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A Stochastic Geometry Model for Cognitive Radio Networks

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We propose a probabilistic model based on stochastic geometry to analyze cognitive radio in a large wireless network with randomly located users sharing the medium with carrier sensing multiple access. Analytical results are derived on the impact of the interaction between primary and secondary users, on their medium access probability, coverage probability and throughput. These results can be seen as the continuation of the theory of priorities in queueing theory to spatial processes. They give insight on the guarantees which can be offered to primary users and more generally on the possibilities offered by cognitive radio to improve the effectiveness of spectrum utilization in such networks.

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1. INTRODUCTION

Bandwidth is well known to become more and more scarce with the explosion of wireless communications. Measurements nevertheless show that at any specific time and location, most of the spectrum is vastly underutilized [1]. This manifests itself through voids in either time, space or spectrum. Cognitive radio aims at exploiting these voids in order to accommodate extra radio devices (referred to as secondary users) in a network whose first function is to serve a population of primary users, e.g. licensed to use the spectrum. The key requirement is that the primary users ought to be as little affected as possible by the presence of secondary users.

There have been many efforts in implementing cognitive radio to exploit voids in time and spectrum, see [2], [3] and [4]. The present paper concentrates on exploiting voids in *space* by considering cognitive radio in a wireless network with users distributed in the Euclidean plane. The physical layer for cognitive radio networks has already been considered in e.g. [19] and [18]. Several Medium Access Control (MAC) layer protocols were proposed for this setting like Cognet [11], [12], AMAC [13] and C-CSMA/CA [10]. Other schemes meant to ensure the preeminence of primary transmissions are discussed in [15], [14]. If some first models were proposed in e.g. [3] and [20], it is fair to say that there is a need for a comprehensive mathematical framework for analyzing such cognitive radio networks. Our main contribution is a new probabilistic framework based on stochastic geometry [8], [7] for analyzing the performance of MAC protocols within this context.

The protocols we focus on in the present paper are those using Carrier Sensing as a mechanism for Secondary Users (SUs) to exploit the spectrum left over by Primary Users (PUs). This can be explained in simple geometric terms as follows: each primary transmitter has a protection zone. If a user located in this zone transmits at the same time, a collision occurs. Any user located in this protection zone is hence called a contender of this transmitter. In most cases, the union of all primary transmitter protection zones does not cover the whole space. Thus, one can accommodate secondary users in the remaining space to better utilize the spatial resource.

Using the stochastic geometry framework alluded to above, a quantification of this interaction between the two classes of users is obtained. In fact, this approach leads to closed form analytical expressions for the main performance metrics of interest here, like the Medium Access Probability (MAP), the Coverage Probability (COP) and the density of throughput (defined below). The formulas in question are reminiscent of those in priority queues [22], although the scheduling is here for space/spectrum rather than for time/CPU as in the queuing context. The difference here is that, due to the pervasive nature of radio transmission, the presence of SUs is not anymore transparent to PUs as e.g. in preemptive priority queuing. In fact, if the number of nearby SUs is large enough, PUs can still suffer from non-negligible degradation. The models discussed in the present paper aim at quantifying this degradation and suggesting policies which can be used to limit it. The main interest of the derived closed form formulas is that they allow one to quantify and hence control the degradation incurred by primary users due to the presence of secondary ones as well as to evaluate the rate obtained by secondary users.

In particular, we will consider the following three models:

- Single primary user: This is the simplest model considered in this paper. It focuses on a particular primary transmitter-receiver pair and can hence be used as a basic building block for any cognitive radio network. It is also quite well studied in the literature. We consider this simple model first for the sake of clear exposition and to prepare readers who are not acquainted with stochastic geometry to more involved models.

  The model features a single primary transmitter located at the center of the plane, together with a population of secondary users. Each user in this network has an intended receiver. To protect the primary transmission, the SUs are only allowed to transmit when they are not too close to the primary receiver. This can be guaranteed by Carrier Sensing (CS): The primary receiver uses a beacon to announce its presence to the SUs. Whenever an SU wants to transmit, it should first sense the channel in order to detect the primary signals. If the received primary signal is weak enough, the SU then assumes that it is far from the primary receiver and that it can hence transmit.

- Multicast primary user: This model is one step closer to certain practical cognitive radio networks such as a TV network or a cellular network.

  It features a single primary transmitter at the center of the plane, which has multiple receivers. In a TV network, one can think of the primary transmitter and its receivers as the TV station and the TV receiver antennas respectively. In cellular networks, the former is the cell base station and the latter are end-user devices respectively. There is also a population of secondary wireless devices which try to use the unused spectrum. As in the single primary user model, we use CS to protect the transmissions of PUs. To this end, we consider two cases: The first one is the *passive mode*, where the primary receivers have no beaconing functionality to announce their presence (think of TV antennas for example). In this case, the SUs have to rely on the signal from the primary transmitter to do the carrier sensing. If an SU senses the signal from the primary transmitter to it very weak, then it can assume that it is far away compared to the transmission range of the primary transmitter. Hence, it is allowed to transmit. The second case is the *active mode*, where the primary receivers can transmit beacons. An example is provided by cellular networks, where the uplink channels from the end-user devices to the base station can be used to transmit beacons. The active mode is expected to be much more efficient than the passive one.

- Cognitive-CSMA wireless networks: This model focuses on the scenario where we have a primary mobile ad-hoc network, together with a secondary one which tries to utilize the unused spectrum. Each user in each network has an intended receiver. CS is used to guarantee that no SU generates too much interference to *any* primary receiver.

  Furthermore, CS is also used in both (primary and secondary) networks to control the level of interference in each class. This results in the cognitive-CSMA protocol, which is based on the following principles: Whenever a user in a network (either primary or secondary) wants to transmit, it has to use CS to detect the presence of other users in the network. If this user senses no other user nearby, it is then allowed to transmit. Otherwise, it has to wait before transmitting.

  As in [7], this cognitive-CSMA protocol is modeled by a Matérn type model, which is defined as follows. Let us start with the primary network: Each PU samples an independently and identically distributed (i.i.d.) random variable (r.v.), which is uniformly distributed in $[0,1]$. These r.v.s will be referred to as timers below. Two users are said to contend with each other if either of the two senses the presence of the other. In this case, they are not allowed to transmit at the same time; otherwise they would interfere and this would corrupt the reception at each receiver. This exclusion rule is implemented as follows: a tagged primary user is allowed to transmit by the CSMA protocol if and only if (iff) it has the smallest timer amongst its contenders. For the secondary network, in order to respect the primary transmissions, a SU is forbidden to transmit (or blocked) whenever it senses the presence of a PU. The SUs that sense the presence of no PU then compete to access the medium; this is done as above, namely using random timers and CS.

  In this setting, we also consider both the *active mode* and the *passive mode*. In the *active mode*, the receivers can transmit beacons so that the CS can be done more efficiently. This can for example be implemented by the Request To Send-Clear To Send (RTS-CTS) handshaking technique: Whenever a user wants to transmit, it sends an RTS message to its receiver. Upon receiving this message, the receiver senses other ongoing transmissions in the network. If it senses the medium free, it replies a CTS message to the transmitter and the transmission takes place. The *passive mode* is used when the receivers are not in a position to contribute to the RTS-CTS hand shaking technique or to transmit beacons. In this case, the CS is carried out by making the transmitter listen to the signal of the other transmitters.

For all the models described above, even with the CS based protection, the aggregated interference from *all* the SUs can still have non-negligible impact on the primary performance. Mathematical tools from stochastic geometry are then employed to quantify this *secondary to primary interference* and its impact on the performance of primary users.

The paper is organized as follows. Section 2 introduces the stochastic geometry framework by analyzing the single primary user model. Section 3 extends this framework to the multicast primary user model. Section 4 focuses on the cognitive-CSMA model. Section 5 gathers our conclusions.

Throughout the papper, for all definitions pertaining to point processes, like planar Poisson point processes (PPP), Palm distributions, Slivnyak's theorem, or to stochastic geometry, like shot noise fields, Matérn point processes, the readers could refer to [8] or [7].

