New Algorithms for Static Analysis via Dyck Reachability

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4th Inria/EPFL Workshop

February 15, 2018

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE


REMOTE HOME CONTROL
ONLINE HOME AUTOMATION
ECO MODE
22°C


MEDICAL

- Does it crash?
- Is it efficient?
- Does it leak information?
- Is it safe?
- Is it responsive?

Static Analysis

- Program analysis is hard
- All non-trivial problems undecidable
- Relax ...

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  - . . . your ambitions
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Static analysis

- Lightweight, "clever" scans of the program
- Detect "obvious" bugs

1. Fast
2. On demand
3. Dynamic

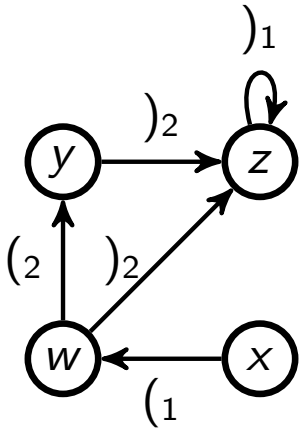
Static Analysis via Dyck Reachability

Dyck Reachability

$$\Sigma = \{(_1,)_1, \ldots (_k,)_k\} \cup \{\epsilon\}$$

$$G = (V, E, \lambda : E \to \Sigma)$$

$$\mathcal{S} \to \mathcal{S}\ \mathcal{S} \mid (_1\ \mathcal{S}\ )_1 \mid \ldots \mid (_k\ \mathcal{S}\ )_k \mid \epsilon$$



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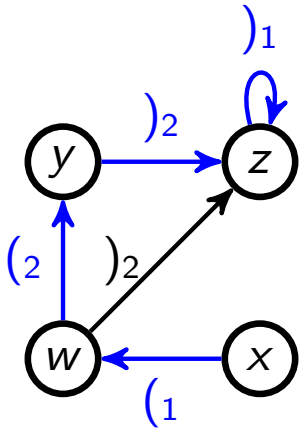

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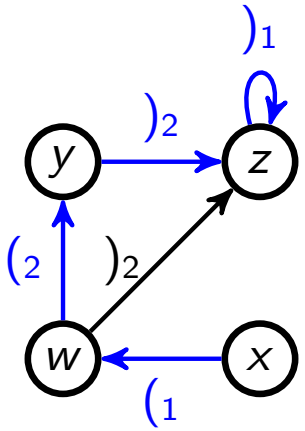

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- Alias analysis
- Data-dependence analysis
- Data-flow analysis
- Shape analysis
- Impact analysis
- Bloat analysis
- Type-based flow analysis
- Program slicing

Algorithms

Q: Is v Dyck-reachable from u?

- Dyck languages are a class of CFL
- Solution similar to CYK for CFL parsing
- $O(n^3)$ [Yannakakis '90]
- $O(n^3/\log n)$ [Chaudhuri '08]
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Can do better if the graph is simple!

1. Low treewidth
2. Bidirected

Static Analysis

- Analyze source code without executing it

Typical paradigm in static analysis: reduce the problem to a graph problem P:
Input: A program of methods M_i

1. Extract control flow graphs G_i
2. Annotate G_i
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Data-dependence analysis

- Which variable depends on which others
- Identify Def-Use chains in a program
- LHS depends on RHS

1	$x \leftarrow 2$
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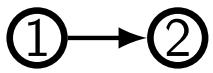

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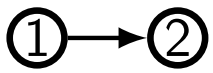
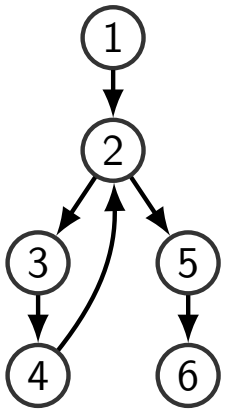


Intra-procedural → Finite Graphs

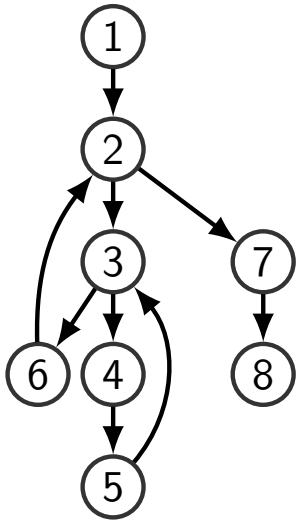
Method 1: dot_vector

```
result ← 0
for i ← 1 to n do
    z ← x[i] · y[i]
    result ← result + z
end
return result
```



Method 2: dot_matrix

```
1 C ← zero matrix of size n × m
2 for i ← 1 to n do
3     for j ← 1 to m do
4         Call dot_vector(A[i, :], B[:, j])
5         C[i, j] ← the value returned by the
            call of line 4
6     end
7 end
8 return C
```



