ c$
	voter 9 :	$c \succ a \succ b \succ d$

- **Borda rule**: a candidate ranked 1st / 2nd / 3rd / last in a vote gets 3 / 2 / 1 / 0 points. The candidate with maximum total number of points wins.

$$a \mapsto (4 \times 3) + 2 = 14 \quad b \mapsto 17 \quad c \mapsto 15 \quad d \mapsto 8$$

$$Borda(P) = b$$

Voting

- $X = \{x_1, \ldots, x_m\}$ set of candidates

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P:	voters 1, 2, 3, 4 :	$c \succ b \succ d \succ a$
	voters 5, 6, 7, 8 :	$a \succ b \succ d \succ c$
	voter 9 :	$c \succ a \succ b \succ d$

- **many other rules!**

Voting

- What should we do in case of ties?

P':	voters 1, 2, 3, 4 :	$c \succ b \succ d \succ a$
	voters 5, 6, 7, 8 :	$a \succ b \succ d \succ c$
	voter 9 :	abstains

- *Who is the plurality winner?*
- *Irresolute plurality*:

$$Plurality(P') = \{a, c\} \text{ (cowinners)}$$

- *Resolute plurality*: compose irresolute plurality with a tie-breaking mechanism T (priority relation over candidates). For instance: age.

$$Plurality_T(P') = a \text{ if } a \text{ is older than } c$$

Voting with incomplete preferences

Or more precisely incomplete knowledge of the voters' preferences

voters 1, 2, 3, 4 :	$c \succ b \succ d \succ a$
voters 5, 6, 7, 8 :	$a \succ b \succ d \succ c$
voter 9 (late) :	$? \succ ? \succ ? \succ ?$

- if the rule is Borda:

$$a : 12 + ? \quad b : 16 + ? \quad c : 12 + ? \quad d : 8 + ?$$

- if the rule is plurality:

$$a : 4 + ? \quad b : 0 + ? \quad c : 4 + ? \quad d : 0 + ?$$

Possible and necessary winners

More generally:

- for each voter: P_i is a partial order on the set of candidates.
- $P = \langle P_1, \ldots, P_n \rangle$ incomplete profile
- completion of P: voting profile

  $$T = \langle T_1, \ldots, T_n \rangle$$

  where each T_i is a linear order extending P_i.
- r (resolute) voting rule

- c is a possible winner if there exists a completion of P for which c is elected.
- c is a necessary winner if c is elected in every completion of P.

  Konczak & L (05); Walsh (07); Xia & Conitzer (08) ...

Possible and necessary winners

	voter 1	voter 2	voter 3
incomplete preferences	$a \to c$, $a \to b$	$b \to a$	$c \to a \to b$
completions	$a \to b \to c$ $a \to c \to b$	$b \to a \to c$ $b \to c \to a$ $c \to b \to a$	$c \to a \to b$

6 profile completions:

$$\langle abc, bac, cab \rangle \quad \langle abc, bca, cab \rangle \quad \langle abc, cba, cab \rangle$$
$$\langle acb, bac, cab \rangle \quad \langle acb, bca, cab \rangle \quad \langle acb, cba, cab \rangle$$

Possible and necessary winners

- c *possible winner* if there exists a completion of P in which c is elected.
- c *necessary winner* if c is elected in every completion of P.

$a \succ b, a \succ c$	$b \succ a$	$c \succ a \succ b$	plurality with tie-breaking $b > a > c$	Borda idem
abc	cba	cab	c	c
abc	bca	cab	$b(ac)$	$b(ac)$
abc	bac	cab	$b(ac)$	a
acb	cba	cab	c	c
acb	bca	cab	$b(ac)$	c
acb	bac	cab	c	a

- possible plurality winners: $\{b, c\}$.
- Possible Borda winners: $\{a, b, c\}$
- no necessary winner (both for Borda and plurality)

Possible and necessary winners

- c *possible winner* if there exists a completion of P in which c is elected.
- c *necessary winner* if c is elected in every completion of P.

	we learn		plurality with	Borda
$a \succ b, a \succ c$	$b \succ a \succ c$	$c \succ a \succ b$	tie-breaking $b > a > c$	idem
abc	cba	cab	c	c
abc	bca	cab	b(ac)	b(ac)
abc	bac	cab	b(ac)	a
acb	cba	cab	c	c
acb	bca	cab	b(ac)	c
acb	bac	cab	c	a

- possible plurality winners: $\{b, c\}$.
- necessary Borda winner: a

Possible and necessary winners

In which contexts do we get such incomplete preferences?

1. Missing votes
2. Missing candidates
3. Incomplete lists
4. Truncated ballots

Possible and necessary winners

In which contexts do we get such incomplete preferences?

