The 2T bosonic qutrit

A two-mode qutrit inspired from cat qudits

Aurélie Denys[1] Anthony Leverrier[1]

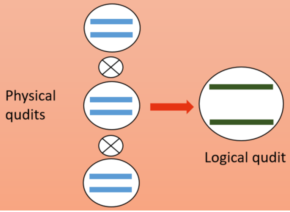
[1]Inria Paris, France

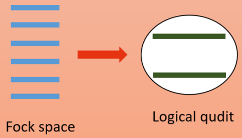
Défi EQIP - 30/11/22

Quantum error correction

- Main idea: Introduce redundancy by encoding a Hilbert space into another one of higher dimension.
- Two solutions: multi-qudit formalism and bosonic codes (approach followed by Amazon, Alice&Bob).

Multi-qudit QEC	**Bosonic QEC**
A logical qudit is encoded into several physical qudits.	A logical qudit is encoded into one or several modes (an infinite dimensional space)

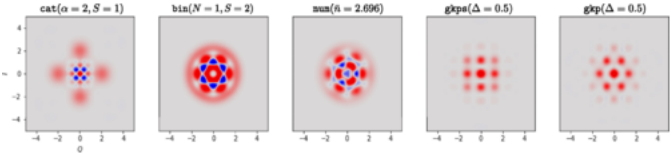




Bosonic quantum error correction

Single-mode codes

Encoding a qubit (or qudit) into one mode (corresponding to a Fock space $Span(\{|n\rangle : n \in \mathbb{N}\})$): single-mode codes
Examples: cat-codes, binomial codes, GKP codes



Wigner function of $\frac{1}{2}(|\bar{0}\rangle\langle\bar{0}| + |\bar{1}\rangle\langle\bar{1}|)$
V. Albert et al, PRA 2019

Goal: Design a "good" two-mode code

Multimode codes

Multi mode codes (living in tensor products of several Fock spaces) expected to give better performances.
Examples: multi-mode GKP, pair-cat, etc...

Goal

Design a "good" multimode code.

Code space definition

Consider a finite constellation of coherent states $\{|\alpha_1\rangle, \dots, |\alpha_n\rangle\}$, where each α_i is a string of complex numbers (two complex numbers if working with two modes), and define codes whose logical states are given by superpositions of these coherent states.

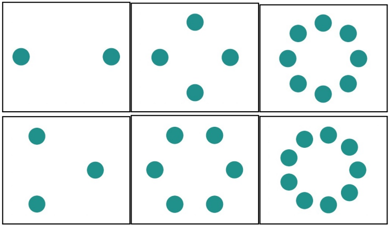
Why?

- The cat code is an example of such a code and it has nice properties.
- Get analytical formulas for the action of the loss channel on the code space.

Choice of the constellation

Cat codes constellation

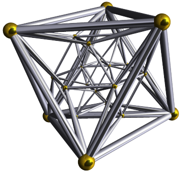
$$|\bar{k}\rangle \propto \sum_{\ell=0}^{dM-1} e^{-\frac{2i\pi k\ell}{d}} \left|\alpha e^{2i\frac{\ell\pi}{d}}\right\rangle$$



The phase-shift keying modulations forming regular polygons

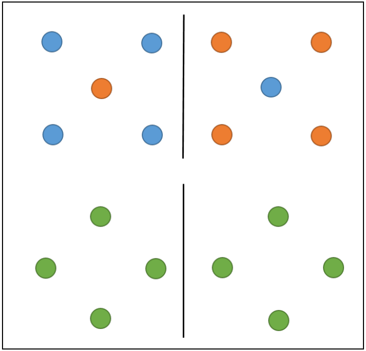
The 2T-constellation

A regular polytope in dimension 4? Look at the subgroups of the units of the quaternions, among which the binary tetrahedral group $2T$.