Inter-procedural → Recursive Graphs

Method 1: $f_1()$

```
1 ...
2 a ← g()
3 ...
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Method 3: $g()$

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1 ...
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Method 2: $f_2()$

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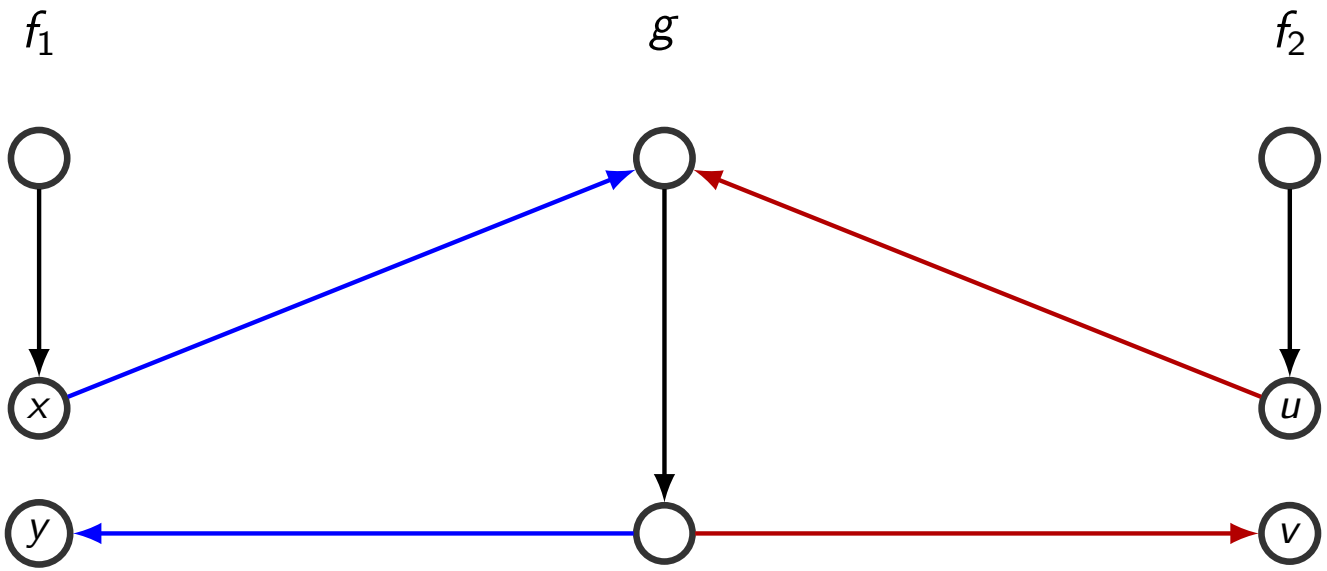

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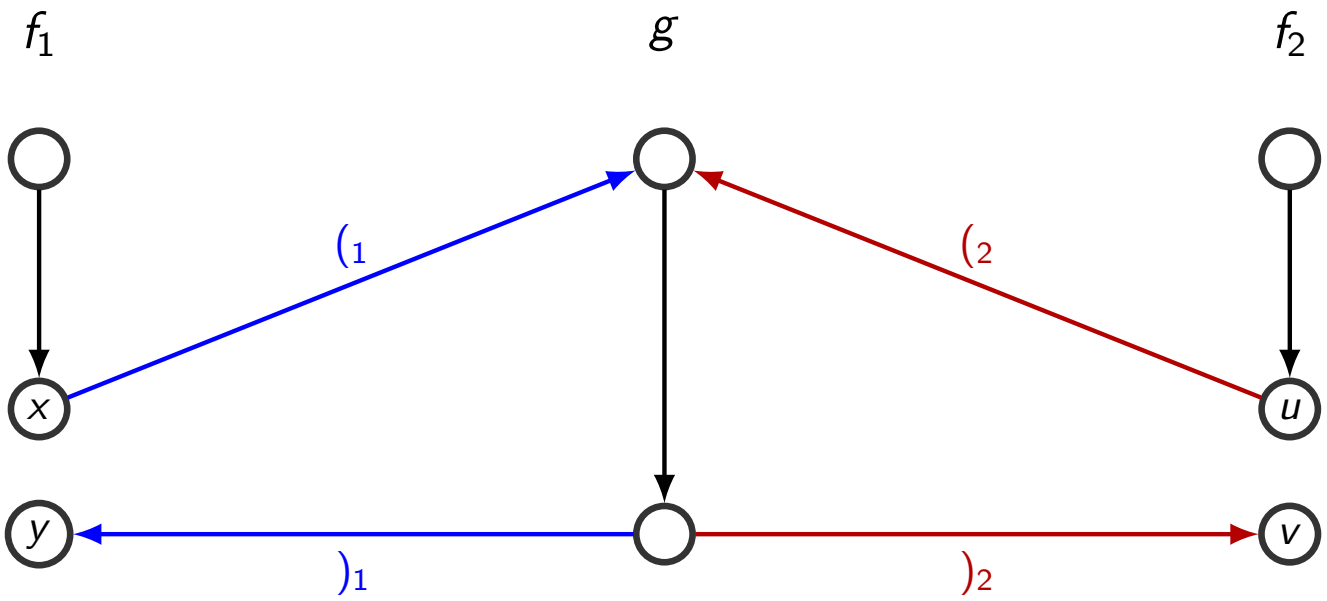

Filter out interprocedurally invalid paths

Inter-procedural → Recursive Graphs

Method 4: dot_vector

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Method 5: dot_matrix

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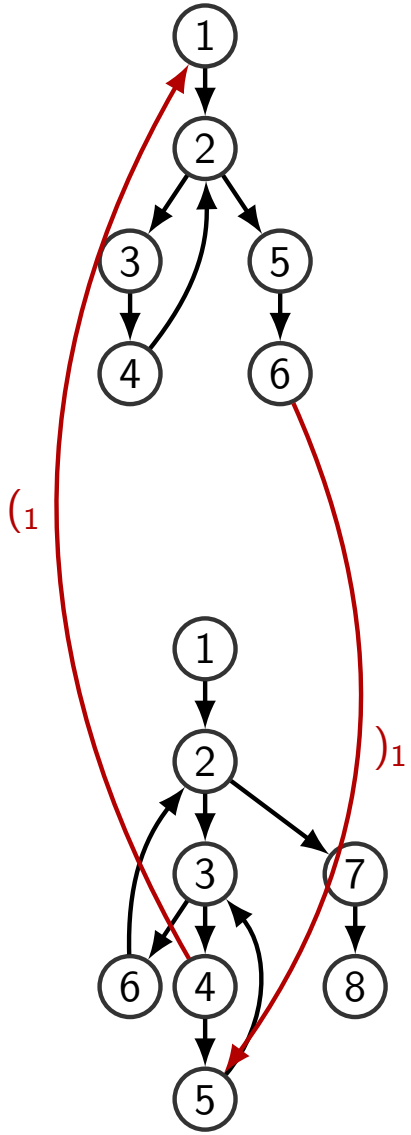
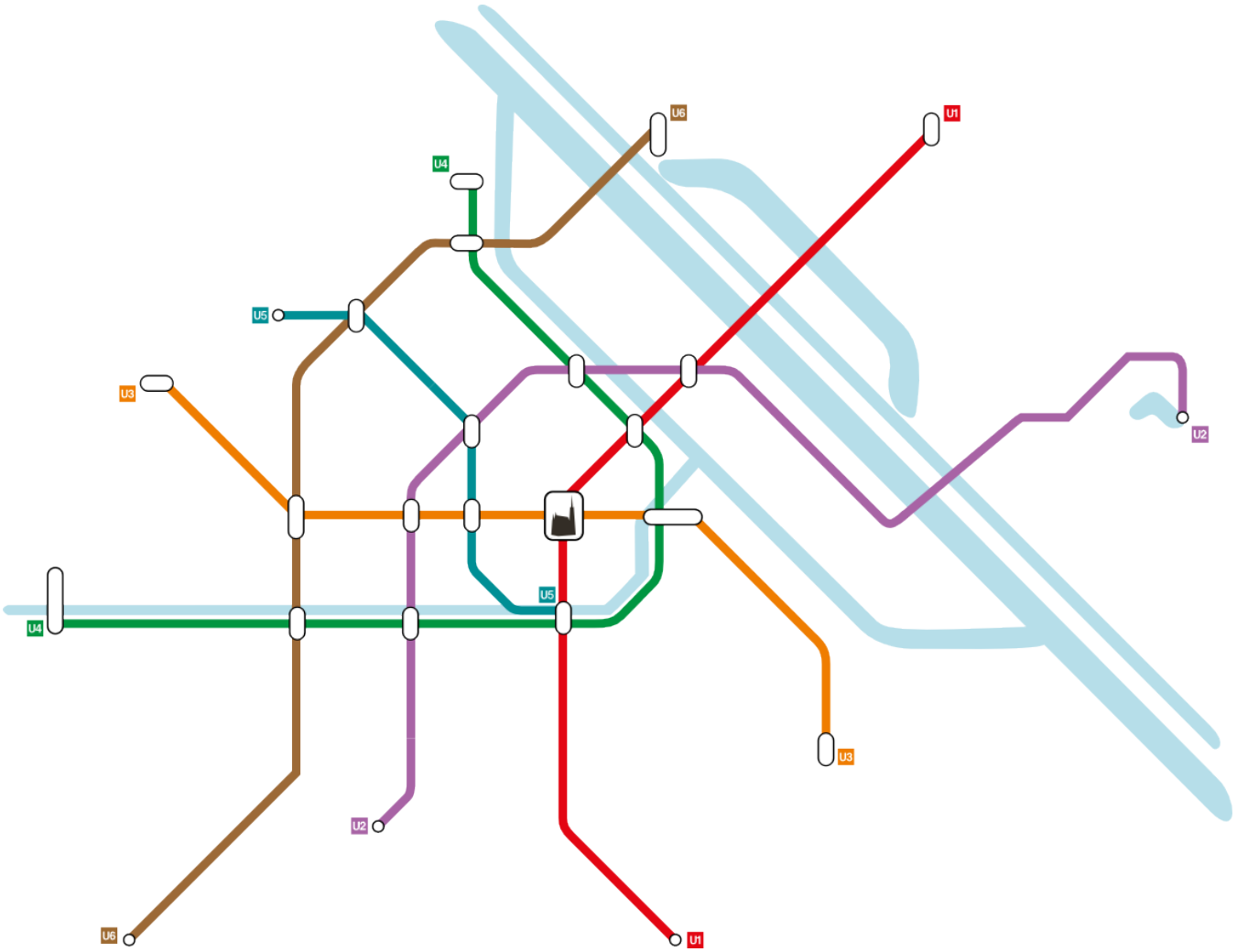


- Control-flow graphs are simple
- "Similar to trees"
- Precise notion: treewidth

Treewidth

Cops and Robber



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The **treewidth** of the metro graph is the largest t from which you can always escape

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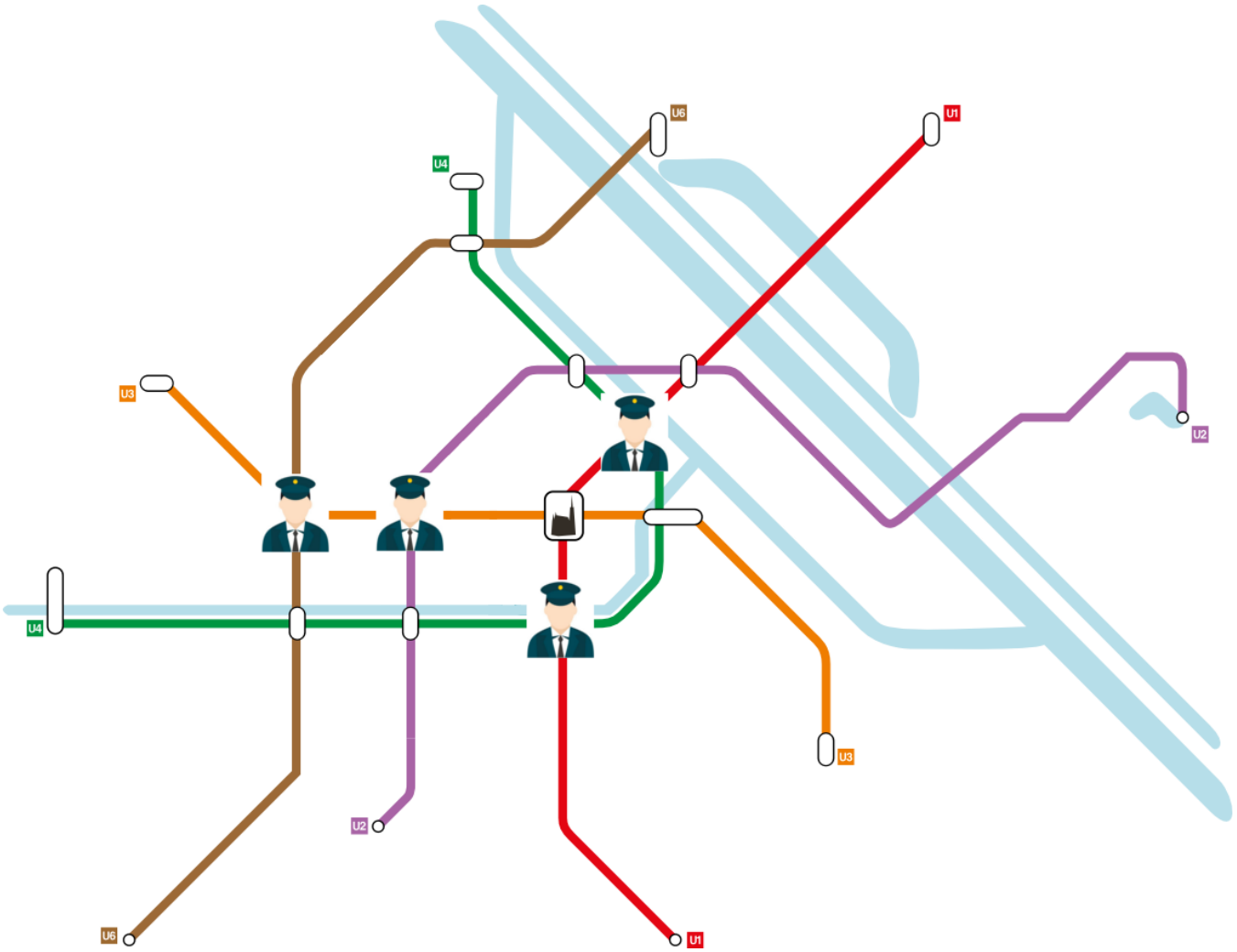