2. SINGLE PRIMARY USER

In this section we consider a cognitive radio network model with a single primary user and a population of secondary users. This models a network where the primary population is very sparse. Nevertheless, the presence of secondary users still causes interference to the primary user due to the pervasive nature of radio communication. The goal of this section is to quantify this 'secondary to primary' interference and its impact on the performance of the primary user.
In this model, the secondary users employ an ALOHA based MAC protocol within their class. Each secondary user independently tosses a coin. If the result is head, then this user senses the medium to check whether there is any primary contender. The tagged secondary user transmits only if it sees the network free of primary contenders. The bias of these coins is a pre-set parameter of the protocol.

2.1. User Model

The model features a primary user which is a transmitter-receiver pair T_0^I, R_0^I and a population of secondary users (transmitter-receiver pairs) $\{T_i^{II}, R_i^{II}\}$. By abuse of notation we will use also this notation for the positions of transmitters and receivers. We assume:

- T_0^I is at the center of the plane.
- $|R_0^I| = R$.
- θ_0^I is the angular position of R_0^I.
- The set of secondary transmitters who access the network is here determined by the ALOHA rule, i.e. it is the subset of transmitters tossing a head.
- The point process of secondary transmitters $\Phi^{II} = \{T_i^{II}\}$ is assumed to form a realization of a homogeneous PPP of intensity λ^{II}. Below λ^{II} will denote the intensity of active secondary transmitters; namely if one denotes by p the bias of the coin used by secondary transmitters and by ν^{II} the intensity of the total population of secondary transmitters, then $\lambda^{II} = p\nu^{II}$.
- Each secondary receiver is assumed to be uniformly distributed on a circle of radius r centered at its transmitter: $R_i^{II} = T_i^{II} + rl(\theta_i^{II})$, where θ_i^{II} is uniformly, identically and independently distributed (i.i.d) in $[0, 2\pi]$, and $l(\theta) = (cos(\theta), sin(\theta))$.
- $F_{0,i}^{I-II}$ ($F_{i,0}^{II-I}$, $F_{i,j}^{II-II}$ and $F_{0,0}^{I-I}$ resp.) denotes the fading of the channel from T_0^I to R_i^{II} (T_i^{II} to R_0^I, T_i^{II} to R_j^{II} and from T_0^I to R_0^I resp.). All the fading variables are assumed to be i.i.d. and to follow the cumulative distribution function (c.d.f.) $G(.) = \mathbb{P}(F < .)$. We assume Rayleigh fading, i.e. $G(x) = 1 - e^{-\mu x}$.

2.2. Retain and transmission model

We assume a deterministic threshold, so that the indicator variable that secondary user i belongs to the primary user's protection zone is

$$U_i = \mathbf{1}_{F_{i,0}^{II-I}|T_i^{II} - R_0^I|^{-\alpha} > \rho},$$

where ρ is the pre-set threshold.
Interference is treated as noise and the transmission is successful if the SINR is larger than a pre-set constant T. The SINRs of the primary user and secondary users are defined as:

$$\mathrm{SINR}_0^I = \frac{F_{0,0}^{I-I}/R^\alpha}{W(R_0^I) + I_{\Phi_M^{II}}(R_0^I)}$$

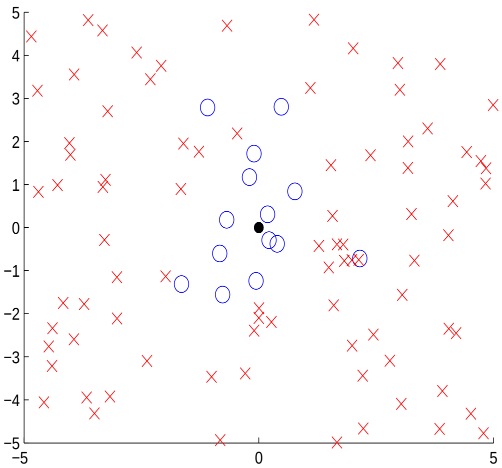
$$\mathrm{SINR}_i^{II} = \frac{F_{i,i}^{II-II}/r^\alpha}{W(R_i^{II}) + F_{0,i}^{I-II}/|R_i^{II}|^\alpha + I_{\Phi^{II} \setminus T_i^{II}}(R_i^{II})},$$

where W is the power of the thermal noise, $\Phi_M^{II} = \{T_j^{II} \text{ s.t } U_j = 1\}$ and $I_\Xi(x)$ is the Shot-Noise associated with the point process Ξ at point x, defined by:

$$I_\Xi(x) = \sum_{X_i \in \Xi} f(|X_i - x|),$$

with f some response function. Particular cases of interest are:

$$\begin{aligned} I_{\Phi_M^{II}}(R_0^I) &= \sum_{j \text{ s.t. } U_j = 1} F_{j,i}^{II-I}/|T_j^{II} - R_i^I|^\alpha \\ &= \sum_j U_j F_{j,0}^{II-I}/|T_j^{II} - R_0^I|^\alpha \\ I_{\Phi^{II} \setminus T_i^{II}}(R_i^{II}) &= \sum_{j \neq i \text{ s.t. } U_j = 1} F_{j,i}^{II-I}/|T_j^{II} - R_i^{II}|^\alpha \\ &= \sum_{j \neq i} U_j F_{j,i}^{II-I}/|T_j^{II} - R_i^{II}|^\alpha. \end{aligned}$$



The path loss exponent is $\alpha = 3$; the fading random variables are exponential with parameter $\mu = 10$; the retained secondary users are red crosses while the blocked ones are blue circles.

Note that the Shot-noise interference can be defined with any p.p. Ξ. In the special case where Ξ is Poisson, we ave the following useful result, the proof of which can be found in [7] or [8]:

THEOREM 2.1. *Suppose that $\Xi = \{y_i\}$ is a PPP with intensity measure $m(x)dx$. For all functions v from $\mathbb{R}^2$ to $[0,1]$ satisfying $\int_{\mathbb{R}^2}(1-v(x))m(x)dx < \infty$, we have:*

$$\boldsymbol{E}\left[\prod_i v(y_i)\right] = \exp\left\{-\int_{\mathbb{R}^2}(1-v(x))m(x)dx\right\}. \quad (1)$$

2.3. Performance analysis

In this model, the primary user always accesses the channel. Thus we are only interested in the MAP of a secondary user, which is $\mathbf{P}(U_i = 1)$. Furthermore, because of interference, not every transmission attempt is successful. We will hence consider the COP, which is defined as the probability that the SINR of the tagged user is larger than T. We will also consider a more global performance metric, namely the *total throughput* (TT), which is defined as the mean number of successful transmissions in the network per time slot.
The following propositions gather analytical results on the above metrics for this model.

PROPOSITION 1. *Given the position of the primary receiver, and that of the i^{th} secondary user, the MAP of the latter is:*

$$1 - \exp\{-\mu\rho|T_i^{II} - R_0^I|^\alpha\}. \quad (2)$$

Proof.
We have:

$$\begin{aligned}\mathbf{P}(U_i = 1) &= \mathbf{P}\left(\frac{F_{0,i}^{I-II}}{|T_i^{II} - R_0^I|^\alpha} < \rho\right)\\ &= \mathbf{P}(F_{0,i}^{I-II} < \rho|T_i^{II} - R_0^I|^\alpha)\\ &= 1 - \exp\{-\mu\rho|T_i^{II} - R_0^I|^\alpha\}.\end{aligned}$$

□

Since each SU senses the network independently, the SUs who gain access to the medium forms an independent thinning of the process of all SUs with a thinning probability which only depends on the location of the SU. As the latter is an homogeneous PPP of intensity λ^{II}, the former is an inhomogeneous PPP of intensity measure $\lambda^{II}\left(1 - \exp\{-\mu\rho|x - R_0^I|^\alpha\}\right)dx$.