The 24-cell whose vertices correspond to elements of $2T$

The $2T$-constellation



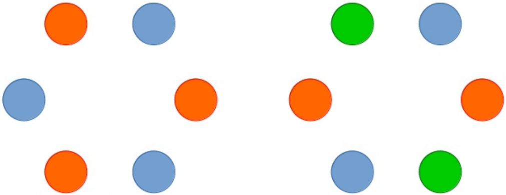
$$
\begin{aligned}
\mathcal{H}_{2T} := \mathrm{Span}(\{ & |\sqrt{2}e^{i(2k+1)\pi/4}\alpha\rangle|0\rangle, \\
& |0\rangle|\sqrt{2}e^{i(2k+1)\pi/4}\alpha\rangle, \\
& |e^{ik\pi/2}\alpha\rangle|e^{i\ell\pi/2}\alpha\rangle \\
& : 0 \leq k, \ell \leq 3\})
\end{aligned}
$$

The $dM-$component cat qudit

- $\mathbb{U}_M = \{e^{\frac{2ik\pi}{M}} : k \in [\![0, M-1]\!]\}$ is a normal subgroup of $\mathbb{U}_{dM}$.
- $\forall k \in [\![0, d-1]\!]$,

$$|\phi_k\rangle \propto \sum_{u \in \omega^k \mathbb{U}_M} |u\rangle$$

  where $\omega = e^{\frac{2ik\pi}{dM}}$, $|u\rangle := |(a+ib)\alpha\rangle$ for $u = a + ib$.
- Code space $= Span(\{|\phi_k\rangle : k \in [\![0, d-1]\!]\})$



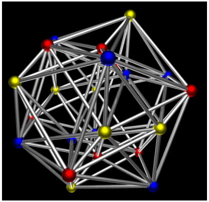
The $2T$-qutrit: A qutrit in the $2T$-constellation

- $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ is a normal subgroup of $2T$.
- $\forall \mathrm{k} \in \{0, 1, 2\}$,

$$|\phi_k\rangle \propto \sum_{q \in \omega^k Q} |q\rangle$$

where $\omega = -\frac{1}{2}(1 + i + j + k)$ $|q\rangle := |(a+ib)\beta\rangle\,|(c-id)\beta\rangle$ with $\beta = \sqrt{2}\alpha e^{i\frac{\pi}{4}}$ for $q = a + ib + jc + kd$.

- Code space $=$ $Span(|\phi_0\rangle, |\phi_1\rangle, |\phi_2\rangle)$



Author: UtilisateurTheon, CC-BY-SA-4.0, Commons

Orthonormal basis

The $dM-$component cat qudit

$$|\bar{k}\rangle \propto \sum_{\ell=0}^{2} \zeta^{-k\ell}|\phi_\ell\rangle,$$

for $k \in \{0,1,2\}$ where $\zeta = e^{\frac{2i\pi}{d}}$.

The 2T-qutrit

$S\,|\phi_i\rangle = |\phi_{i-1mod3}\rangle$ where S is a symplectic matrix.

$$|\bar{k}\rangle \propto \sum_{\ell=0}^{2} \zeta^{-k\ell}|\phi_\ell\rangle,$$

for $k \in \{0,1,2\}$ where $\zeta = e^{\frac{2i\pi}{3}}$.

$$S|\bar{k}\rangle = \zeta^{k}|\bar{k}\rangle$$

S acts as a logical Pauli $\bar{Z}$.

Stabilisers

Jump operators for the $2T$-constellation

$$F_1 = (\hat{a}^4 + \hat{b}^4 + \alpha^4)^2 - 9\alpha^8,$$
$$F_2 = 6\hat{a}^4\hat{b}^4 - \alpha^4(\hat{a}^4 + \hat{b}^4) - 4\alpha^8.$$

Stabilisers

$$S_1 = F_1 + 1$$
$$S_2 = F_2 + 1$$
$$S_3 = e^{i(\hat{n}_1+\hat{n}_2)\pi/4}$$
$$S_4 = e^{i\hat{n}_1\pi/2}$$
$$S_5 = SWAP$$

Figure of merit

Tool to quantify how close a state is from the corresponding original state after performing a recovery operation?