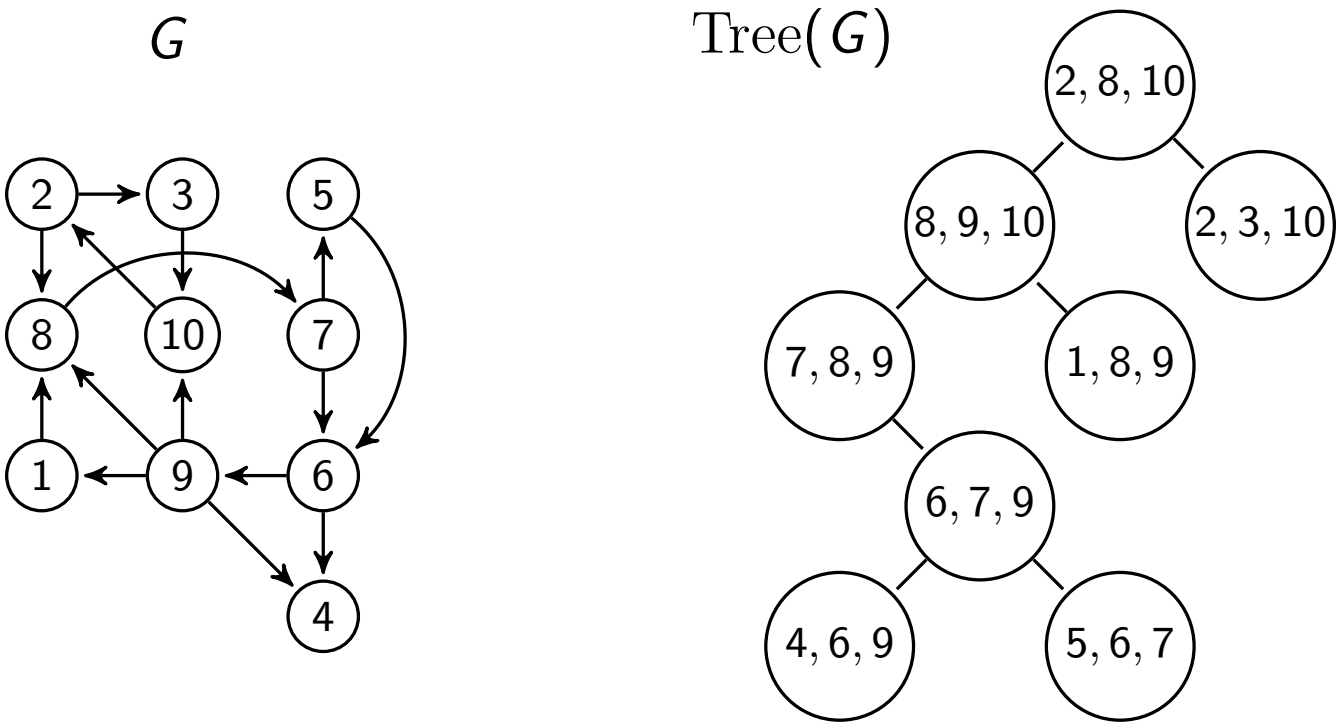


Tree Decompositions

Definition (Tree decomposition)

Given a graph $G = (V, E)$, a **tree-decomposition** $\mathrm{Tree}(G) = (V_T, E_T)$ is a tree of bags $B_i \subseteq V$

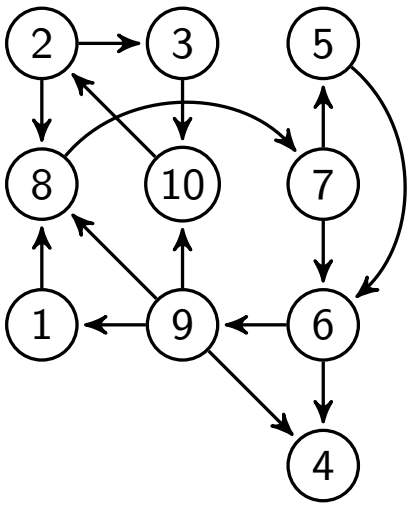


Treewidth

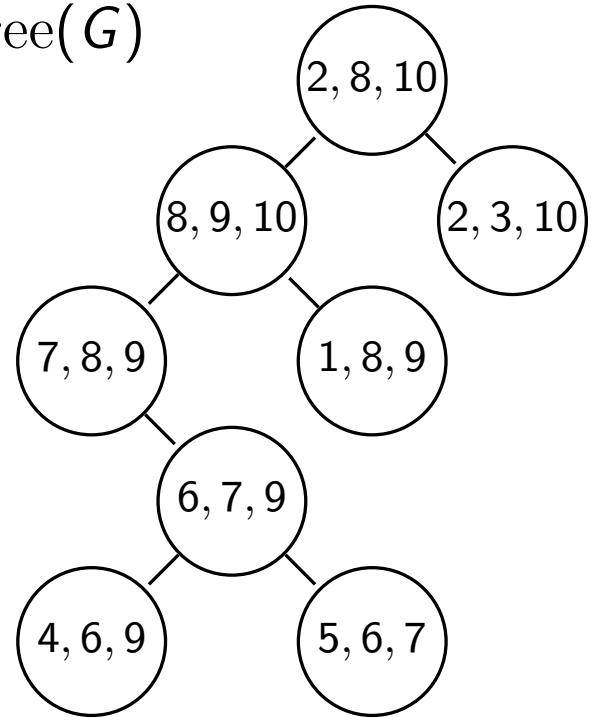
Definition (Treewidth)

The width of $\mathrm{Tree}(G)$ is $\max_i |B_i| - 1$. The **treewidth** of G is the minimum width of a tree decomposition of G.

G



$\mathrm{Tree}(G)$



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CFGs of typical imperative programs have small treewidth

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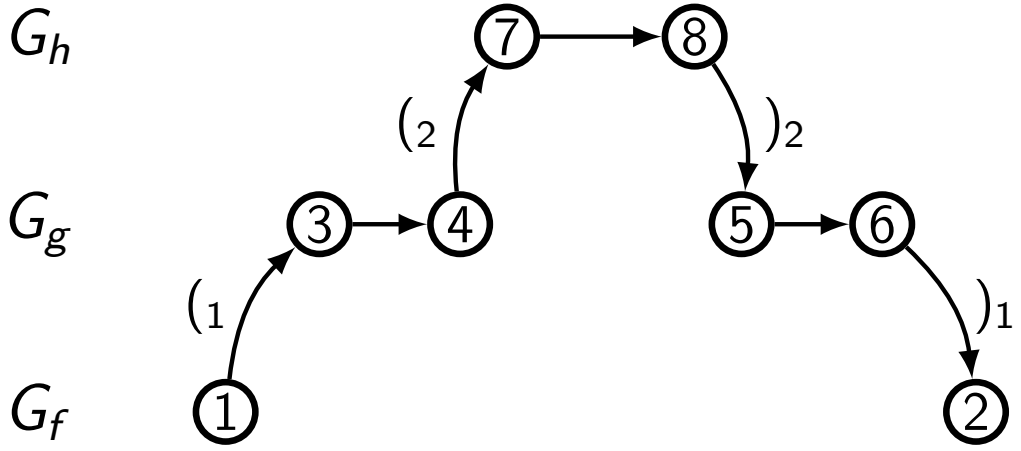
Theorem (Tree decomposition)

For constant treewidth graphs, $\mathrm{Tree}(G)$ *can be constructed in* $O(n)$ *time*

Algorithmic Principle



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2. If Entry $\rightsquigarrow$ Exit
   - Insert summary edge

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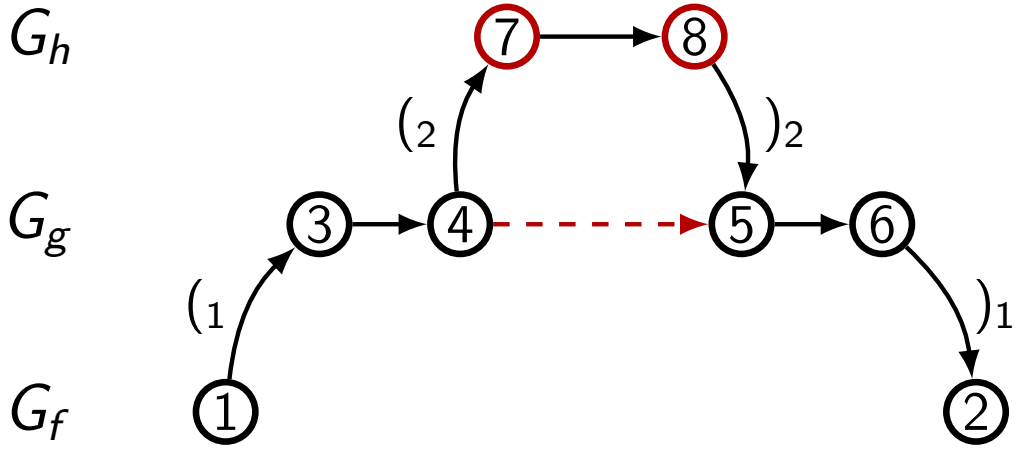

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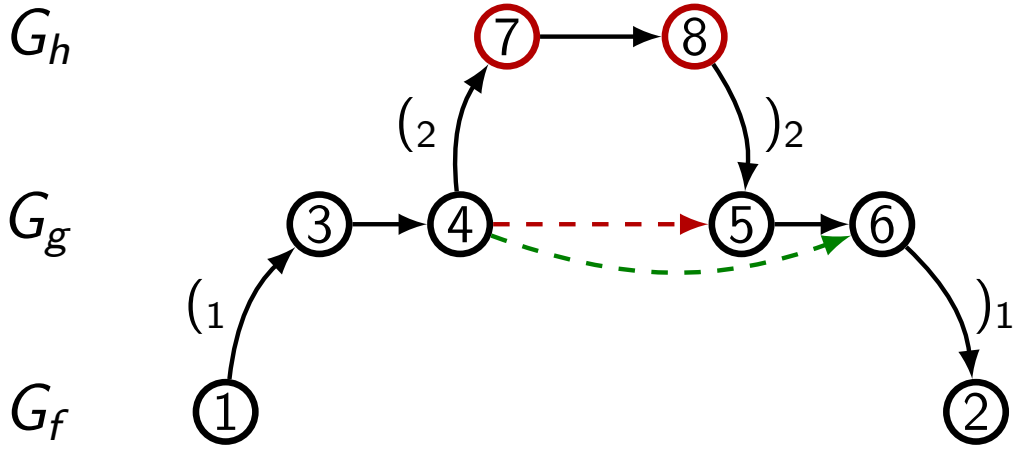