PROPOSITION 2. *Given the location of the primary receiver, the COP of the primary transmission is:*

$$p_{\mathrm{COP}}^I = \mathcal{L}_W(\mu T R^\alpha)\exp\left\{-\lambda^{II}\int_{\mathbb{R}^2} g(x,R)dx\right\}, \quad (3)$$

with

$$g(x,R) = 1 - e^{-\mu\rho|x|^\alpha} - \frac{|x|^\alpha(1 - e^{-\mu\rho(TR^\alpha + |x|^\alpha)})}{TR^\alpha + |x|^\alpha}. \quad (4)$$

and with $\mathcal{L}_W(s)$ the Laplace transform of the thermal noise W.

Proof.
We want to compute the following probability:

$$\mathbf{P}\left(\mathrm{SINR}_0^I > T\right) = \mathbf{P}\left(\frac{F_{0,0}^{I-I}/R^\alpha}{W(R_0^I) + I_{\Phi_M^{II}}(R_0^I)} > T\right).$$

Using the fact that $F_{0,0}^{I-I}$ is an exponential r.v. with parameter μ and $W(.)$ is independent of everything, we have:

$$\begin{aligned}\mathbf{P}\left(\mathrm{SINR}_0^I > T\right) &= \mathbf{E}\left[e^{-\mu T R^\alpha (W(R_0^I) + I_{\Phi_M^{II}}(R_0^I))}\right]\\ &= \mathbf{E}\left[e^{-\mu T R^\alpha W(R_0^I)}\right]\mathbf{E}\left[e^{-\mu T R^\alpha I_{\Phi_M^{II}}(R_0^I)}\right]\\ &= \mathcal{L}_W(\mu T R^\alpha)\mathbf{E}\left[e^{-\mu T R^\alpha I_{\Phi_M^{II}}(R_0^I)}\right].\end{aligned}$$

For the second term in the last equality, we have:

$$\mathbf{E}\left[e^{-\mu T R^\alpha I_{\Phi_M^{II}}(R_0^I)}\right] = \mathbf{E}\left[e^{-\mu T R^\alpha \sum_j U^j F_{j,0}^{II-I}/|T_j^{II} - R_0^I|^\alpha}\right]$$

$$= \mathbf{E}\left[\prod_j \mathbf{E}\left[e^{-\mu T R^\alpha U^j F_{j,0}^{II-I}/|T_j^{II} - R_0^I|^\alpha} \mid \mathcal{G}\right]\right],$$

where $\mathcal{G}$ is the sigma algebra endowed by the positions of secondary users and where the external expectation is conditional on the positions of the primary receiver. Simple calculations give:

$$\mathbf{E}\left[e^{-\mu T R^\alpha U^j F_{j,0}^{II-I}/|T_j^{II}-R_0^I|^\alpha} \mid \mathcal{G}\right]$$
$$= \mathbf{E}\left[e^{-\mu T R^\alpha \mathbf{1}_{F_{j,0}^{II-I}<\rho|T_j^{II}-R_0^I|^\alpha} F_{j,0}^{II-I}/|T_j^{II}-R_0^I|^\alpha} \mid \mathcal{G}\right]$$
$$= 1 - g(T_j^{II} - R_0^I, R).$$

Then, using Theorem 2.1 we have:

$$\mathbf{E}\left[e^{-\mu T R^\alpha I_{\Phi_M^{II}}(R_0^I)}\right] = \exp\left\{-\lambda^{II}\int_{\mathbb{R}^2} g(x - R_0^I, R)dx\right\}.$$

Applying a change variable from x to $x + R_0^I$ gives us the desired result.

□

Proposition 3. *Given the position of the primary receiver, the COP of a secondary transmitter at position y is:*

$$p_{\mathrm{COP}}^{II}(y,\lambda^{II}) = \frac{\mathcal{L}_W(\mu T r^\alpha)}{2\pi}\int_0^{2\pi}\frac{|y+rl(\theta)|^\alpha}{|y+rl(\theta)|^\alpha + Tr^\alpha} e^{-\lambda^{II}\int_{\mathbb{R}^2}\frac{Tr^\alpha(1-e^{-\mu\rho|x-R_0^I|^\alpha})}{Tr^\alpha+|x-y-rl(\theta)|^\alpha}dx}d\theta. \quad (5)$$

Proof.
Let $\mathbf{P}_{y,z}$ denote the Palm distribution of the secondary transmitters at y and its receiver at z, which can be interpreted as the conditional probability of the model conditioned to having a tagged secondary transmitter at y and its receiver at z. Without loss of generality (w.l.o.g.), we can assume that this tagged user is numbered 1. We have:

$$\mathbf{P}_{y,z}(\mathrm{SINR}_1^{II} > T)$$
$$= \mathbf{P}_{y,z}\left[F_{1,1}^{II-II} > Tr^\alpha\left(W(z) + F_{0,1}^{I-II}/|z|^\alpha + I_{\Phi^{II}\setminus y}(z)\right)\right]$$
$$= \mathbf{E}_{y,z}\left[\exp\left\{-\mu Tr^\alpha\left(W(z) + F_{0,1}^{I-II}/|z|^\alpha + I_{\Phi^{II}\setminus y}(z)\right)\right\}\right]$$
$$= \mathbf{E}_{y,z}[e^{-\mu Tr^\alpha W(z)}]\mathbf{E}_{y,z}[e^{-\mu Tr^\alpha F_{0,1}^{I-II}/|z|^\alpha}] \mathbf{E}_{y,z}[e^{-\mu Tr^\alpha I_{\Phi^{II}\setminus y}(z)}].$$

The first term of the last product is:

$$\mathbf{E}_{y,z}[e^{-\mu Tr^\alpha W(z)}] = \mathcal{L}_W(\mu Tr^\alpha).$$

For the second term, using the fact that $F_{0,1}^{I-II}$ is an exponential r.v. with parameter μ we have::

$$\mathbf{E}_{y,z}[e^{-\mu Tr^\alpha F_{0,1}^{I-II}/|z|^\alpha}]$$
$$= \mathcal{L}_{F_{0,1}^{I-II}}\left(\mu\frac{Tr^\alpha}{|z|^\alpha}\right) = \frac{|z|^\alpha}{|z|^\alpha + Tr^\alpha}.$$

For the last term:

$$\mathbf{E}_{y,z}[e^{-\mu Tr^\alpha I_{\Phi^{II}\setminus y}(z)}]$$
$$= \mathbf{E}_{y,z}\left[\prod_{i\neq 1}\mathbf{E}\left[e^{-\mu Tr^\alpha\frac{F_{i,1}^{II-II}}{|T_i^{II}-z|^\alpha}U_i^{II}} \mid \mathcal{H}\right]\right],$$

where $\mathcal{H}$ is the sigma algebra endowed by the secondary positions. Using Proposition 1, the term inside of the product can be computed as:

$$\mathbf{E}\left[e^{-\mu Tr^\alpha\frac{F_{i,1}^{II-II}}{|T_j^{II}-z|^\alpha}U_i^{II}} \mid \mathcal{H}\right]$$
$$= 1 - \mathbf{E}[U_i^{II}] + \mathbf{E}[U_i^{II}]\frac{|T_i^{II}-z|^\alpha}{|T_i^{II}-z|^\alpha + Tr^\alpha}$$
$$= 1 - (1 - e^{-\mu\rho|T_i^{II}-R_0^I|^\alpha})\frac{Tr^\alpha}{|T_i^{II}-z|^\alpha + Tr^\alpha}.$$

Then using Theorem 2.1, we have:

$$\mathbf{E}_{y,z}\left[e^{-\mu Tr^\alpha I_{\Phi^{II}\setminus y}(z)}\right] = e^{-\lambda^{II}\int_{\mathbb{R}^2}\frac{Tr^\alpha(1-e^{-\mu\rho|x-R_0^I|^\alpha})}{Tr^\alpha+|x-z|^\alpha}dx}.$$

Putting these terms together and using the fact that z is uniformly distributed in a circle of radius r centered at y gives us the desired result.

□

It is easy to see that the total throughput of the secondary users on the whole plane is infinite. Thus we consider only the secondary users in a region C which is a disk of radius $R_{max} \gg \max\{R, r\}$ centered at T_0^I. The intuition behind this choice of C is that, for secondary users outside C, the interactions with the primary user are negligible, so that these secondary users behave like in a network without primary users.