Missing votes $n - k$ voters have reported a full ranking; the other k have not reported anything.

voter 1	...	voter n-k	voter n-k+1	...	voter n
● ↓ ⋮ ↓ ●	...	● ↓ ⋮ ↓ ●	$\emptyset$	...	$\emptyset$

Strategic interpretation:

- x is a possible winner if the last k voters have a way of casting their votes such that x wins: constructive manipulation for x.

Possible winners and manipulation

- Borda rule
- a single voter hasn't voted yet
  - 4 voters:

$$\begin{array}{c} a \succ b \succ d \succ c \succ e \\ b \succ a \succ e \succ d \succ c \\ c \succ e \succ a \succ b \succ d \\ d \succ c \succ b \succ a \succ e \end{array}$$

  - Current Borda scores

$$a \mapsto 10 \quad b \mapsto 10 \quad c \mapsto 8 \quad d \mapsto 7 \quad e \mapsto 5$$

Can the last voter find a vote so that the winner is ... a?

Possible winners and manipulation

- Borda rule
- a single voter hasn't voted yet
  - 4 voters:

$$\begin{array}{c} a \succ b \succ d \succ c \succ e \\ b \succ a \succ e \succ d \succ c \\ c \succ e \succ a \succ b \succ d \\ d \succ c \succ b \succ a \succ e \end{array}$$

  - Current Borda scores

$$a \mapsto 10 \quad b \mapsto 10 \quad c \mapsto 8 \quad d \mapsto 7 \quad e \mapsto 5$$

Can the last voter find a vote so that the winner is ... a?

- $a \succ \dots$
- yes

Possible winners and manipulation

- Borda rule
- a single voter hasn't voted yet
  - 4 voters:

$$\begin{array}{c} a \succ b \succ d \succ c \succ e \\ b \succ a \succ e \succ d \succ c \\ c \succ e \succ a \succ b \succ d \\ d \succ c \succ b \succ a \succ e \end{array}$$

  - Current Borda scores

$$a \mapsto 10 \quad b \mapsto 10 \quad c \mapsto 8 \quad d \mapsto 7 \quad e \mapsto 5$$

Can the last voter find a vote so that the winner is ... c?

Possible winners and manipulation

- Borda rule
- a single voter hasn't voted yet
  - 4 voters:

$$\begin{array}{c} a \succ b \succ d \succ c \succ e \\ b \succ a \succ e \succ d \succ c \\ c \succ e \succ a \succ b \succ d \\ d \succ c \succ b \succ a \succ e \end{array}$$

  - Current Borda scores

$$a \mapsto 10 \quad b \mapsto 10 \quad c \mapsto 8 \quad d \mapsto 7 \quad e \mapsto 5$$

Can the last voter find a vote so that the winner is ... c?

- $c \succ e \succ d \succ b \succ a$
- scores: $c \mapsto 12$, $a \mapsto 10$, $b \mapsto 11$, $d \mapsto 9$, $e \mapsto 8$
- yes

Possible winners and manipulation

- Two voters haven't voted yet

- Borda rule
- Tie-breaking priority $a > b > c > d > e > f$.
- Current Borda scores:

$$a \mapsto 12 \quad b \mapsto 10 \quad c \mapsto 9 \quad d \mapsto 9 \quad e \mapsto 4 \quad f \mapsto 1$$

- Do the last two voters have a constructive manipulation for e?
- Homework!
- A simple greedy algorithm like before does not work.

Possible winners and manipulation

Existence of a manipulation for the Borda rule:

- *for a single voter* : in P
  - Bartholdi, Tovey & Trick, 1989
- for a *coalition of at least two voters* : NP-complete
  - Betzler, Niedermeyer & Woeginger, 2011
  - Davies, Katsirelos, Narodytska & Walsh, 2011

- Lots of results of this kind
- Complexity provides a computational barrier to manipulation

Possible winners: new candidates

In which contexts do we get such incomplete preferences?

New candidates The voters have expressed their votes on a set of candidates, and then some new candidates come in.

- Doodle: agents vote on a first set of dates, and then new dates become possible
- Recruiting committee: a preliminary vote is done before the last applicants are interviewed

voter 1	voter 2	...	voter n
c	b		b
$\downarrow$	$\downarrow$		$\downarrow$
a	c	...	a
$\downarrow$	$\downarrow$		$\downarrow$
b	a		c
$(d, e?)$	$(d, e?)$	...	$(d, e?)$

Possible winners: new candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win?*

- 12 voters; initial candidates : $X = \{a, b, c\}$; one new candidate y.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners?

a	5
b	4
c	3
y	

initial scores (before y is taken into account)

Possible winners: new candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win?*

- 12 voters; initial candidates : $X = \{a, b, c\}$; one new candidate y.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners?

a	5	$\rightarrow$ **5**
b	4	$\rightarrow 4$
c	3	$\rightarrow 3$
y		$\rightarrow 0$

nobody votes for y

Possible winners: new candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win?*