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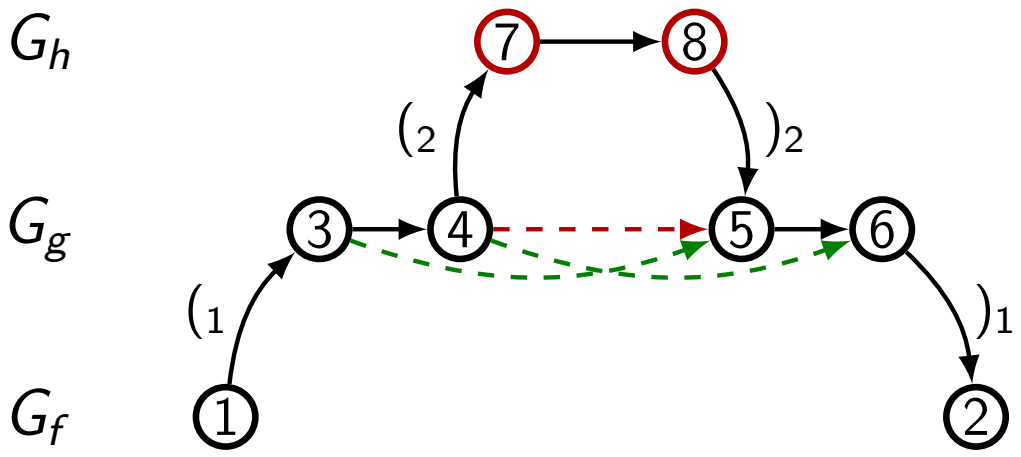

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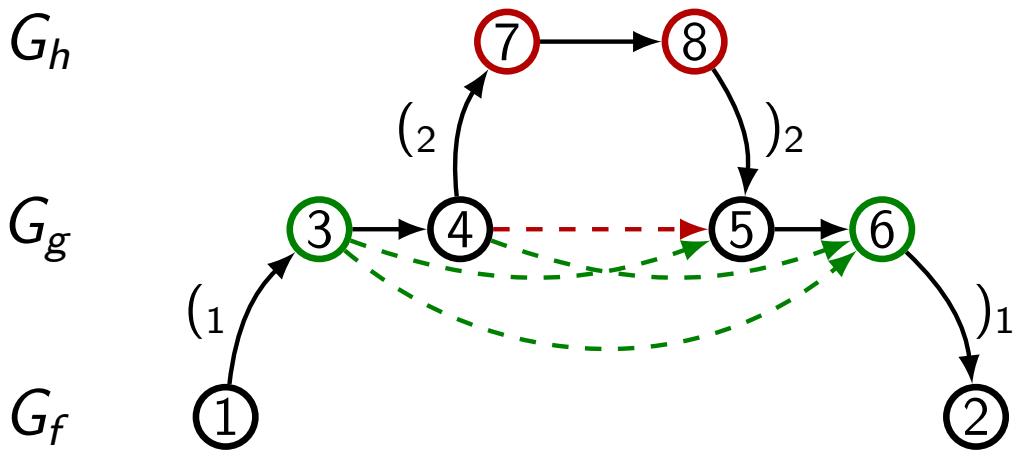

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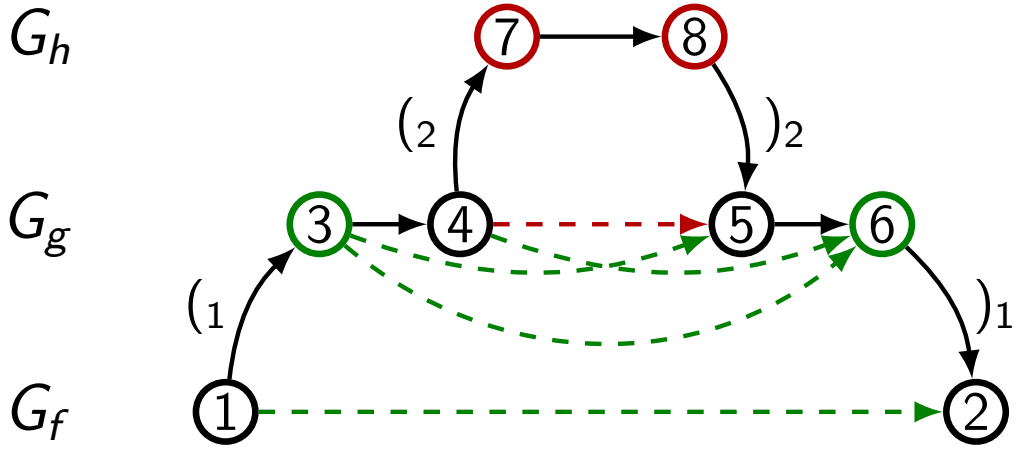

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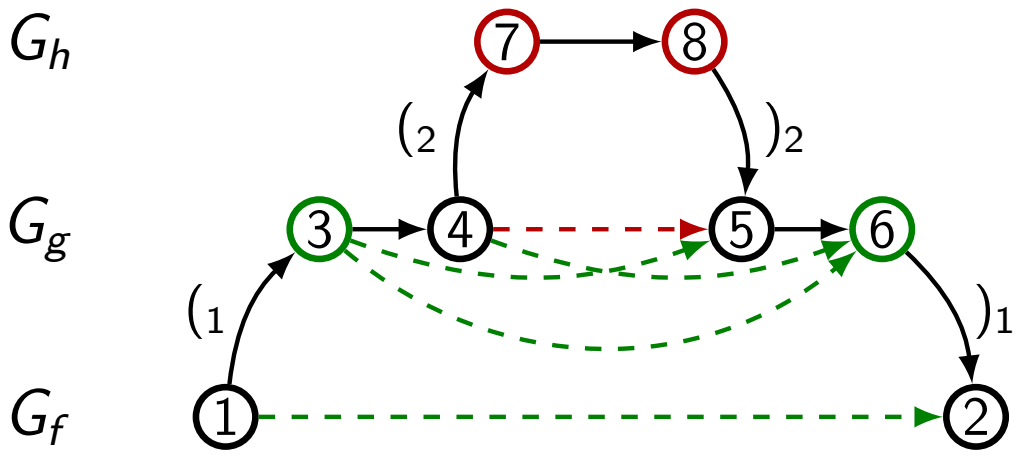

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- Most time spent in (1)
- Incremental reachability
- Low treewidth

Reachability on low-treewidth graphs

Reachability [ESA'16]

Given a graph $G = (V, E)$ of n nodes and bounded treewidth

Preprocessing time	**Space**	**Pair query time**	**Single-source query time**
–	$O(n)$	$O(n)$	$O(n)$
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Interprocedural Dataflow Analysis on Recursive Graphs of Low Treewidth

Interprocedural Dataflow Analysis

Input:

- Recursive graph G with n nodes
- Finite graphs G_i of n_i nodes and bounded treewidth
  - Every G_i has a unique entry and exit

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  - Weighted graph