Proposition 4. *Given the position of the primary receiver, the TT of secondary users within C is:*

$$S^{II}(\lambda^{II}) = \lambda^{II}\int_C p_{\mathrm{COP}}^{II}(y,\lambda^{II})(1 - e^{-\mu\rho|y-R_0^I|^\alpha})dy. \quad (6)$$

Proof.
This is just a corollary of Campbell's formula (see [8] or [7]) and the fact that U_i and SINR_i^{II} are independent.

□

2.4. Cognitive radio guarantees

In this subsection we discuss the policies that secondary users have to comply with in order to provide some guarantees to the primary user. From (3) we can see that $p_{\mathrm{COP}}^I(\lambda^{II})$ decreases exponentially fast to 0 as λ^{II} goes to ∞. Thus, for $1 > L > 0$, there exists a unique λ^{up} such that $p_{\mathrm{COP}}^I(\lambda^{up}) = L$. If one wishes to have a stochastic guarantee that the

COP of the primary user is at least L, then one ought to limit the density of secondary users below λ^{up}. Within this constraint, the secondary users seek to optimize their TT, i.e. maximize $S^{II}(\lambda^{II})$. Let $\lambda^{max} = \arg\max\{S^{II}(.)\}$; the optimal operation point in this context consists in setting the secondary intensity equal to $\lambda^* = \min\{\lambda^{up}, \lambda^{max}\}$. This can be done in a distributed way by requiring each secondary user to adjust its ALOHA coin tossing bias such that the intensity of secondary users accessing the network is λ^*.

3. MULTICAST PRIMARY USER

In this section we investigate a cognitive radio network model featuring a multicast primary user, i.e. a primary transmitter with a population of primary receivers, and a population of secondary users. As in the above model, the other primary base stations are assumed to be so far that the inter-cell interference is negligible. Thus, the only factor that has negative impact on the primary user performance is the 'secondary to primary' interference. As usual, SUs must use CS to guarantee that they do not cause excessive interference to PUs.

3.1. User model

We use the same notation for nodes and position of nodes as in the previous model. We assume:

- The primary transmitter T_0^I is at the center of the plane.
- The process of primary receivers $\{R_i^I\}$ forms a realization of a PPP of intensity $\lambda^I \mathbf{1}_C$, where C is the cell of the base station. We assume that C is a disk of radius R centered at T_0^I.
- The process of secondary transmitters $\Phi^{II} = \{T_i^{II}\}$ forms a realization of a PPP of intensity $\lambda^{II}\mathbf{1}_C$. Thus any secondary user outside C belongs to other cells and is not considered.
- The secondary receiver R_i^{II} is assumed to be uniformly distributed on the circle of radius r centered at T_i^{II}, i.e. $R_i^{II} = T_i^{II} + rl(\theta_i^{II})$, where $l(.)$ is defined as above and θ_i^{II} is i.i.d. in $[0, 2\pi]$.
- $F_{0,i}^{I-I}$ ($F_{0,i}^{I-II}$, $F_{i,j}^{II-I}$ and $F_{i,j}^{II-II}$ resp.) is the fading of the channel from T_0^I to R_i^I (from T_0^I to R_i^{II}, T_i^{II} to R_j^I and from T_i^{II} to R_j^{II}). The fading variables are i.i.d exponential of parameter μ.
- G_i is the fading from T_0^I to T_i^{II} which is assumed to be identically distributed to $F_{0,i}^{I-II}$ and independent of everything else in the system. This fading will be used for the CS in passive mode.

3.2. Retain and transmission model

In this context, a secondary user is allowed to transmit if it does not cause too much interference to *any* of the primary receivers. Namely, the retain indicator of the i^{th} secondary user is:

$$U_{p.i} = \mathbf{1}_{G_i/|T_i^{II} - T_0^I|^\alpha < \xi} \qquad \text{in passive mode,}$$

or

$$U_{a,i} = \prod_j \mathbf{1}_{F_{i,j}^{II-I}/|T_i^{II} - R_j^I|^\alpha < \rho} \qquad \text{in active mode,}$$

where ρ and ξ are pre-set parameters. As above, the transmission scheme treats interference as noise and a transmission is successful if the SINR is higher than T. The SINR is defined for primary and secondary receivers as:

- In passive mode:

$$\mathrm{SINR}_{p,i}^I = \frac{\frac{F_{0,i}^{I-I}}{|R_i^I|^\alpha}}{W(R_i^I) + I_{\Phi_p^M}(R_i^I)}$$

$$\mathrm{SINR}_{p,i}^{II} = \frac{\frac{F_{i,i}^{II-II}}{r^\alpha}}{W(R_i^{II}) + \frac{F_{0,i}^{I-II}}{|R_i^{II}|^\alpha} + I_{\Phi_p^M \setminus T_i^{II}}(R_i^{II})}.$$

- In active mode:

$$\mathrm{SINR}_{a,i}^I = \frac{\frac{F_{0,i}^{I-I}}{|R_i^I|^\alpha}}{W(R_i^I) + I_{\Phi_a^M}(R_i^I)}$$

$$\mathrm{SINR}_{a,i}^{II} = \frac{\frac{F_{i,i}^{II-II}}{r^\alpha}}{W(R_i^{II}) + \frac{F_{0,i}^{I-II}}{|R_i^{II}|^\alpha} + I_{\Phi_a^M \setminus T_i^{II}}(R_i^{II})}.$$

In the above formulas, $\Phi_p^M = \{T_i^{II} \text{ s.t. } U_{p,i} = 1\}$ and $\Phi_a^M = \{T_i^{II} \text{ s.t. } U_{a,i} = 1\}$ are the point processes of retained secondary transmitters in passive and active mode respectively. The Shot-noise fields representing interferences are defined as:

$$I_{\Phi_p^M}(R_i^I) = \sum_j U_{p,j} F_{j,i}^{II-I}/|T_j^{II} - R_i^I|^\alpha$$

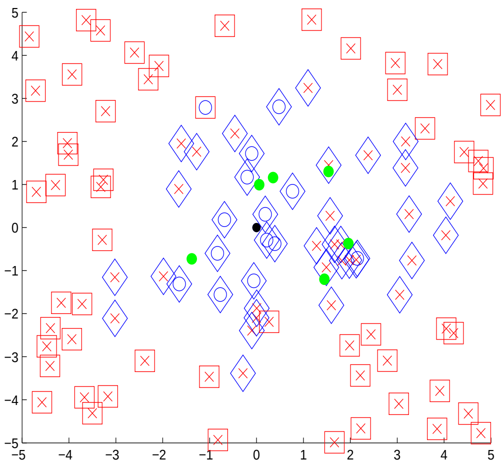
$$I_{\Phi_p^M \setminus T_i^{II}}(R_i^{II}) = \sum_{j \neq i} U_{p,j} F_{j,i}^{II-II}/|T_j^{II} - R_i^{II}|^\alpha.$$

$$I_{\Phi_a^M}(R_i^I) = \sum_j U_{a,j} F_{j,i}^{II-I}/|T_j^{II} - R_i^I|^\alpha$$

$$I_{\Phi_a^M \setminus T_i^{II}}(R_i^{II}) = \sum_{j \neq i} U_{a,j} F_{j,i}^{II-II}/|T_j^{II} - R_i^{II}|^\alpha.$$

3.3. Performance analysis

In this subsection, the performance metrics of interest are still the MAP, COP and TT, the definitions of which are provided in Section 2.



The primary receivers are the green dots. The red crosses are retained secondary users in active mode, blue circles are blocked ones in active mode. The red square are retained secondary users in passive mode, blue diamonds are blocked ones in passive mode. For the sake of comparison, we put the same sensing threshold for both passive and active mode, which is equal to 1.

3.3.1. Passive mode:

Proposition 5. *In passive mode, the MAP of a secondary transmitter at position y is:*

$$1 - \exp\{-\mu\xi|y|^\alpha\}.$$

The proof is similar to that of Proposition 1 and is hence omitted.