- 12 voters; initial candidates : $X = \{a, b, c\}$; one new candidate y.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners?

a	5	$\rightarrow$ 3	
b	4	$\rightarrow$ **4**	2 who voted for a
c	3	$\rightarrow$ 3	now vote for y
y		$\rightarrow$ 2	

Possible winners: new candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win?*

- 12 voters; initial candidates : $X = \{a, b, c\}$; one new candidate y.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners?

a	5	$\rightarrow 2$	3 who voted for a
b	4	$\rightarrow 2$	and 2 who voted for b
c	3	$\rightarrow 3$	now vote for y, who wins!
y		$\rightarrow \mathbf{5}$	c cannot win

Possible winners: new candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win?*

- 12 voters; initial candidates : $X = \{a, b, c\}$; two new candidates y_1, y_2
- plurality with tie-breaking priority $a > b > c > y_1 > y_2$
- Who are the possible winners?

a	5	2	
b	4	2	
c	3	3	c wins
y		3	
y'		2	

- characterization and computation of possible winners for many voting rules

  Chevaleyre, L, Maudet and Monnot (2010); Xia, L and Monnot (2011); Chevaleyre, L, Maudet, Monnot and Xia (2012)

Possible winners: truncated ballots

In which contexts do we get such incomplete preferences?

Incomplete lists The voters rank only the candidates they know (the films they have seen, the candidates they have interviewed etc.)

voter 1	voter 2	...	voter n
$c \rightarrow a \rightarrow b$	$d \rightarrow a$	...	$c \rightarrow e \rightarrow d \rightarrow f$

Possible winners: truncated ballots

In which contexts do we get such incomplete preferences?

Truncated ballots The voters are asked to rank only their top k candidates (to limit the amount of communication)

voter 1	voter 2	...	voter n
$\bullet$	$\bullet$		$\bullet$
$\downarrow$	$\downarrow$		$\downarrow$
$\bullet$	$\bullet$	...	$\bullet$
$\downarrow$	$\downarrow$		$\downarrow$
?	?		?

Possible winners: truncated ballots

- plurality: $k = 1$ is enough for the true winner to be determined!
- Borda, tie-breaking priority $a > b > c > d > e$

voter 1	voter 2	voter 3
a	b	c
$\downarrow$	$\downarrow$	$\downarrow$
?	?	?

- possible winners: all
- no necessary winner

Possible winners: truncated ballots

- plurality: $k = 1$ is enough for the true winner to be determined!
- Borda, tie-breaking priority $a \rhd b \rhd c \rhd d \rhd e$

voter 1	voter 2	voter 3
a	b	c
$\downarrow$	$\downarrow$	$\downarrow$
d	d	d
$\downarrow$	$\downarrow$	$\downarrow$
?	?	?

- possible winners: a, b, c, d
- no necessary winner

Possible winners: truncated ballots

- plurality: $k = 1$ is enough for the true winner to be determined!
- Borda, tie-breaking priority $a \rhd b \rhd c \rhd d \rhd e$

voter 1	voter 2	voter 3
a	b	c
$\downarrow$	$\downarrow$	$\downarrow$
d	d	d
$\downarrow$	$\downarrow$	$\downarrow$
e	a	b
$\downarrow$	$\downarrow$	$\downarrow$
?	?	?

- d necessary winner
- stop!

Kalech, Kraus, Kaminka and Goldman (2011); Baumeister, Faliszewski, L and Rothe (2012); and more papers.

Incomplete knowledge of preferences

- possible and necessary winners: epistemic notions
- every ordinal notion in social choice or game theory can be 'modalized' this way.
- another example: *fair division*...

Fair Division of Indivisible Items

- $N = \{1, \ldots, n\}$ set of agents
- $O = \{o_1, \ldots, o_m\}$ indivisible items
- allocation: maps each item to an agent

  $\pi = (\pi_1 | \ldots | \pi_n)$ where π_i is the share of agent i;

  $\pi = [o_1 o_2 | o_3 | o_4 o_5]$: 1 receives $\{o_1 o_2\}$, 2 receives $\{o_3\}$, 3 receives $\{o_4, o_5\}$.
- $\succeq_i$ preference relation of agent i over 2^O

Envy-freeness π is envy-free if for all i, j, $\pi_i \succeq_i \pi_j$ (i does not envy j)

Pareto efficiency π is Pareto-efficient if there is no π' such that

- $\pi'_i \succeq_i \pi_i$ for all i
- $\pi'_i \succ_i \pi_i$ for some i

Fair Division

$$\succ_{Amedeo}: \quad abc \succ ab \succ ac \succ a \succ bc \succ b \succ c$$
$$\succ_{Catherine}: \quad abc \succ ab \succ ac \succ bc \succ c \succ a \succ b$$

- $[a|bc]$ both envy-free and (Pareto-)efficient

Fair Division

A slightly different example:

$$\begin{array}{ll} \succ_{Amedeo}: & abc \succ ab \succ ac \succ b \succ a \succ bc \succ c \\ \succ_{Catherine}: & abc \succ ab \succ ac \succ bc \succ a \succ b \succ c \end{array}$$

- $[a|bc]$ envy-free, but not efficient
- $[b|ac]$ efficient, but not envy-free because Amedeo envies Catherine

Fair Division

- m items $\rightarrow$ each agent must rank 2^m subsets of items

The combinatorial trap...