  - Semiring distance on lattice over D

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Theorem (POPL'15)

For low-treewidth graphs we construct dynamic reachability oracles:

- **Preprocess** G_i *in* $O(n_i \cdot |D|^3)$ *time*
- **Query** *for the distance* $d(u, v)$ *in* $O(|D|^3 \cdot \log n_i)$ *time*
- **Update** *the weight of an edge* (u, v) *in* $O(|D^3| \cdot \log n_i)$ *time*

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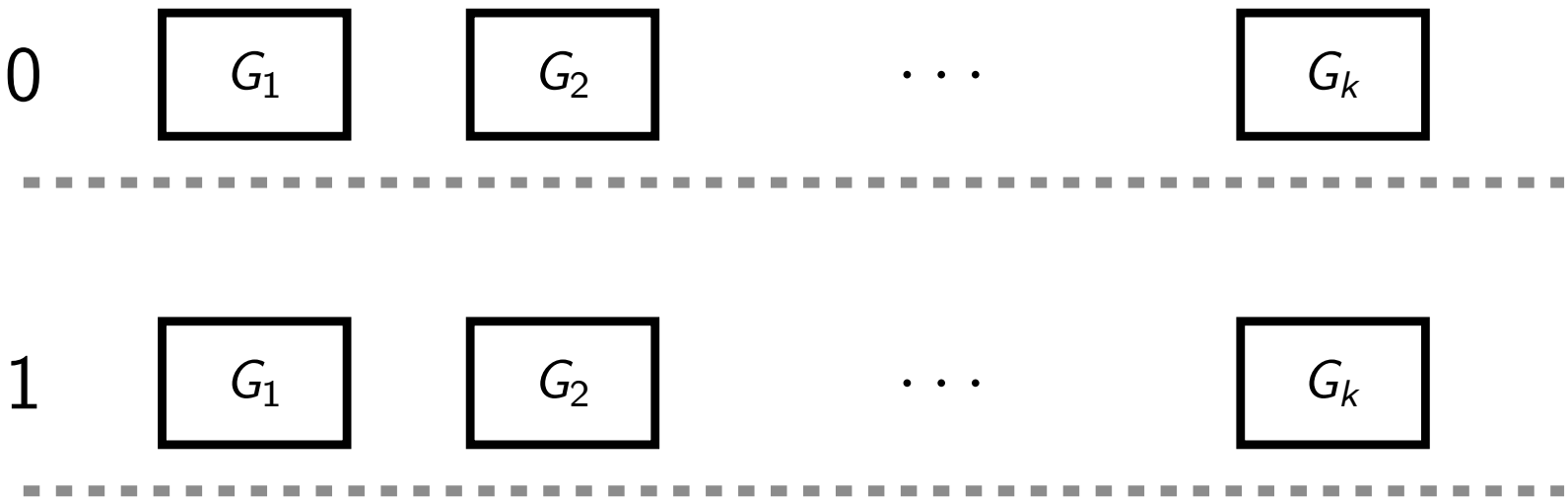

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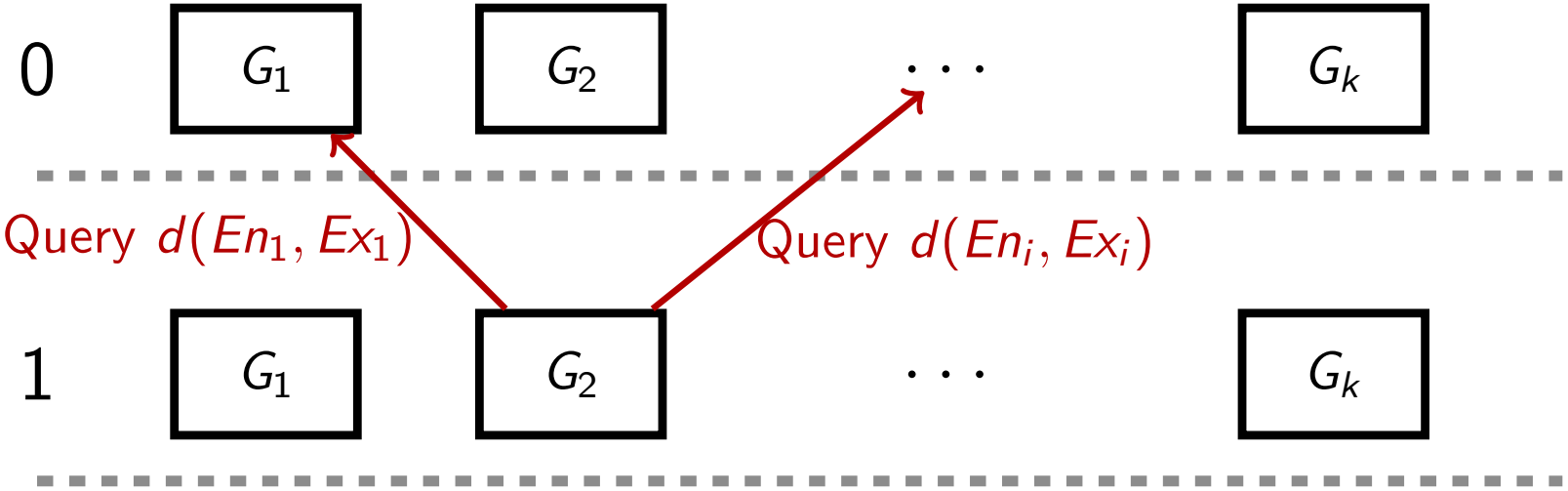

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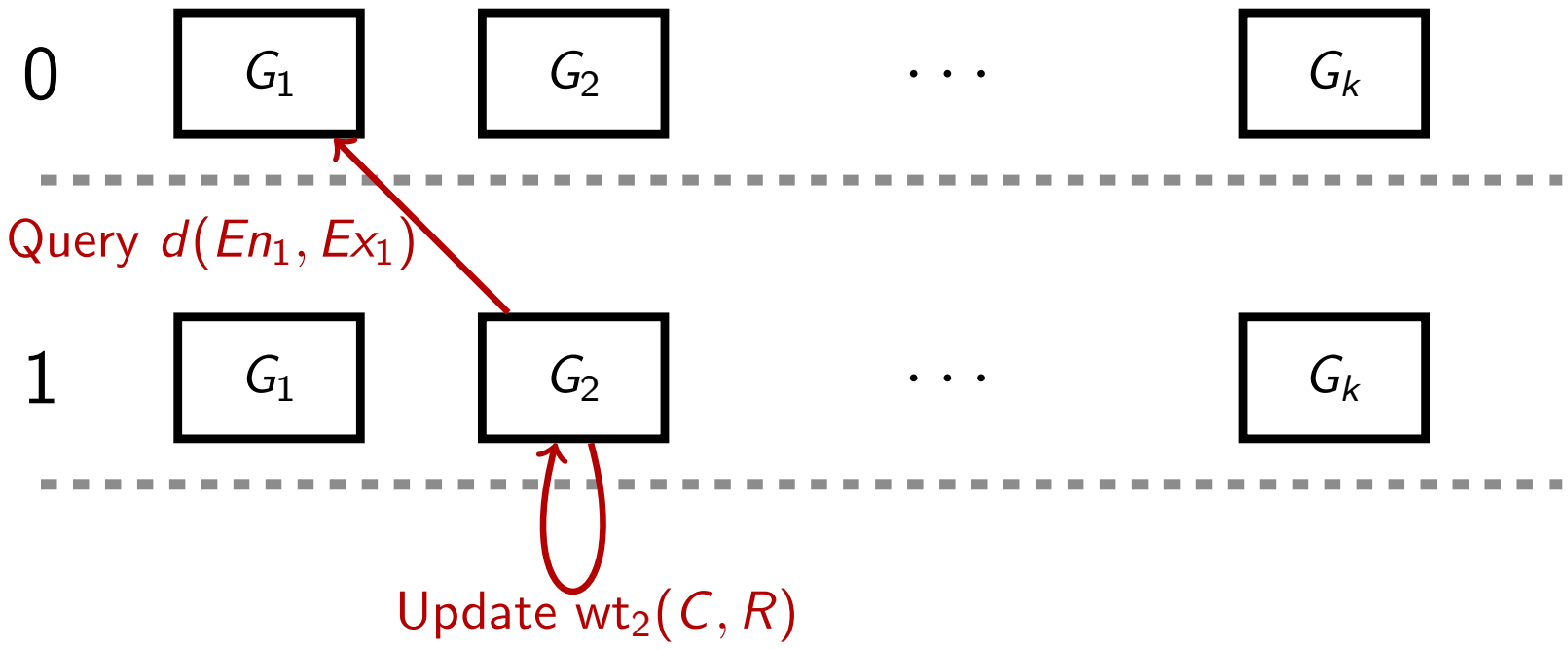

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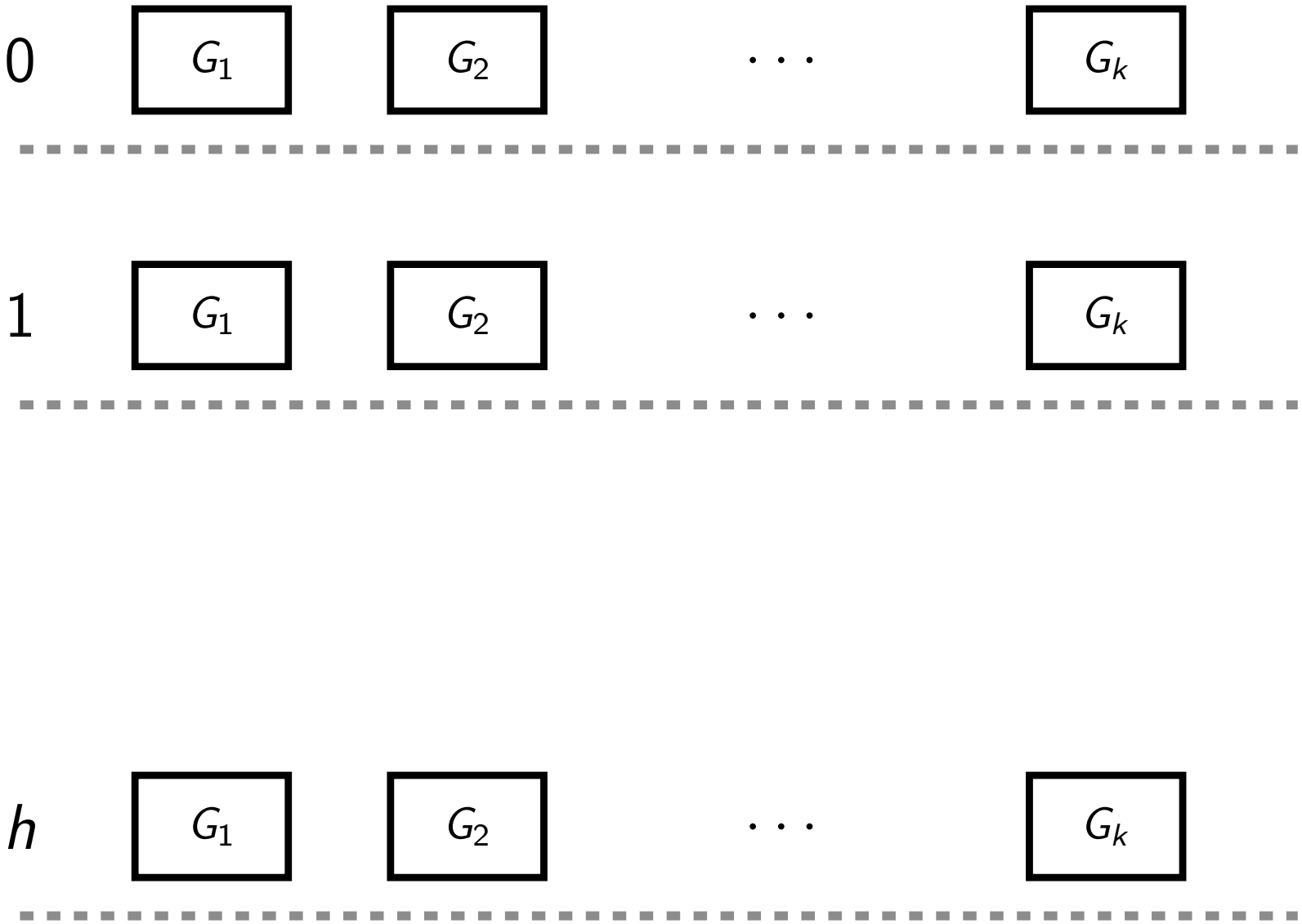

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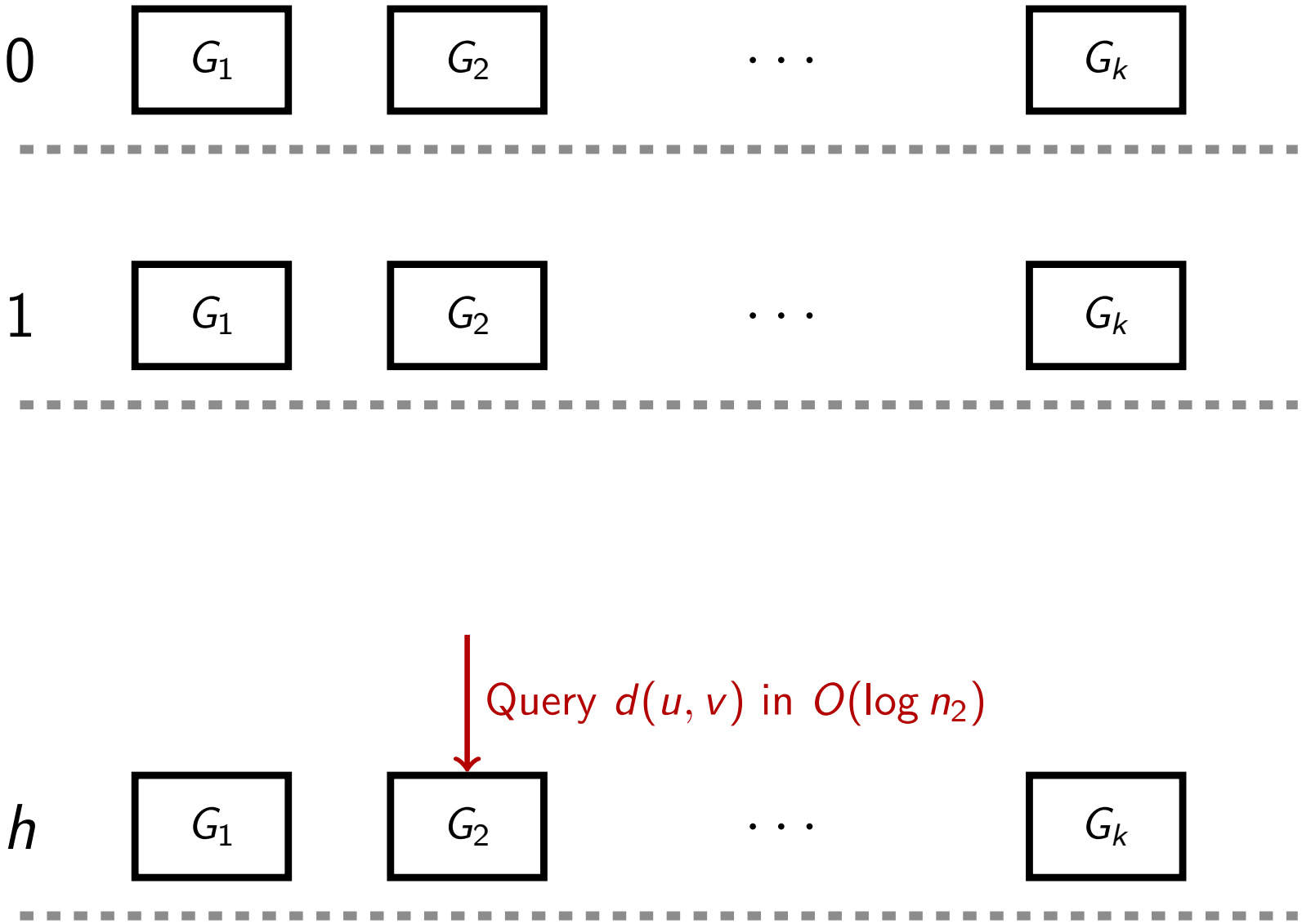

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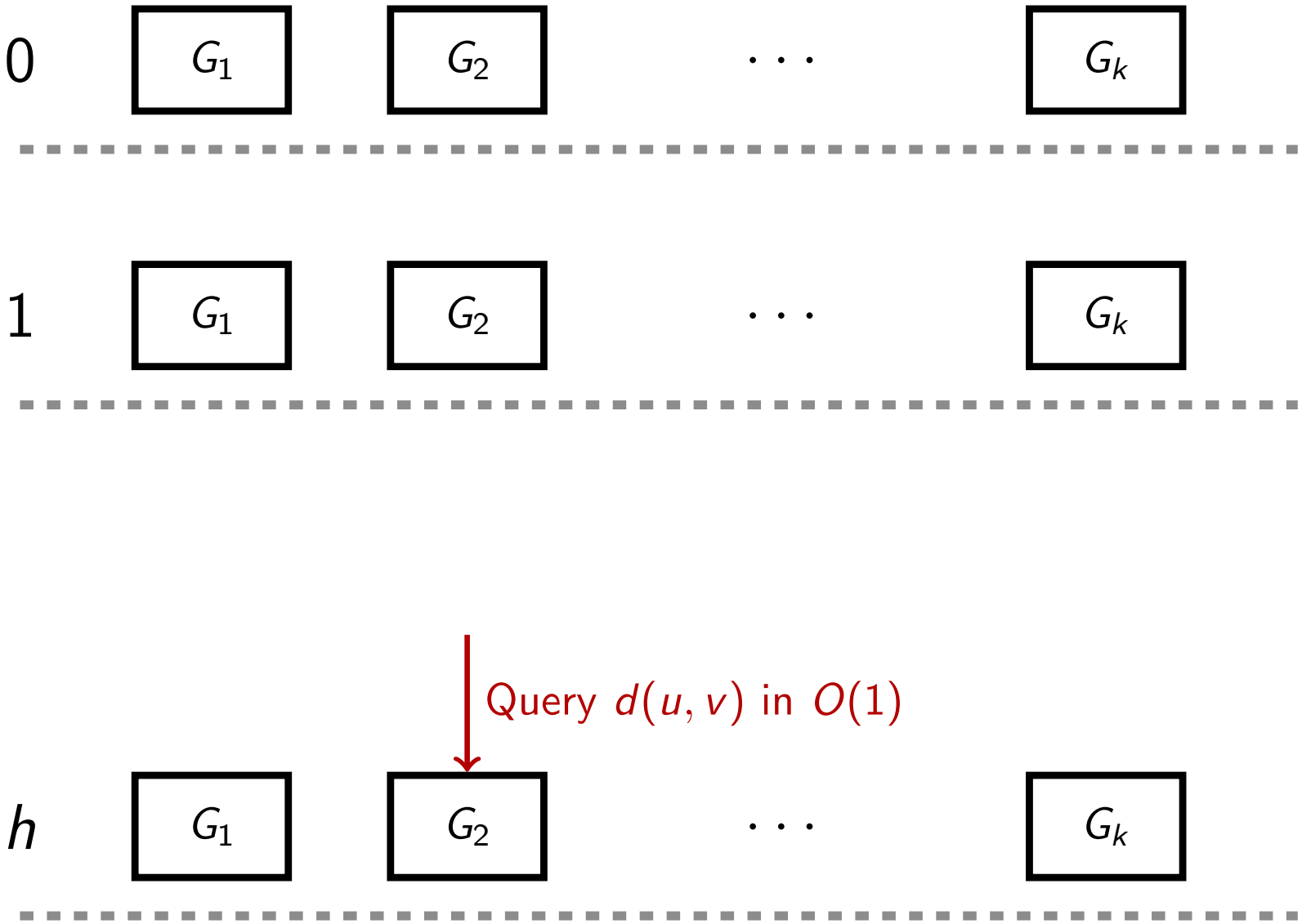

Dataflow Analysis [POPL'15]

Input graph with n nodes, D data fact set, $|D| = \Omega(\log n)$

	Preprocessing time	**Space**	**Query**	
			Single-source	**Pair**