Proposition 6. *In passive mode, the COP of a primary receiver at position y is:*

$$p^I_{p,\mathrm{COP}}(\lambda^{II}, y) = \mathcal{L}_W(\mu T|y|^\alpha)$$
$$\exp\left\{-\lambda^{II}\int_{\mathbb{R}^2}\frac{T|y|^\alpha}{T|y|^\alpha + |x-y|^\alpha}(1-e^{-\mu\xi|x|^\alpha})dx\right\}.$$

Proof.
First using the formula for the SINR and denoting by $\mathbf{P}_{0,y}$ the Palm probability at 0 w.r.t. the primary transmitter and y w.r.t. the point process of primary receivers, we have:

$$\mathbf{P}_{0,y}\left(\frac{\frac{F^{I-I}_{0,1}}{|y|^\alpha}}{W(y) + I_{\Phi^M_p}(y)} > T\right)$$
$$= \mathbf{E}_{0,y}\left[e^{-\mu T|y|^\alpha(W(y)+I_{\Phi^M_p}(y))}\right]$$
$$= \mathbf{E}_{0,y}\left[e^{-\mu T|y|^\alpha W(y)}\right]\mathbf{E}_{0,y}\left[e^{-\mu T|y|^\alpha I_{\Phi^M_p}(y)}\right].$$

The first term is $\mathcal{L}_W(\mu T|y|^\alpha)$. For the second term:

$$\mathbf{E}_{0,y}\left[e^{-\mu T|y|^\alpha(I_{\Phi^M_p}(y))}\right]$$
$$= \mathbf{E}_{0,y}\left[\prod_j \mathbf{E}\left[e^{-\mu T|y|^\alpha U_{p,j}\frac{F^{II-I}_{j,1}}{|T^{II}_j - y|^\alpha}}\right] \mid \mathcal{F}\right],$$

with $\mathcal{F}$ the sigma algebra endowed with the positions of secondaries. Now, using the independence of $U^{II}_j, F^{II-I}_{j,1}$, we can compute the term inside the product as:

$$\mathbf{E}\left[e^{-\mu T|y|^\alpha U^{II}_j\frac{F^{II-I}_{j,i}}{|T^{II}_j - y|^\alpha}} \mid \mathcal{F}\right]$$
$$= 1 - (1 - e^{-\mu\xi|T^{II}_j|^\alpha})\frac{T|y|^\alpha}{T|y|^\alpha + |T^{II}_j - y|^\alpha}.$$

Then using Theorem 2.1 gives us the wanted result.

□

Proposition 7. *In passive mode, the COP of a secondary users at position y is:*

$$p^{II}_{p,\mathrm{COP}}(\lambda^{II}, y) = \mathcal{L}_W(\mu T|y|^\alpha)\int_0^{2\pi}\frac{|y+lr(\theta)|^\alpha}{Tr^\alpha + |y+lr(\theta)|^\alpha}$$
$$e^{-\lambda^{II}\int_{\mathbb{R}^2}\frac{Tr^\alpha}{T|y|^\alpha + |x-y-rl(\theta)|^\alpha}(1-e^{-\mu\xi|x|^\alpha})dx}d\theta.$$

Proof. Let $\mathbf{Q}_{0,y,z}$ denote the Palm probability at 0 w.r.t. the primary transmitter at y w.r.t. the secondary transmitter and at z w.r.t. its receiver (w.l.o.g. we assume that the transmitter-receiver pair at (y,z) is numbered 1. We proceed as in the proof of Proposition 6:

$$\mathbf{Q}_{0,,z}(\mathrm{SINR}^{II}_{p,1} > T)$$
$$= \mathbf{Q}_{0,y,z}\left(\frac{\frac{F^{II-II}_{1,1}}{r^\alpha}}{W(z) + \frac{F^{I-II}_{0,1}}{|z|^\alpha} + I_{\Phi^M_p\setminus y}(z)}\right)$$
$$= \mathbf{E}_{0,y,z}\left[e^{-\mu Tr^\alpha\left(W(z)+\frac{F^{I-II}_{0,1}}{|z|^\alpha}+I_{\Phi^M_p\setminus y}(z)\right)}\right]$$
$$= \mathcal{L}_W(\mu Tr^\alpha)$$
$$\mathcal{L}_{F^{I,II}_{0,1}}\left(\mu T\frac{r^\alpha}{|z|^\alpha}\right)\mathcal{L}_{I_{\Phi^M_p\setminus y}(z))}(\mu Tr^\alpha),$$

where $\mathcal{L}_V$ denotes the Laplace transform of the r.v. V. Since $F^{I-II}_{0,1}$ is exponential:

$$\mathcal{L}_{F^{I-II}_{0,1}}\left(\mu T\frac{r^\alpha}{|z|^\alpha}\right) = \frac{|z|^\alpha}{|z|^\alpha + Tr^\alpha}.$$

For the last term, using Theorem 2.1, we have:

$$\mathcal{L}_{I_{\Phi^M_p\setminus y}(z))}(\mu Tr^\alpha) = e^{-\lambda^{II}\int_{\mathbb{R}^2}\frac{Tr^\alpha}{T|y|^\alpha + |x-R^{II}_i|^\alpha}(1-e^{-\mu\xi|x|^\alpha})dx}.$$

Putting these together and using the fact that R^{II}_1 is uniformly distributed on the circle of radius r centered at y gives us the wanted result.

□

PROPOSITION 8. *Under the passive mode, the TT is*

$$S_p^I(\lambda^I,\lambda^{II}) = \lambda^I \int_C p_{p,\text{COP}}^I(y,\lambda^{II})dy, \qquad (7)$$

for primary users and

$$S_p^{II}(\lambda^I,\lambda^{II}) = \lambda^{II} \int_C p_{p,\text{COP}}^{II}(y,\lambda^{II})(1-e^{-\mu\xi|y|^\alpha})dy, \qquad (8)$$

for secondary users.

Proof. This proof is similar to that of Proposition 4.

□

3.3.2. Active mode:

PROPOSITION 9. *The MAP of a secondary transmitter at location y is:*

$$\exp\{-\lambda^I N(y)\}, \qquad (9)$$

where:

$$N(y) = \int_C \exp\{-\mu\rho|y-x|^\alpha\}dx. \qquad (10)$$

Proof. Denoting by $\mathbf{P}_{0,y}$ the Palm probability at 0 w.r.t. the primary transmitter and at y w.r.t. the secondary transmitter point process, we have:

$$\begin{aligned}
&\mathbf{P}_{0,y}(U_{a,1}=1) = \mathbf{E}_y[U_{a,1}] \\
&= \mathbf{E}_{0,y}\left[\prod_j (\mathbf{1}_{F_{1,j}^{II-I}/|y-R_j^I|^\alpha<\rho})\right] \\
&= \mathbf{E}\left[\prod_j (1-e^{-\mu\rho|y-R_j^I|^\alpha})\right] \\
&= \exp\left\{-\lambda^I \int_C e^{-\mu\rho|x-y|^\alpha}dx\right\} = \exp\{-\lambda^I N(y)\},
\end{aligned}$$

where we have used Theorem 2.1.

□

PROPOSITION 10. *The COP of a primary receiver at position y is:*

$$p_{a,\text{COP}}^I(y,\lambda^I,\lambda^{II}) = \mathcal{L}_W(\mu T|y|^\alpha) \exp\Big\{-\lambda^{II}\int_C g(x,y)e^{-\lambda^I N(x)}dx\Big\}. \qquad (11)$$

Proof.
This proof is the same as the proof of Proposition 6; we only have to replace $U_{p,j}$ by $U_{a,j}$.

□

PROPOSITION 11. *The COP for a secondary user at position y is:*

$$p_{a,\text{COP}}^{II}(y,\lambda^I,\lambda^{II}) = \frac{\mathcal{L}_W(\mu T r^\alpha)}{2\pi}\int_0^{2\pi} \frac{|y+lr(\theta)|^\alpha}{Tr^\alpha+|y+lr(\theta)|^\alpha} \exp\left\{-\lambda^{II}\int_C \frac{Tr^\alpha}{Tr^\alpha+|x-y-lr(\theta)|^\alpha}e^{-\lambda^I N(x)}dx\right\}d\theta. \qquad (12)$$

Proof. This proof is the same as that of Proposition 7; we only have to replace $U_{p,j}$ by $U_{a,j}$.