Twenty binary variables...

$o_8 o_5 \succ o_5 o_3 o_9 \succ o_8 \succ \varnothing \succ o_5 \succ o_8 o_5 o_3 o_9 \succ o_8 o_3 \succ o_5 o_9 \succ o_3 o_9 \succ$
$o_8 o_9 \succ o_8 o_3 o_9 \succ o_5 o_3 \succ o_9 \succ o_3 \succ o_8 o_5 o_9 \succ o_8 o_5 o_3 o_1 o_2 o_5 o_8 o_9 \succ$
$o_1 o_5 o_6 \succ o_7 \succ o_2 o_3 o_4 o_5 o_6 o_7 o_8 \succ o_1 o_2 o_3 o_4 o_5 \succ o_1 o_3 \succ o_2 \succ$
$o_1 o_3 o_7 o_9 \succ o_1 o_5 \succ o_1 o_7 o_8 o_9 \succ o_2 \succ o_4 \succ o_6 \succ o_1 o_7 \succ o_1 o_2 o_3 \succ$
$o_1 o_2 \succ o_2 o_5 o_4 \succ o_1 \succ o_2 \succ o_1 o_2 o_5 o_4 \succ o_1 o_5 \succ o_2 o_4 \succ o_5 o_4 \succ$
$o_1 o_4 \succ o_1 o_5 o_4 \succ o_2 o_5 \succ o_4 \succ o_5 \succ o_1 o_2 o_4 \succ o_1 o_2 o_5 \succ o_1 o_5 \succ$
$o_5 o_3 o_9 \succ o_1 \succ \varnothing \succ o_5 \succ o_1 o_5 o_3 o_9 \succ o_1 o_3 \succ o_5 o_9 \succ o_3 o_9 \succ o_1 o_9 \succ$
$o_1 o_3 o_9 \succ o_5 o_3 \succ o_9 \succ o_3 \succ o_1 o_5 o_9 \succ o_1 o_5 o_3 o_9 o_6 o_5 o_1 o_9 \succ o_9 o_5 o_6 \succ$
$o_7 \succ o_6 o_3 o_4 o_5 o_6 o_7 o_1 \succ o_9 o_6 o_3 o_4 o_5 \succ o_9 o_3 \succ o_6 \succ o_9 o_3 o_7 o_9 \succ$
$o_9 o_5 \succ o_9 o_7 o_1 o_9 \succ o_6 \succ o_4 \succ o_6 \succ o_9 o_7 \succ o_9 o_6 o_3 \succ o_9 o_6 \succ$
$o_6 o_5 o_4 \succ o_9 \succ o_6 \succ o_9 o_6 o_5 o_4 \succ o_9 o_5 \succ o_6 o_4 \succ o_5 o_4 \succ o_9 o_4 \succ$

$\rightarrow$ 1048575 subsets $\rightarrow$ the expression takes more than 12 days.

Fair Division and Incomplete Preferences

- m items $\rightarrow$ each agent must rank 2^m subsets of items
- a solution: agents rank *single items*: $\rhd_i$ on O
- preference relation (transitive + irreflexive) $\succ_i$ on 2^O extending $\rhd_i$
- $\succ_i$ is the smallest preference relation such that

monotonicity for all $X \subseteq O$ and $o \notin X$, $X \cup \{o\} \succ X$

I'm happier if one more item is added to my bundle

responsiveness for all $X \subseteq O$ and $o, o' \notin X$,
if $o \rhd o'$ then $X \cup \{o\} \rhd X \cup \{o'\}$

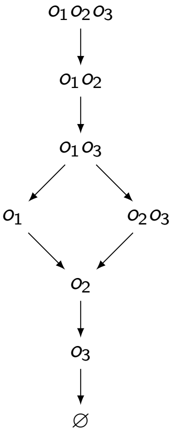
I'm happier if one item of my bundle is changed into an item I prefer.

