Shot-Noise Based Dynamics of Point Processes

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Structure of the Lecture

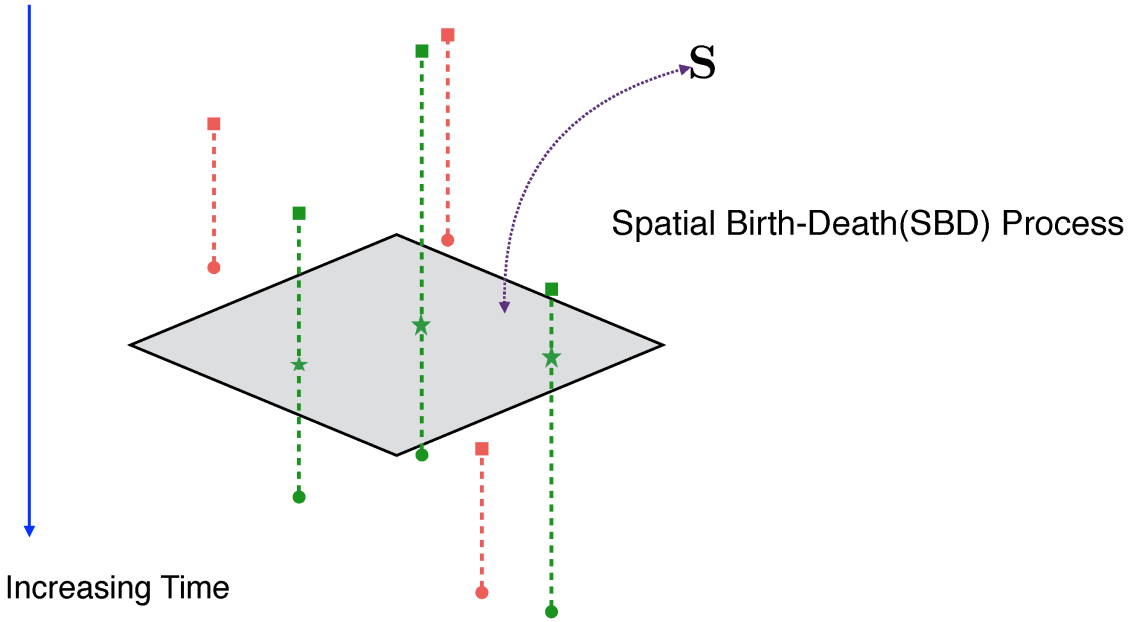
- **Motivations**
- **Rate Functions**
- **Compact versus Infinite Phase Space**
- **Stability**
- **Rate Conservation Principle**
- **Computational Tools**

Motivations

- **Shot-Noise of a point process**
  1. **Interference created by the PP in a wireless network**
  2. **Bit rate received by a point from the PP in a P2P network**
  3. **Infection rate of a susceptible point in epidemics on PP**

- **Example of Shot-Noise based dynamics**
  - **spatial birth and death process with death rate of a point function of shot-noise**
  - **spatial on-off process with transition rates function of shot-noise**

Spatial Birth-and-Death Processes

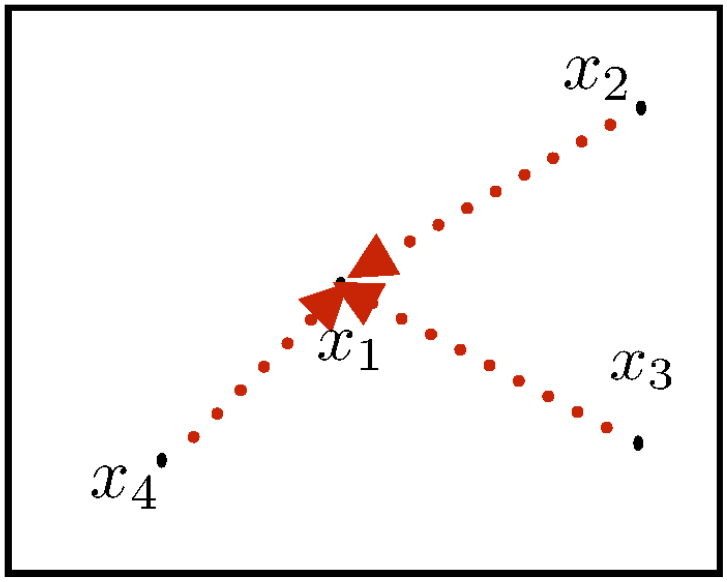


Rate Functions

- **Markov dynamics on the space of counting measures**
- **Rate Function Examples**
  e.g. birth rate, death rate, transition rate
  - **Proportional to SN: infection rate in epidemics**
  - **Inversely Proportional to SN:**
    Low Signal to Interference Shannon bit rate
  - **Proportional to $\log(1+\mathbf{SINR})$ with I=Shot-Noise**
    Shannon bit rate