Entanglement fidelity

$$f(\mathcal{R} \circ \mathcal{N} \circ \mathcal{E}) = \langle\phi| \left(I_A \otimes (\mathcal{R} \circ \mathcal{N} \circ \mathcal{E})\right)(|\phi\rangle\langle\phi|)\,|\phi\rangle$$

of the channel given by the composition of the encoding $\mathcal{E}$, the noise $\mathcal{N}$, and the recovery $\mathcal{R}$ channels, for an input state

$$|\phi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d} |k\rangle\,|k\rangle$$

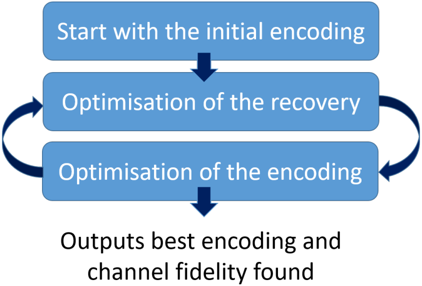
the maximally entangled state.

Noise model

Pure-loss (main source of noise in bosonic systems)

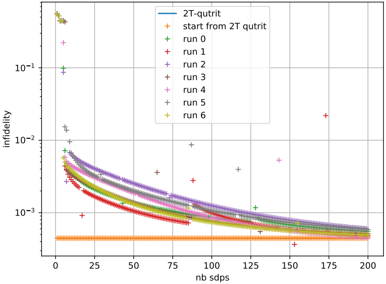
Maximising the figure of merit

Maximising simultaneously over both the encoding and the recovery channels is hard but an iterative optimisation, via semi-definite programs (SDPs), works well in practice.

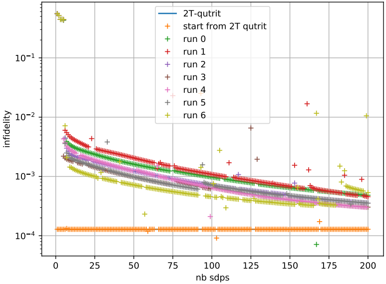


Performances of the $2T$-qutrit against loss

Comparison with best numerical qutrit found



$\gamma = 0.01, \alpha = 1.5$

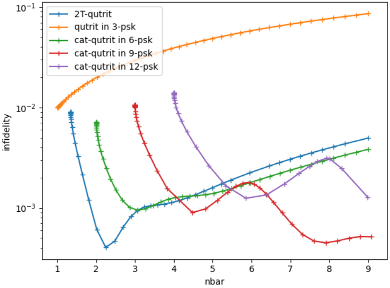


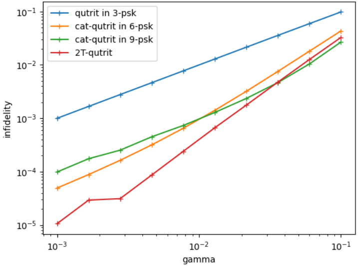
$\gamma = 0.005, \alpha = 1.5$

For small loss strengths γ, the biconvex optimisation does not identify better qutrits than the $2T$-qutrit.

Performances of the 2T-qutrit against loss

Comparison with cat qutrits





$\gamma = 0.01$ $\quad$ $0.25 \leq \alpha \leq 2$

For small loss strengths γ, the $2T$-qutrit performs better than cat qutrits.

Conclusion

Summary

- We introduced a new two-mode bosonic qutrit: the $2T$-qutrit.
- Advantages: Underlying group structure similar to that of cat qudits (the formalism and techniques can be leveraged to this new code), better performance against loss for small noise strengths.

Open questions

- Find a universal gate set and how to implement it.
- Describe a physical error correction procedure.