Fair Division and Incomplete Preferences

$m = 2,\ o_1 \rhd o_2$

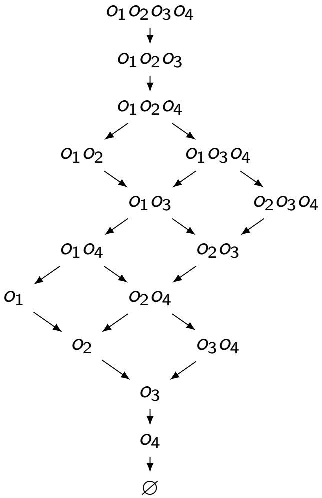
Ranking single items

$m = 3,\ o_1 \rhd o_2 \rhd o_3$



Ranking single items

$m = 4,\ o_1 \rhd o_2 \rhd o_3 \rhd o_4$



Ranking single items

- $P = (\rhd_1, \ldots, \rhd_n)$
  $\mapsto R = (\succ_1, \ldots, \succ_n)$ collection of partial orders on 2^O
- π *possibly envy-free*
  if it is envy-free for some complete extension of R
- π *necessarily envy-free*
  if it is envy-free for all complete extensions of R
- *possible Pareto efficiency*, *necessary Pareto efficiency*: defined similarly.

Ranking single items

Simple characterization:

- π is necessarily envy-free if for all agents i, j, and all $k \leq |\pi_i|$, i prefers his kth best item in π_i to the kth best item in π_j
- π is possibly envy-free if for all agents i, j, either $|\pi_i| > |\pi_j|$ or for some $k \leq |\pi_i|$, i prefers his kth best item in π_i to the kth best item in π_j.

$$\begin{array}{ll} \rhd_{Amedeo}: & \mathbf{a} \rhd b \rhd c \rhd \mathbf{d} \rhd e \rhd f \\ \rhd_{Catherine}: & \mathbf{b} \rhd a \rhd c \rhd \mathbf{e} \rhd f \rhd d \\ \rhd_{Yannick}: & \mathbf{c} \rhd d \rhd \mathbf{f} \rhd b \rhd e \rhd d \end{array}$$

necessarily efficient, necessarily envy-free

Ranking single items

Simple characterization:

- π is necessarily envy-free if for all agents i, j, and all $k \leq |\pi_i|$, i prefers his kth best item in π_i to the kth best item in π_j
- π is possibly envy-free if for all agents i, j, either $|\pi_i| > |\pi_j|$ or for some $k \leq |\pi_i|$, i prefers his kth best item in π_i to the kth best item in π_j.

$$\rhd_{Amedeo}: \quad \mathbf{a} \rhd b \rhd \mathbf{c} \rhd \mathbf{d} \rhd e \rhd f$$
$$\rhd_{Catherine}: \quad \mathbf{b} \rhd a \rhd c \rhd \mathbf{e} \rhd \mathbf{f} \rhd d$$

possibly (but not necessarily) envy-free, necessarily efficient

Characterizations and computation of possibly/necessarily envy-free and possibly/necessarily efficient allocations: Bouveret, Endriss and L (2010); Aziz, Gaspers, Mackenzie & Walsh (2015).

Plan

Expressing Preferences

- Domains of solutions in social choice often have a *combinatorial* structure
$$A = D_1 \times \ldots \times D_p$$
where D_i = finite set of values associated with a variable X_i.
- Such problems are (generally) computationally hard.
- How can we represent decision making problems in a more compact, more modular, more intuitive way?
- How can we solve these complex decision making problems?

A first example: prioritized goals

- $G = \langle G_1, \ldots, G_q \rangle$
- G_i set of goals φ_i^j of priority i – each being a propositional formula
- G_1= set of highest priority goals, then G_2 etc.
- leximin ordering: $x \succ_{leximin} y$ if there is a $k \leq q$ such that
  - for each $i < k$, x and y satisfy the same number of goals in G_i
  - x satisfies more goals in G_k than y.

Multiple Referenda

Nancéiens called to urns:

- should we build a new university campus or not? (c or $\neg c$)
- should we build a new tram or not? (t or $\neg t$)
- should we build a zoo or not? (z or $\neg z$)

- Amedeo's prioritized goals:

$$G_1 = \{\neg(c \wedge t \wedge z)\}, G_2 = \{c\}, G_3 = \{t\}$$

- Amedeo's induced preference relation:

$$\begin{array}{c} ct\overline{z} \\ \downarrow \\ c\overline{t}z \sim c\overline{t}\overline{z} \\ \downarrow \\ \overline{c}tz \sim \overline{c}t\overline{z} \\ \downarrow \\ \overline{c}\overline{t}z \sim \overline{c}\overline{t}\overline{z} \\ \downarrow \\ ctz \end{array}$$

Multiple Referenda

- Amedeo: $G_1 = \{\neg(c \wedge t \wedge z)\}$, $G_2 = \{c\}$, $G_3 = \{t\}$

  $ct\bar{z} \succ \ldots$

- Yannick: $G_1 = \{\neg(c \wedge t \wedge z)\}$, $G_2 = \{t\}$, $G_3 = \{z\}$

  $\bar{c}tz \succ \ldots$

- Catherine: $G_1 = \{\neg(c \wedge t \wedge z)\}$, $G_2 = \{z\}$, $G_3 = \{c\}$

  $c\bar{t}z \succ \ldots$

If we vote separately on each issue, the following outcome may occur:

- Catherine and Amedeo vote for c, Yannick against;
- Amedeo and Yannick vote for t, Catherine against;
- Catherine and Yannick vote for z, Amedeo against
- Outcome: ctz – is it good?