No preprocessing	-	$O(n \cdot \lvert D\rvert^2)$	$O(n \cdot \lvert D\rvert^3)$	$O(n \cdot \lvert D\rvert^3)$
Complete preprocessing	$O(n^2 \cdot \lvert D\rvert^3)$	$O(n^2 \cdot \lvert D\rvert^2)$	$O(n \cdot \lvert D\rvert^2)$	$O(\lvert D\rvert^2)$
Ours	$\mathbf{O(n \cdot \lvert D\rvert^3)}$	$\mathbf{O(n\lvert D\rvert^2)}$	$\mathbf{O(n \cdot \lvert D\rvert^2)}$	$\mathbf{O(\lvert D\rvert^2)}$

Dataflow Analysis [POPL'15]

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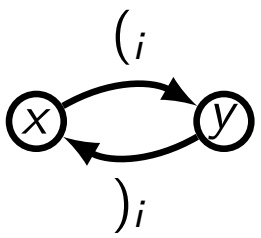
Dyck Reachability on Bidirected Graphs

Bidirected graphs

$$\forall u, v \in V : \ \lambda(u, v) = (_i \quad \Longleftrightarrow \quad \lambda(v, u) =)_i$$



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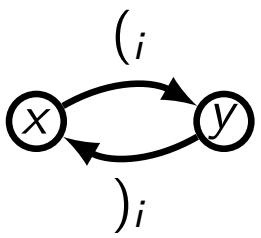

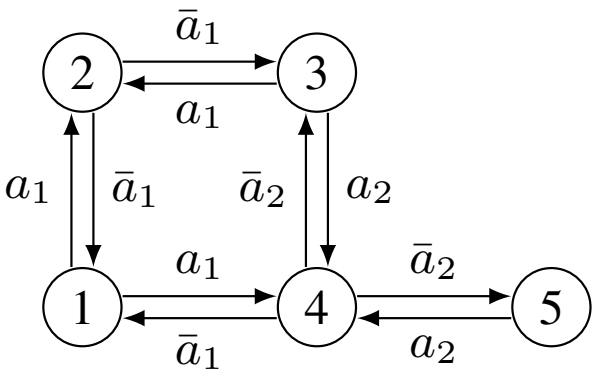

- Demand-driven field-sensitive alias analysis
- [YXR, ISSTA '11] [ZLYS, PLDI '13]

Reachability is Equivalence

Theorem (ZLYS '13)

Dyck reachability on bidirected graphs is an equivalence relation.

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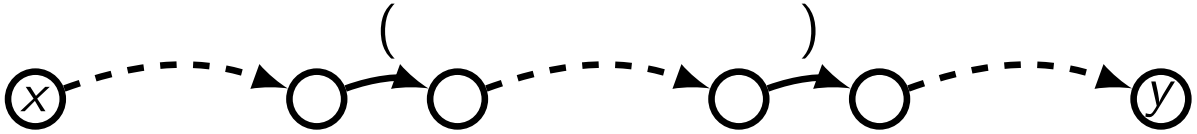