□

PROPOSITION 12. *Under active mode, the TT is*

$$S_a^I(\lambda^I,\lambda^{II}) = \lambda^I \int_C p_{a,\text{COP}}^I(y,\lambda^I,\lambda^{II})dy, \qquad (13)$$

for primary users and

$$S_a^{II}(\lambda^I,\lambda^{II}) = \lambda^{II} \int_C p_{a,\text{COP}}^{II}(y,\lambda^I,\lambda^{II})e^{-\lambda^I N(y)}dy, \qquad (14)$$

for secondary users.

Proof. This proof is similar to that of Proposition 4.

□

3.4. Cognitive radio guarantees

As in Subsection 2.4, we seek for an operation point that complies with performance guarantees for primary users and at the same time maximizes the performance of secondary users. In this section, we only consider the active mode; the principle is the same for the passive mode. Here, instead of considering the local COP of each primary receiver, we consider the global performance metric: the total throughput. From (13), we have two important remarks. First, the TT $S_a^I(\lambda^I,\lambda^{II})$ of primary users increases almost linearly in the primary receivers intensity λ^I. This comes from 2 reasons: increasing λ^I makes the MAPs of secondary users decrease exponentially fast as shown in (9) and thus decreases inter-class interference; increasing λ^I also increases the number of primary receivers and makes the TT increase almost linearly. The second important remark is that $S_a^I(\lambda^I,\lambda^{II})$ decreases exponentially fast to 0 as λ^{II} goes to ∞. This means that in spite of the protection zones, inter-class interference from an over-crowded area of secondary users can destroy any primary transmission. Thus, a limitation on the secondary users intensity, which is similar to that in Subsection 2.4, ought to be applied to stochastically guarantee an acceptable performance for primary users.

More precisely, for $L > 0$, there exists a unique λ^{up} such that $S_a^I(\lambda^I,\lambda^{up}) = L$. One wants to guarantee a minimum TT L for primary users; thus the secondary users intensity must be smaller than λ^{up}. Within this constraint, ones should tune the intensity of secondary users to maximize the secondary total throughput $S^{II}(\lambda^I, \lambda^{II})$. This can be enforced in an almost distributed way by requiring each secondary user to adjust the ALOHA coin bias similarly to that presented in Subsection 2.4. However, this time the scheme is only 'almost' distributed since the tuning of the coin bias requires that each secondary user knows the 'global' parameter λ^I .

4. COGNITIVE-CSMA

4.1. Probabilistic model

We now focus on a model featuring two mobile ad hoc networks (MANETs): a primary and a secondary one. MANETs are self-configuring wireless network such that each user in the network also acts as a relay to route the traffic of other users. Hence, there can be many users in the network which transmit at the same time and cause excessive interference. For this reason, each MANET in this model employs the CSMA MAC protocol, a brief description of which is provided in Section 1, to control the level of interference. To control secondary to primary interference, CS is employed in a way which is similar to that of the two previous models.

4.1.1. General model

The user locations in this network are assumed to be realizations of two independent marked PPPs $\tilde{\Phi}^I = \{T_i^I, t_i^I\}$ and $\tilde{\Phi}^{II} = \{T_i^{II}, t_i^{II}\}$ with intensity λ^I and λ^{II} respectively, on $\mathbb{R}^2$. The model also features infinite matrices $\mathbf{F}^{I-I}$, $\mathbf{F}^{I-II}$, $\mathbf{F}^{II-I}$, $\mathbf{F}^{II-II}$, $\mathbf{G}^{I-I}$, $\mathbf{G}^{II-II}$ and $\mathbf{G}^{II-I}$.

(i) $\Phi^I = \{T_i^I\}$ denotes the position of the PUs. Each user has an intended receiver R_i^I uniformly distributed in the circle of radius r centered at this user.

(ii) $\Phi^I = \{T_i^{II}\}$ denotes the position of the SUs. Each user has an intended receiver R_i^{II} uniformly distributed in the circle of radius r centered at this user.

(iii) $\{t_i^I\}$, $\{t_i^{II}\}$ are i.i.d. r.v.s representing the timers of primary and secondary users, which are used by the CSMA protocol. They are uniformly distributed in $[0, 1]$.

(iv) $\mathbf{F}^{I-II} = \{F_{i,j}^{I-II}\}$, $\mathbf{F}^{II-I} = \{F_{i,j}^{II-I}\}$, $\mathbf{F}^{I-I} = \{F_{i,j}^{I-I}\}$, $\mathbf{F}^{II-II} = \{F_{i,j}^{II-II}\}$: $F_{i,j}^{I-II}$ ($F_{i,j}^{II-I}$, $F_{i,j}^{I-I}$, $F_{i,j}^{II-II}$) is the fading of the channel from transmitter T_i^I to receiver R_j^{II} (from transmitter T_i^{II} to receiver R_j^I, from transmitter T_i^I to receiver R_j^I, from transmitter T_i^{II} to receiver R_j^{II} respectively).

(v) $\mathbf{G}^{I-I} = \{G_{i,j}^{I-I}\}$, $\mathbf{G}^{II-II} = \{G_{i,j}^{II-II}\}$, $\mathbf{G}^{I-II} = \{G_{i,j}^{I-II}\}$: $G_{i,j}^{I-I}$ ($G_{i,j}^{II-II}$, $G_{i,j}^{I-II}$) is the fading of the channel from transmitter T_i^I to transmitter T_j^I (from transmitter T_i^{II} to transmitter T_j^{II}, from transmitter T_i^I to transmitter T_j^{II} respectively). These fading variables are used in passive mode.

4.1.2. Retaining model

Passive mode:

We first consider the passive mode, where the receivers cannot send beacons to announce their presence. In this case, the condition that a transmitter is allowed to transmit is encoded in the following retain indicators:

(i) For a primary transmitter T_i^I:

$$U_{p,i}^I = \prod_{j \neq i} \left(\mathbf{1}_{t_i^I \leq t_j^I} + \mathbf{1}_{t_i^I > t_j^I} \mathbf{1}_{\frac{G_{i,j}^{I-I}}{|T_i^I - T_j^I|^\alpha} < \rho} \right). \quad (15)$$

(ii) For a secondary transmitter T_i^{II}:

$$U_{p,i}^{II} = \left(\prod_j \mathbf{1}_{\frac{G_{j,i}^{I-II}}{|T_j^I - T_i^{II}|^\alpha} < \rho} \right) \prod_{j \neq i} \left(\mathbf{1}_{t_i^{II} \leq t_j^{II}} + \mathbf{1}_{t_i^{II} > t_j^{II}} \mathbf{1}_{\frac{G_{j,i}^{II-II}}{|T_j^{II} - T_i^{II}|^\alpha} < \rho} \right). \quad (16)$$

In the above formulas, ρ is a preset parameter determining how *sensitive* the CS should be. In (15), the first term inside the product corresponds to the case where the timer of T_i^I is smaller than that of T_j^I. The second term corresponds to the case where the timer of T_i^I is larger than that of T_j^I, but the signal from T_j^I sensed by T_i^I is smaller than the threshold ρ. For (16), the first product corresponds to the condition that a secondary user must not sense the presence of any primary user. The explanation for the second product is the same as that of (15).