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If we vote separately on each issue, the following outcome may occur:

- Catherine and Amedeo vote for c, Yannick against;
- Amedeo and Yannick vote for t, Catherine against;
- Catherine and Yannick vote for z, Amedeo against
- Outcome: ctz – is it good?

Need for more sophisticated methods!

Preference logics

von Wright (1963):

- formulas built up from preference statements $\alpha \rhd \beta$
- $\alpha \wedge \neg\beta$-worlds preferred to $\beta \wedge \neg\alpha$-worlds, *ceteris paribus*
- here *ceteris paribus* means that all variables not appearing in α or β must be interpreted identically
- $\mathit{tram} \rhd \mathit{zoo}$:
  - implies $(\mathit{tram}, \neg \mathit{zoo}, \mathit{campus}) \succ (\neg \mathit{tram}, \mathit{zoo}, \mathit{campus})$
  - implies $(\mathit{tram}, \neg \mathit{zoo}, \neg \mathit{campus}) \succ (\neg \mathit{tram}, \mathit{zoo}, \neg \mathit{campus})$
  - $(\mathit{tram}, \mathit{zoo}, \mathit{campus}) \succ (\mathit{tram}, \neg \mathit{zoo}, \neg \mathit{campus})$ incomparable
- $\mathit{campus} \wedge \mathit{tram} \rhd \mathit{campus} \wedge \neg \mathit{tram}$,
  [shorthand $\mathit{campus} : \mathit{tram} \rhd \neg \mathit{tram}$]
  $\neg \mathit{campus} \wedge \neg \mathit{tram} \rhd \neg \mathit{campus} \wedge \mathit{tram}$
  - $(\mathit{campus}, \mathit{tram}, \neg \mathit{zoo}) \succ (\mathit{campus}, \neg \mathit{tram}, \neg \mathit{zoo})$
  - $(\neg \mathit{campus}, \neg \mathit{tram}, \neg \mathit{zoo}) \succ (\neg \mathit{campus}, \mathit{tram}, \neg \mathit{zoo})$
  - etc.

Preference logics

- 'Modern' preference logics: Hansson (2001), van Benthem, Roy and Girard (2009), Bienvenu, L and Wilson (2010).
- formulas are Boolean combinations of preference statements of the form

$$\alpha \rhd \beta \mid\mid F$$

  α, β propositional formulas, F a set of propositional formulas
- α preferred to β when F is held constant; other formulas can vary
- formally: $\succ$ satisfies $(\alpha \rhd \beta \mid\mid F)$ if $\omega \succ \omega'$ holds for all ω, ω' such that
  - $\omega \models \alpha$
  - $\omega' \models \beta$
  - forall $\varphi \in F$: $\omega \models \varphi$ if and only if $\omega' \models \varphi$.
- $\mathit{campus} \rhd \neg\mathit{campus} \mid\mid \varnothing$:
  - $(\mathit{campus}, \neg\mathit{tram}, \neg\mathit{zoo}) \succ (\neg\mathit{campus}, \mathit{tram}, \mathit{zoo})$
- $\mathit{zoo} \rhd \neg\mathit{zoo} \mid\mid \{\mathit{campus}\}$
  - $(\mathit{campus}, \mathit{tram}, \mathit{zoo}) \succ (\mathit{campus}, \neg\mathit{tram}, \neg\mathit{zoo})$

Preference Logics and Multiple Referenda

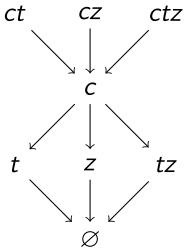
Catherine's preferences:

- better a campus than not, and this preference overrides everything else

$$c \rhd \neg c \mid\mid \varnothing$$

- better a tram or a zoo than neither of them

$$t \vee z \rhd \neg t \wedge \neg z \mid\mid \{c\}$$



Preference Logics and Multiple Referenda

Catherine's preferences:

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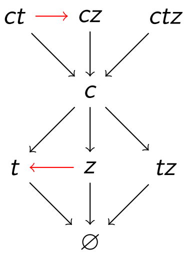
$$c \rhd \neg c \mid\mid \varnothing$$