Wireless Example

Configuration
$\mathbf{\Phi = \{x_1, \ldots, x_n\}}$



Attenuation function $\mathbf{l : \mathbb{R}^+ \to \mathbb{R}^+}$
Interference seen at $\mathbf{x_i}$**:**

$$\mathbf{I(\Phi, x_i) = \sum_{y \in \Phi \setminus \{x_i\}} l(||y - x_i||)}$$

Rate of file transfer at receiver $\mathbf{x_i}$**:**

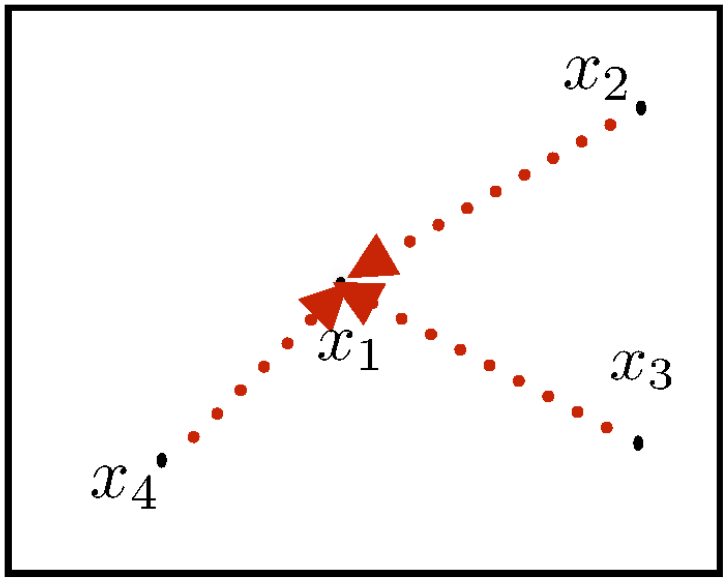
$$\mathbf{R(\Phi, x_i) = \frac{s}{I(\Phi, x_i)}}$$

If file exponential mean 1, this is also the death rate of $\mathbf{x_i}$ **in** Φ

Peer-to-Peer Example

Configuration

$\mathbf{\Phi = \{x_1, \ldots, x_n\}}$



TCP rate as a function of distance:

$$\mathbf{l} : \mathbb{R}^+ \to \mathbb{R}^+$$

Bit Rate received by $\mathbf{x_i}$:

$$\mathbf{R}(\mathbf{\Phi}, \mathbf{x_i}) = \sum_{\mathbf{y} \in \mathbf{\Phi} \setminus \{\mathbf{x_i}\}} \mathbf{l}(||\mathbf{y} - \mathbf{x_i}||)$$

If file exponential mean 1, this is also the death rate of $\mathbf{x_i}$ in Φ

Compact versus Infinite Phase Space

- **Compact phase space case**
  - **Typically large torus**
  - **Uniformizable Markov process**

- **$\mathbb{R}^d$ case**
  - **Construction is elaborate even for finite time Graphical methods**
  - **Interest in translation invariant dynamics**

Stability - Non Degeneracy

- **Tightness**
- **Convergence of the dynamics as time goes to infinity**
- **Translation invariant and time-stationary solutions**
- **Phase Diagram**

Analysis of Steady State

- **Rate Conservation Principle**
- **Higher order Moment Measure Hierarchy**
- **Multiple Stationary Regimes**
- **Tail Behavior**

Computational Heuristics

- **Factorization of third moment measures**
- **Mean Fields**

Lectures

1. **Wireless Birth and Death Processes and Interference Queuing Networks (2019)**
2. **Spatial Birth and Death Processes with Poisson Births and Shot-Noise Deaths (2017)**
3. **Contact Processes on Point Processes (2020-preprint)**