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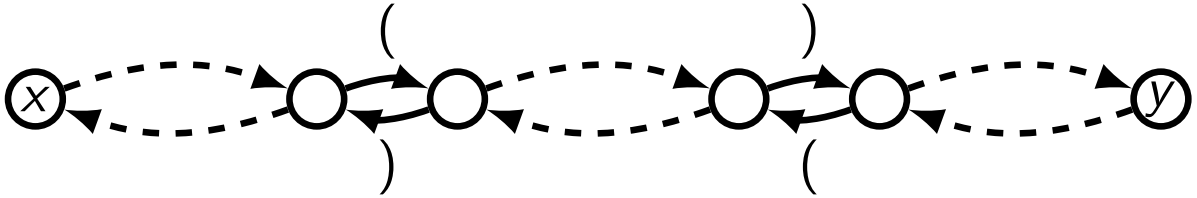

Dyck Reachability on Bidirected Graphs [POPL'18]

Graphs of n nodes, $\alpha(n)$ the inverse Ackermann

Source	**Worst-case Time**	**Average-case Time**
[ZLYS '13]	$O\left(n^2\right)$	$O\left(n \cdot \log n\right)$
Our Result	$\mathbf{O(n \cdot \alpha(n))}$	$\mathbf{O(n)}$

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	optimal!	optimal!

Hardness?

Complexity of Dyck Reachability [POPL'18]

- $O(n^3)$
- 2NPDA-hard
  - Conditional cubic lower bound

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Theorem

Dyck reachability on low-treewidth graphs is Boolean Matrix Multiplication - hard.

Can we do $O(n^3/\log^2 n)$?

Complexity on Bidirected Graphs [POPL'18]

Theorem

Dyck reachability on constant-treewidth, bidirected graphs requires $\Omega(n \cdot \alpha(n))$ *time.*

Complexity on Bidirected Graphs [POPL'18]

Theorem

Dyck reachability on constant-treewidth, bidirected graphs requires $\Omega(n \cdot \alpha(n))$ time.

Compare with

Theorem (ZLYZ '13)

Dyck reachability on bidirected trees solvable in $O(n)$ time.