- better a tram or a zoo than neither of them

$$t \vee z \rhd \neg t \wedge \neg z \mid\mid \{c\}$$

- if campus than better a tram than a zoo, otherwise better a zoo

$$c : t \rhd z \mid\mid \{c\} \quad \neg c : z \rhd t \mid\mid \{c\}$$



Preference Logics and Committee Elections

- two seats to fill for the department managing committee
- candidates: A, B, C, D, E

	woman	man
group 1	A,E	B
group 2	C	D

- preferences of voter 1:
  - 1M+1W $\rhd$ 2M $\sim$ 2W $\|$ $\emptyset$
    where: 1M+1W $=$
    $(A \wedge B \wedge \neg C \wedge \neg D \wedge \neg E) \vee (E \wedge B \wedge \neg A \wedge \neg C \wedge \neg D) \vee (\ldots)$
    gender equilibrium more important than everything else
  - 1G1+1G2 $\rhd$ 2G2 $\rhd$ 2G1 $\|$ {1M+1W, 2M, 2W}
    group equilibrium most important thing after gender equilibrium
  - $A \rhd B \rhd C \rhd D \rhd E \,\|\,$ {1M+1W, 2M, 2W, 1G1+1G2,2G1,2G2}
    (*ceteris paribus*)

Committee Elections

	woman	man
group 1	A,E	B
group 2	C	D

- $1M{+}1W \rhd 2M \sim 2W \;||\; \emptyset$
- $1G1{+}1G2 \rhd 2G2 \rhd 2G1 \;||\; \{1M{+}1W,\ 2M,\ 2W\}$
- $A \rhd B \rhd C \rhd D \rhd E \;||\; \{1M{+}1W,\ 2M,\ 2W,\ 1G1{+}1G2, 2G1, 2G2\}$

Induced preference relation for voter 1:

AD ↘

DE → CD → AB → BE → AC → BD → CE → AE

BC ↗

Voter 1's preferred committee is *AD* or *BC* – we don't have enough information to know which one.

Committee Elections

- Voter 1's preferred committee: AD or BC
- Voter 2's preferred committee: AE or BE
- Voter 3's preferred committee: BD

Standard rule for multiwinner approval voting:

- each voter votes for her preferred committee
- k number of winners (here $k = 2$)
- the k candidates that appear most often on the votes are elected
- tie-breaking priority = age: $D > E > A > B > C$

	$1 : AD$	$1 : BC$
$2 : AE$	$2_A 1_B 0_C 2_D 1_E \mapsto AD$	$1_A 2_B 1_C 1_D 1_E \mapsto BD$
$2 : BE$	$1_A 2_B 0_C 2_D 1_E \mapsto BD$	$0_1 3_B 1_C 1_D 1_E \mapsto BD$

- D is a necessary winner
- A and B (and of course D) are possible winners

Preference Revision in Voting

Nanceians called to urns again.

- should we build a new tram or not?
- should we build a zoo or not?

Amedeo's preferences:

- before all, better one facility than none, and better none than two
- I prefer the tram to the zoo
- preference order $\succ_P$:

$$t\bar{z} \succ_P \bar{t}z \succ_P \bar{t}\bar{z} \succ_P tz$$

However:

- I believe that Nanceians will vote against the zoo (they already have a small one in the *Parc de la Pépinière*)
- I have no idea about the outcome for the tram
- normality order $\succ_N$:

$$\bar{t}\bar{z} \sim_N t\bar{z} \succ_N \bar{t}z \sim_N tz$$

◇ normal situation: majority against the zoo

◇ exceptional situation: majority for the zoo

Preference Revision and Voting

preference order		normality order		
$t\bar{z}$	most preferred			
$\downarrow$				
$\bar{t}z$		$\bar{t}\bar{z}$	$t\bar{z}$	normal
$\downarrow$		$\downarrow$		
$\bar{t}\bar{z}$		$\bar{t}z$	tz	exceptional
$\downarrow$				
tz	least preferred			

Preference Revision and Voting

preference order		normality order		
$t\bar{z}$	most preferred			
$\downarrow$				
$\bar{t}z$		$\bar{t}\bar{z}$	$t\bar{z}$	normal
$\downarrow$			$\downarrow$	
$\bar{t}\bar{z}$		$\bar{t}z$	tz	exceptional
$\downarrow$				
tz	least preferred			

most normal t-world $\quad t\bar{z}$

$\downarrow_P \quad (\top :) t \succ_P \neg t$

most normal $\neg t$-world $\quad \bar{t}\bar{z}$

$\varphi : t \succ_P \neg t$ if typical $\varphi \wedge t$-worlds preferred to typical $\varphi \wedge \neg t$-worlds

L and van der Torre (2008)