Active mode:

In active mode, the receivers can send beacons and therefore the CS can be carried out more efficiently. The retain indicators in this case are:

(i) For a primary transmitter T_i^I:

$$U_{p,i}^I = \prod_{j \neq i} \left(\mathbf{1}_{t_i^I \leq t_j^I} + \mathbf{1}_{t_i^I > t_j^I} \mathbf{1}_{\frac{F_{i,j}^{I-I}}{|T_i^I - R_j^I|^\alpha} < \rho} \mathbf{1}_{\frac{F_{j,i}^{I-I}}{|T_j^I - R_i^I|^\alpha} < \rho} \right). \quad (17)$$

(ii) For a secondary transmitter T_i^{II}:

$$U_{p,i}^{II} = \left(\prod_j \mathbf{1}_{\frac{F_{i,j}^{II-I}}{|T_i^{II} - R_j^I|^\alpha} < \rho} \mathbf{1}_{\frac{F_{j,i}^{I-II}}{|T_j^I - R_i^{II}|^\alpha} < \rho} \right) \prod_{j \neq i} \left(\mathbf{1}_{t_i^{II} \leq t_j^{II}} + \mathbf{1}_{t_i^{II} > t_j^{II}} \mathbf{1}_{\frac{F_{i,j}^{II-II}}{|T_i^{II} - R_j^{II}|^\alpha} < \rho} \mathbf{1}_{\frac{F_{j,i}^{II-II}}{|T_j^{II} - R_i^{II}|^\alpha} < \rho} \right). \quad (18)$$

The explanations of these formulas are the same as in the passive mode, except for the sensed signal. Take for example (17); the second term inside the product says that whenever the timer of T_i^I is larger than that of T_j^I, the signal from T_i^I sensed by R_j^I and the signal from T_j^I sensed by R_i^I should not exceed ρ. The other formula can be explained in the same way.

4.1.3. Transmission model

As in the previous models, transmission is successful iff its SINR is larger than a preset threshold T. The SINR of each user is:

(i) For primary user T_i^I in passive mode:

$$\mathrm{SINR}_{p,i}^I = \frac{F_{i,i}^{I-I}/|T_i^I - R_i^I|^\alpha}{W(R_i^I) + \Gamma_p^I(R_i^I) + \Gamma_p^{II}(R_i^I)}.$$

(ii) For secondary user T_i^{II} in passive mode:

$$\mathrm{SINR}_{p,i}^{II} = \frac{F_{i,i}^{II-II}/|T_i^{II} - R_i^{II}|^\alpha}{W(R_i^{II}) + \Gamma_p^I(R_i^{II}) + \Gamma_p^{II}(R_i^{II})}.$$

(iii) For primary user T_i^I in active mode:

$$\mathrm{SINR}_{a,i}^I = \frac{F_{i,i}^{I-I}/|T_i^I - R_i^I|^\alpha}{W(R_i^I) + \Gamma_a^I(R_i^I) + \Gamma_a^{II}(R_i^I)}.$$

(iv) For secondary user T_i^{II} in active mode:

$$\mathrm{SINR}_{a,i}^{II} = \frac{F_{i,i}^{II-II}/|T_i^{II} - R_i^{II}|^\alpha}{W(R_i^{II}) + \Gamma_a^I(R_i^{II}) + \Gamma_a^{II}(R_i^{II})}.$$

Here, $W(.)$ is the thermal noise process, which is assumed to be independent of all other elements of the model. $\Gamma_p^I(.)$, $\Gamma_a^I(.)$, $\Gamma_p^{II}(.)$, $\Gamma_a^{II}(.)$ are the aggregated interference from primary users in active mode, from primary users in passive mode, from secondary users in active mode, from secondary users in passive mode respectively. These interferences can be expressed as:

$$\Gamma_p^I(R_i^I) = \sum_{j\neq i} U_{p,j}^I F_{j,i}^{I-I}/|T_j^I - R_i^I|^\alpha,$$
$$\Gamma_p^I(R_i^{II}) = \sum_{j} U_{p,j}^I F_{j,i}^{I-II}/|T_j^I - R_i^{II}|^\alpha,$$
$$\Gamma_p^{II}(R_i^I) = \sum_{j} U_{p,j}^{II} F_{j,i}^{II-I}/|T_j^{II} - R_i^I|^\alpha,$$
$$\Gamma_p^{II}(R_i^{II}) = \sum_{j\neq i} U_{p,j}^{II} F_{j,i}^{II-II}/|T_j^{II} - R_i^{II}|^\alpha,$$
$$\Gamma_a^I(R_i^I) = \sum_{j\neq i} U_{a,j}^I F_{j,i}^{I-I}/|T_j^I - R_i^I|^\alpha,$$
$$\Gamma_a^I(R_i^{II}) = \sum_{j} U_{a,j}^I F_{j,i}^{I-II}/|T_j^I - R_i^{II}|^\alpha,$$
$$\Gamma_a^{II}(R_i^I) = \sum_{j} U_{a,j}^{II} F_{j,i}^{II-I}/|T_j^{II} - R_i^I|^\alpha,$$
$$\Gamma_a^{II}(R_i^{II}) = \sum_{j\neq i} U_{a,j}^{II} F_{j,i}^{II-II}/|T_j^{II} - R_i^{II}|^\alpha.$$

4.2. Performance analysis

The aim of this section is again to investigate the MAP, the COP and the TT. We begin with the *passive mode*.

4.2.1. Passive mode

MAP

In Aloha-like protocols, the MAP is a preset constant. In Cognitive-CSMA, this probability is not given a priori and has to be determined. For a typical primary and secondary users, these probabilities in passive mode are

$$p_{p,\mathrm{MAP}}^I(\lambda^I, \lambda^{II}) = \mathrm{P}[U_{p,i}^I = 1]$$
$$p_{p,\mathrm{MAP}}^{II}(\lambda^I, \lambda^{II}) = \mathrm{P}[U_{p,i}^{II} = 1],$$

respectively. The independently marked homogeneous PPP is both stationary and ergodic. Hence this probability is also the proportion of primary and secondary users in the network who gain access to the medium. If there is enough mobility, this is also the proportion of time, namely the frequency, with which each given user accesses the channel (see the high mobility discussion in [7]). As stated before, a user is granted access to the carrier iff it has the smallest timer among its contenders.

Proposition 13. *The MAP for a typical user is*

$$p_{p,\mathrm{MAP}}^I(\lambda^I, \lambda^{II}) = \frac{1 - e^{-\lambda^I \overline{N}_0}}{\lambda^I \overline{N}_0}, \tag{19}$$

for primary users and

$$p_{p,\mathrm{MAP}}^{II}(\lambda^I, \lambda^{II}) = \frac{1 - e^{-\lambda^{II} \overline{N}_0}}{\lambda^{II} \overline{N}_0} e^{-\lambda^I \overline{N}_0}. \tag{20}$$

for secondary users, with

$$\overline{N}_0 = \int_{\mathbb{R}^2} e^{-\mu\rho|x|^\alpha} dx. \tag{21}$$

Proof.

W.l.o.g. we can assume that the typical user is located at the origin and that the point process of the other users forms a Poisson point process process (Slivnyak's Theorem). Consider first the case of a primary user T_0^I. Then, under the Palm distribution

$\mathbf{P}_0$,

$$\begin{aligned}
\mathbf{P}_0(U_{p,0}^I = 1) &= \mathbf{E}_0\left[U_{p,0}^I\right] \\
&= \mathbf{E}_0\left[\prod_{j\neq 0}\left(\mathbf{1}_{t_0^I \le t_j^I} + \mathbf{1}_{t_0^I > t_j^I}\mathbf{1}_{\frac{G_{0,j}^{I-I}}{|T_0^I - T_j^I|^\alpha} < \rho}\right)\right] \\
&= \int_0^1 \mathbf{E}_0\left[\prod_{j\neq 0}\left(\mathbf{1}_{t\le t_j^I} + \mathbf{1}_{t > t_j^I}\mathbf{1}_{G_{0,j}^{I-I} < \rho|T_j^I|^\alpha}\right)\right] dt \\
&= \int_0^1 \mathbf{E}_0\left[\prod_{j\neq 0}\mathbf{E}\left[\mathbf{1}_{t\le t_j^I} + \mathbf{1}_{t > t_j^I}\mathbf{1}_{G_{0,j}^{I-I} < \rho|T_j^I|^\alpha}\right]\right] dt \\
&= \int_0^1 \mathbf{E}_0\left[(1-t) + t(1 - e^{-\mu\rho|T_j^1|^\alpha})\right] dt \\
&= \int_0^1 e^{-\lambda^I \int_{\mathbb{R}^2}\left(1 - \left((1-t) + t(1-e^{-\mu\rho|x|^\alpha})\right)\right)dx} dt \\
&= \int_0^1 e^{-t\lambda^I \int_{\mathbb{R}^2} e^{-\mu\rho|x|^\alpha}dx} dt \\
&= \frac{1 - \exp\{-\lambda^I \overline{N_0}\}}{\lambda^I \overline{N_0}}. \qquad (22)
\end{aligned}$$