Typically complexity on low-treewidth graphs = complexity on trees

Implementation & Experiments

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Alias Analysis

- Implementation in C++
- Compared with [ZLYS, PLDI '13]
- DaCapo-2006 benchmarks
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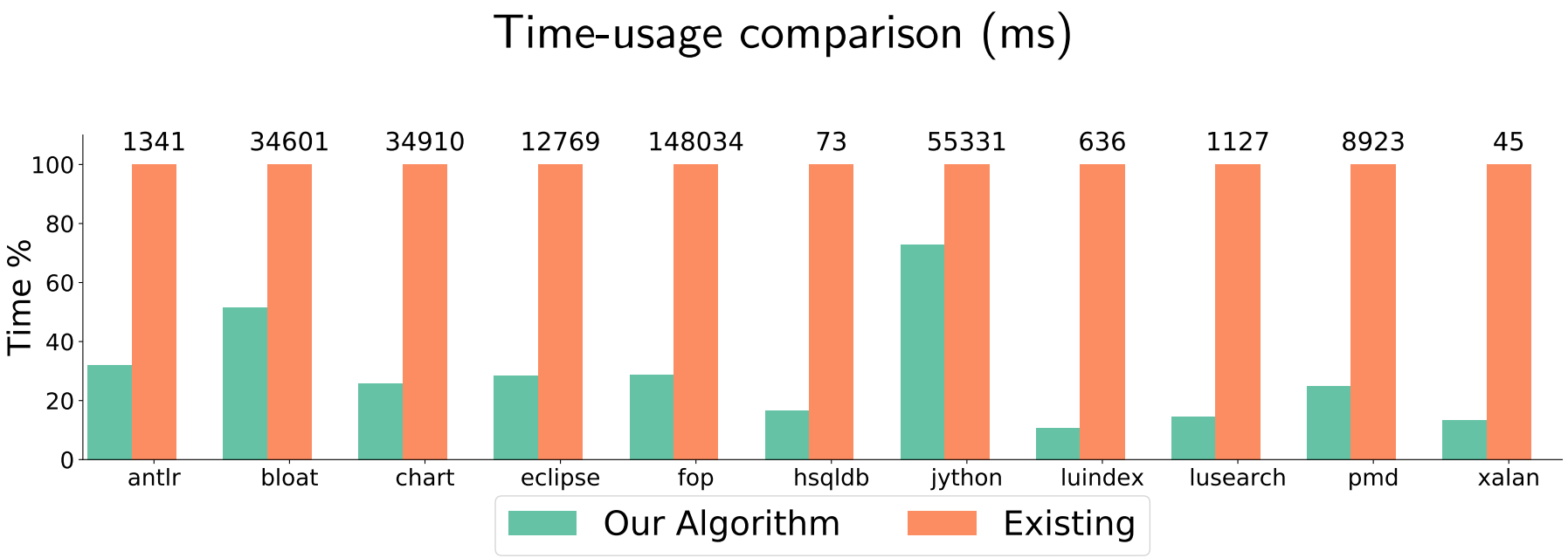
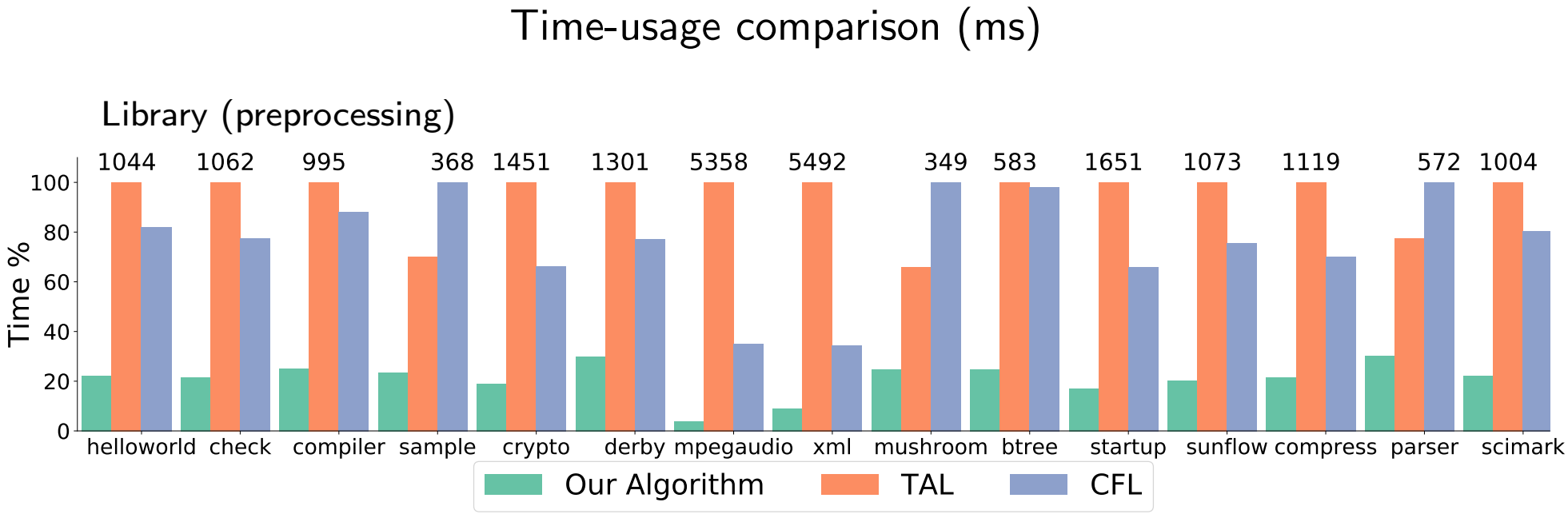
Data-dependence Analysis

- Implementation in Java
- Compared with TAL reachability [TWZXZM, POPL '15]
- Benchmarks:
  - SPECjvm2008
  - 4 randomly chosen GitHub projects

Alias Analysis

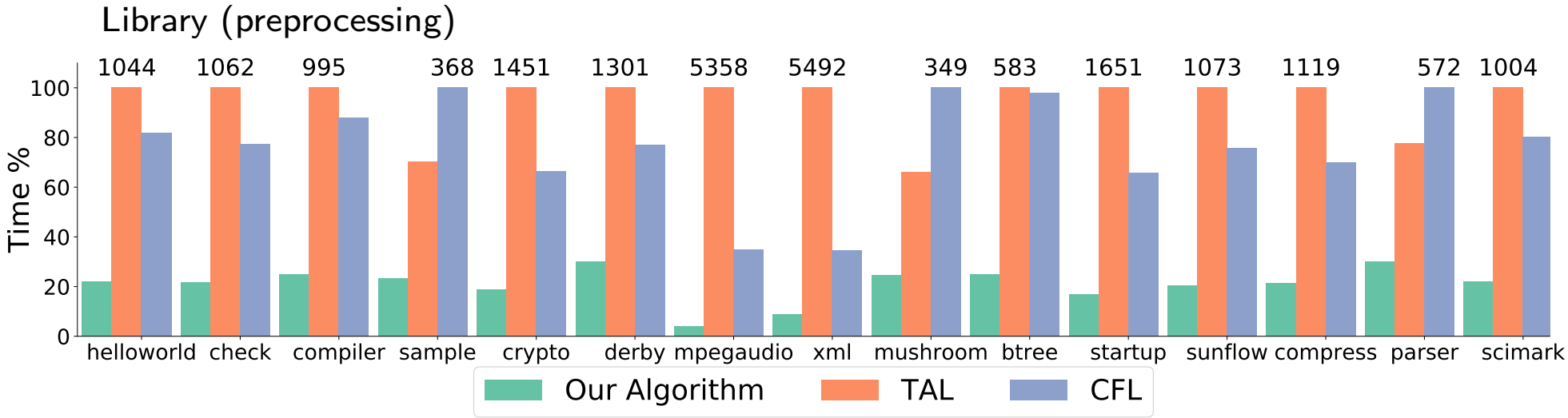


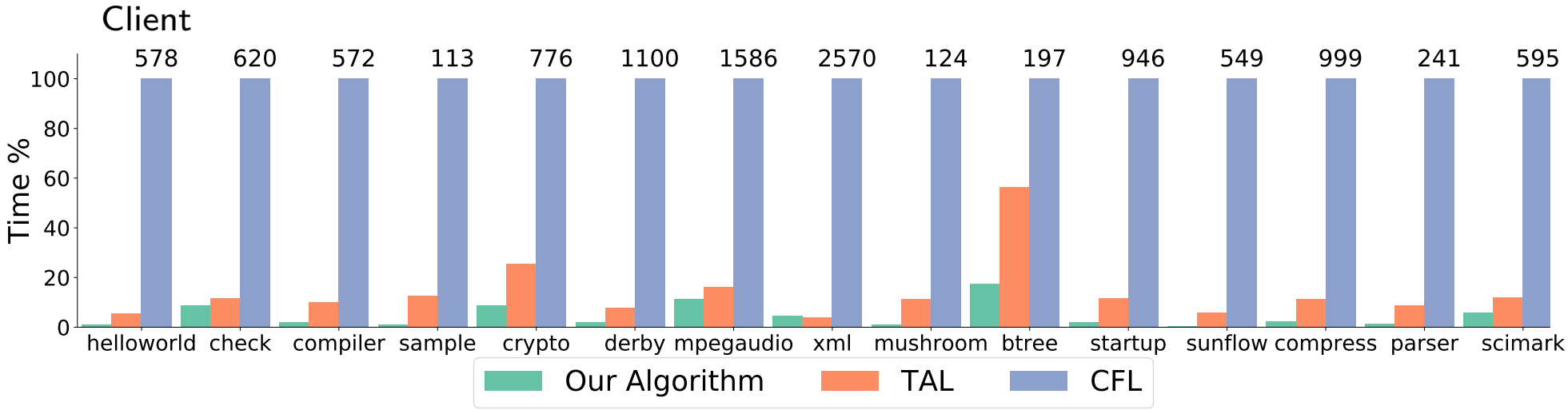
Data-dependence Analysis



Data-dependence Analysis

Time-usage comparison (ms)





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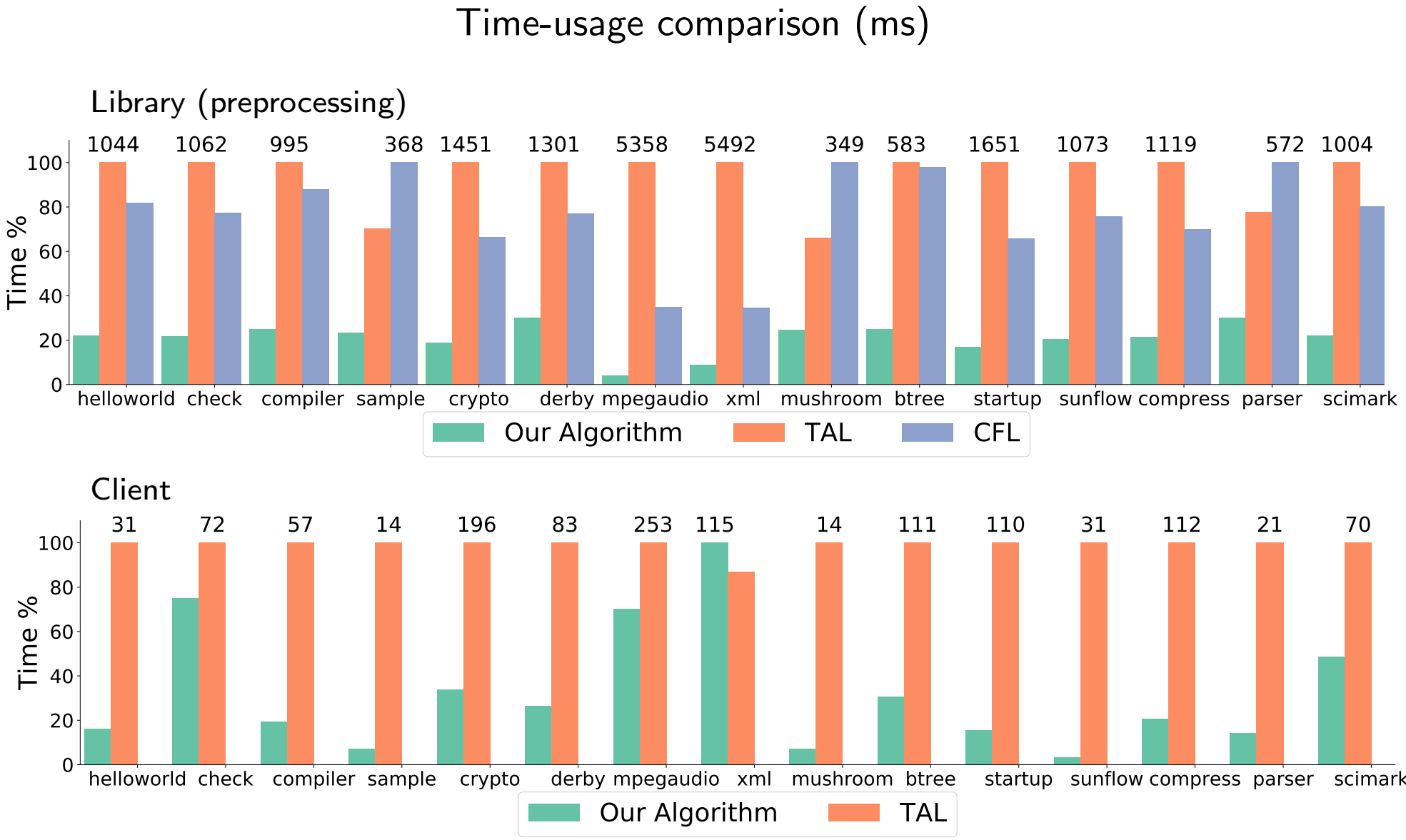

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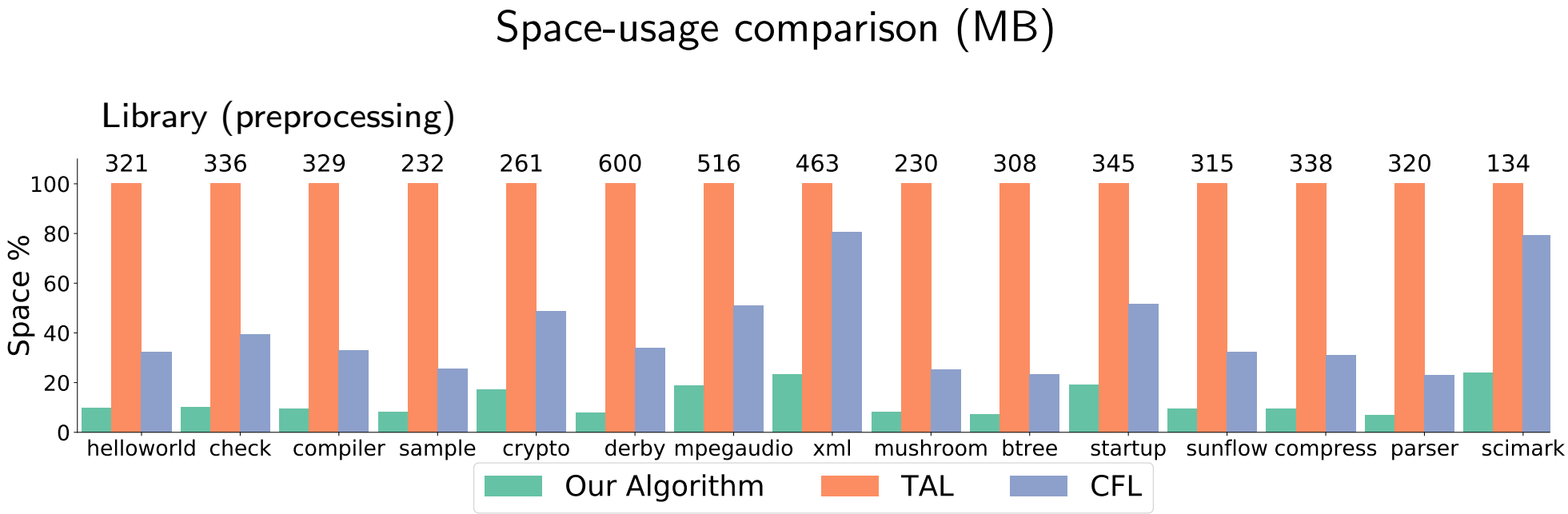
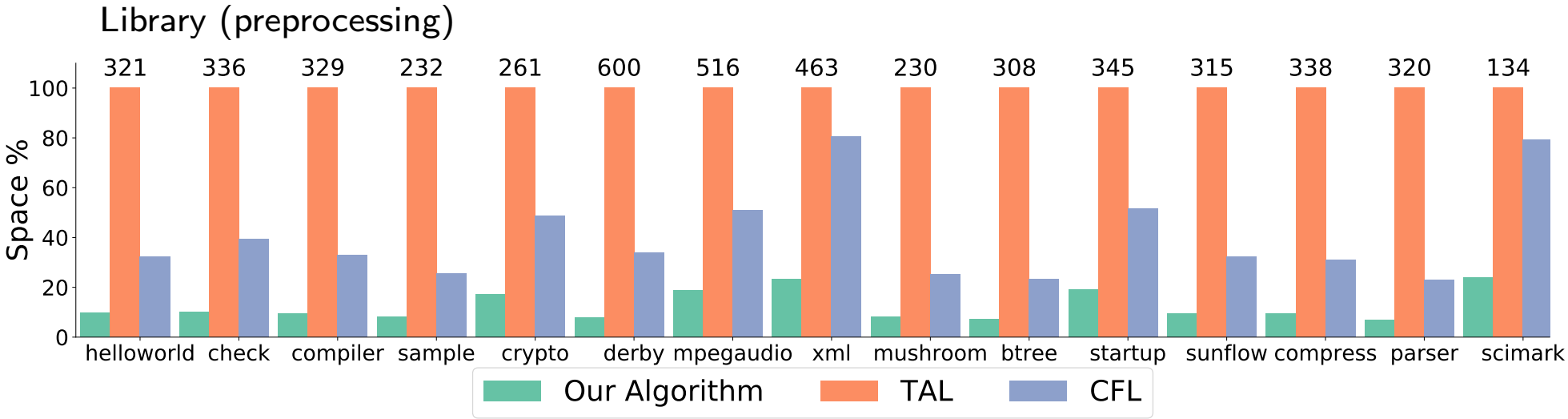


Data-dependence Analysis

	Nodes		Our Algo		TAL		CFL	
Benchmark	Lib.	Cl.	Lib.	Cl.	Lib.	Cl.	Li.	Cl.
serial	69999	468	130	130	>12000	>12000	3964	4314



Thank you!

Questions?