Preference Revision and Voting

preference order		normality order		
$t\bar{z}$	most preferred			
$\downarrow$				
$\bar{t}z$		$\bar{t}\bar{z}$	$t\bar{z}$	normal
$\downarrow$			$\downarrow$	
$\bar{t}\bar{z}$		$\bar{t}z$	tz	exceptional
$\downarrow$				
tz	least preferred			

most normal $z \wedge t$-world $\quad tz$

$\uparrow_P \quad z : \neg t \succ_P t$

most normal $z \wedge \neg t$-world $\quad \bar{t}z$

$\varphi : t \succ_P \neg t$ if typical $\varphi \wedge t$-worlds preferred to typical $\varphi \wedge \neg t$-worlds

L and van der Torre (2008)

Preference Revision and Voting

- should we build a new tram? t or $\overline{t}$
- should we build a zoo? z or $\overline{z}$

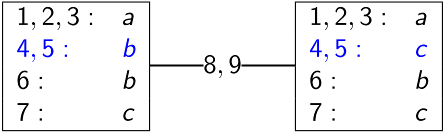
Amedeo's preferences:

- $t \succ \overline{t}$
- $z \succ \overline{z}$
- but $z : \neg t \succ t$
- Amedeo believes that z is very unlikely.
- Therefore he intends to vote for *yes* for z and *yes* for t
- Now, *L'Est Républicain* publishes a poll: it's likely that z will get a slight majority of *yes*!
- Amedeo now votes *yes* for z and *no* for t

Plan

Manipulation under incomplete knowledge

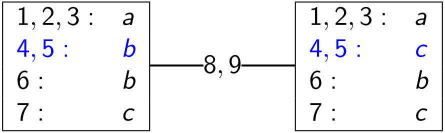
- plurality with tie-breaking priority $a \rhd b \rhd c \rhd d$



- preferences of 8 and 9: $d \succ c \succ b \succ a$
- before 8 and 9 vote:
  - a: 3 points
  - either b: 1 point and c: 3 points, or b: 3 points and c: 1 point
  - d: 0 point
  - how should and 8 and 9 vote?

Manipulation under incomplete knowledge

- plurality with tie-breaking priority $a \rhd b \rhd c \rhd d$



- preferences of 8 and 9: $d \succ c \succ b \succ a$
- before 8 and 9 vote:
  - a: 3 points
  - either b: 1 point and c: 3 points, or b: 3 points and c: 1 point
  - d: 0 point
  - how should and 8 and 9 vote? one for b, one for c
  - a: 3, b: 2 or 4, c: 2 or 4, d: 0
  - winner: b or c
- more generally: manipulation under complex mutual knowledge
- Chopra, Pacuit and Parikh, 04; van Ditmarsch, L and Saffidine, 13; Meir, Lev and Rosenschein, 14; Meir, 15.

Plan

Judgment aggregation

- Instructions from PC chair of the *International Conference on Everything* (ICE-2017, Antarctica):

  accept a paper if and only if it is original and technically valid

- Accept $\leftrightarrow$ Original $\wedge$ Valid

	Original?	Valid?	Accept?
Reviewer 1	Yes	Yes	Yes
Reviewer 2	Yes	No	No
Reviewer 3	No	Yes	No
majority	Yes	Yes	No

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Reviewer 3	No	Yes	No
majority	Yes	Yes	No

- (Metareview). Your paper was judged to be original and technically valid. However, we decided to reject it.
- Judgment aggregation: aggregate opinions about logically interrelated issues...

Judgment aggregation

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majority	Yes	Yes	No

- (Metareview). Your paper was judged to be original and technically valid. However, we decided to reject it.
- Judgment aggregation: aggregate opinions about logically interrelated issues... in a logically consistent way.

Judgment aggregation

- Generalizes preference aggregation

	$a \succ b$?	$b \succ c$?	$a \succ c$?
Reviewer 1	Yes	Yes	Yes
Reviewer 2	Yes	No	No
Reviewer 3	No	Yes	No
majority	Yes	Yes	No

- Resulting judgment set violates transitivity

$$(a \succ b) \wedge (b \succ c) \rightarrow (a \succ c)$$

Judgment aggregation

- Aggregation of equivalence relations: decide how to cluster a, b and c

	$a \sim b$?	$b \sim c$?	$a \sim c$?
Reviewer 1	Yes	Yes	Yes
Reviewer 2	Yes	No	No
Reviewer 3	No	Yes	No
majority	Yes	Yes	No

- Resulting judgment set violates transitivity

$$(a \sim b) \wedge (b \sim c) \rightarrow (a \sim c)$$

- And many more applications (merging ontologies, crowdsourcing etc.)
- see U. Endriss, Judgment Aggregation, in *Handbook of Computational Social Choice* (Cambridge University Press, 2016).

Plan

Finale

Advertising *Handbook of Computational Social Choice*
(Cambridge University Press, 2016, *downloadable for free*)