For the case of a secondary user T_0^{II}, doing similarly, we get:

$$\begin{aligned}
&\mathbf{P}_0(U_{p,0}^{II} = 1) = \mathbf{E}_0\left[U_{p,0}^{II}\right] = \mathbf{E}_0\left[\left(\prod_j \mathbf{1}_{\frac{G_{0,j}^{I-II}}{|T_0^{II} - T_j^I|^\alpha} < \rho}\right)\right. \\
&\left.\prod_{j\neq 0}\left(\mathbf{1}_{t_0^{II} \le t_j^{II}} + \mathbf{1}_{t_0^{II} > t_j^{II}}\mathbf{1}_{\frac{G_{0,j}^{II-II}}{|T_0^{II} - T_j^{II}|^\alpha} < \rho}\right)\right] \\
&= \mathbf{E}_0\left[\prod_j \mathbf{1}_{G_{0,j}^{I-II} < \rho|T_j^I|^\alpha}\right] \\
&\mathbf{E}_0\left[\prod_{j\neq 0}\left(\mathbf{1}_{t_0^{II} \le t_j^{II}} + \mathbf{1}_{t_0^{II} > t_j^{II}}\mathbf{1}_{G_{0,j}^{II-II} < \rho|T_j^{II}|^\alpha}\right)\right].
\end{aligned}$$

In the last equality, we have used the fact that Φ^I and Φ^{II} are independent. The first term in the last expression can be computed as:

$$\begin{aligned}
&\mathbf{E}_0\left[\prod_j \mathbf{1}_{G_{0,j}^{I-II} < \rho|T_j^I|^\alpha}\right] = \mathbf{E}_0\left[\prod_j E\left[\mathbf{1}_{G_{0,j}^{I-II} < \rho|T_j^I|^\alpha}\right]\right] \\
&= \mathbf{E}_0\left[\prod_j\left(1 - e^{-\mu\rho|T_j^I|^\alpha}\right)\right] \\
&= \exp\{-\lambda^I \int_{\mathbb{R}^2}\left(1 - \left(1 - e^{-\mu\rho|x|^\alpha}\right)\right)dx\} \\
&= \exp\{-\lambda^I \overline{N_0}\}.
\end{aligned}$$

For the second term, proceed as in the case of primary user, we get:

$$\begin{aligned}
&\mathbf{E}_0\left[\prod_{j\neq 0}\left(\mathbf{1}_{t_0^{II} \le t_j^{II}} + \mathbf{1}_{t_0^{II} > t_j^{II}}\mathbf{1}_{G_{0,j}^{II-II} < \rho|T_j^{II}|^\alpha}\right)\right] \\
&= \frac{1 - \exp\{-\lambda^{II}\overline{N_0}\}}{\lambda^{II}\overline{N_0}}.
\end{aligned}$$

Combining these gives us:

$$\mathbf{P}_0(U_{p,0}^{II} = 1) = \exp\{-\lambda^I \overline{N_0}\}\frac{1 - \exp\{-\lambda^{II}\overline{N_0}\}}{\lambda^{II}\overline{N_0}}.$$

□

As clear from the physics of Cognitive-CSMA, the MAP of primary users is exactly the same as the MAP of a user in a CSMA network with only the primary users. Equation (20) gives us the impact of primary on secondary users. Simple calculations show that a secondary user always has a smaller MAP than a user in a regular CSMA network of the same intensity. Moreover, if the primary user population is already dense (large λ^{II}), then the loss incurred by secondary users can be quite large.

COP and throughput

Since the p.p.s in our model are stationary, the COP is the same for each primary user (and the same for each secondary user). Hence we can express the COPs using Palm distributions as:

(i) For a typical primary user T_0^I:

$$p_{p,\mathrm{COP}}^I(\lambda^I, \lambda^{II}) = \mathbf{P}^0(\mathrm{SINR}_{p,0}^I > T). \qquad (23)$$

(ii) For a typical secondary user T_0^{II}:

$$p_{p,\mathrm{COP}}^{II}(\lambda^I, \lambda^{II}) = \mathbf{P}^0(\mathrm{SINR}_{p,0}^I > T). \qquad (24)$$

Unfortunately, the distribution of the interference created by the retained nodes is not known in closed form and we have to resort to an approximation to compute this COP. For this approximation, in order to have a more unified view on the network, we consider $\Phi = \Phi^I \cup \Phi^{II}$ as the process of all users in the network, which is a PPP of intensity $\lambda = \lambda^I + \lambda^{II}$. For each user in Φ, we define its virtual timer as:

(i) If this is a primary user T_i^I then the virtual timer is: $t_i^I \lambda^I/\lambda$.
(ii) If this is a secondary user T_i^{II} then the virtual timer is:$\lambda^I/\lambda + t_i^{II}\lambda^{II}/\lambda$

The virtual timers defined in this way are uniformly distributed on $[0,1]$ and this network becomes a CSMA network where primary users play the role of users having a timer smaller than λ^I/λ and secondary users play the role of users having a timer larger than λ^I/λ. Due to the fact that a user is primary or secondary is determined by its virtual timer, we can drop all the superscripts I and II. So, in the new notation, a user T_i in Φ has virtual timer t_i. The fading from T_i to T_j is $G_{i,j}$, which is equal to $G_{j,i}$. The fading from T_i to R_j is $F_{i,j}$. As above, all fading variables are independent exponential r.v.s of parameter μ. The retain indicator and the SINR of T_i are:

$$U_{p,i} = \prod_{j\neq i}(\mathbf{1}_{t_i<t_j} + \mathbf{1}_{t_j\leq t_i}\mathbf{1}_{\frac{G_{i,j}}{|T_i-T_j|^\alpha}<\rho})$$

$$\text{SINR}_{p,i} = \frac{F_{i,i}/r^\alpha}{W(R_i)+\Gamma_p(R_i)}$$

$$\Gamma_p(R_i) = \sum_{j\neq i} U_{p,j}\frac{F_{j,i}}{|T_j-R_i|^\alpha}.$$

The idea now is to approximate the process of retained users by an inhomogeneous PPP For this, we need the following proposition:

Proposition 14. *Assume Rayleigh fading. Conditionally on the fact that the network has two users i and j such that $X_i - X_j = x$, and the fact that user i with virtual timer $t_i = t$ is retained by the Cognitive-CSMA protocol, the probability $h_p(\lambda, x, t)$ that the protocol also retains user j is:*

$$\left(\frac{1-e^{-t\lambda\zeta_p(x)}}{\lambda\zeta_p(x)} + e^{-t\lambda g(x)}\frac{1-e^{(1-t)\lambda\overline{N_0}}}{\lambda\overline{N_0}}\right)\frac{1-e^{-\mu\rho|x|^\alpha}}{1-te^{-\mu\rho|x|^\alpha}}, \tag{25}$$

where $\zeta_p(x)$ is defined as:

$$\zeta_p(x) = \int_{\mathbb{R}^2} e^{-\mu\rho|y|^\alpha}(1-e^{-\mu\rho|y-x|^\alpha})dy. \tag{26}$$

Proof.
The proof can be found in Appendix A.1

□

This suggests to approximate the Palm distribution of Φ_M, conditioned on the timer t of the user at the center, by an inhomogeneous PPP with intensity $\lambda h_p(\lambda, x, t)dx$ and $\lambda = \lambda^I + \lambda^{II}$.

Proposition 15. *If, conditioned on the fact that the network has a user at the center with virtual timer t, we approximate the process of remaining transmitter by an inhomogeneous PPP of intensity measure $\lambda h_p(\lambda, x, t)dx$, the COP of primary and secondary users are respectively:*

(i) *for primary users:*

$$p^I_{p,\text{COP}}(\lambda^I,\lambda^{II}) = \frac{\lambda}{\lambda^I}\mathcal{L}_W(\mu T r^\alpha)\int_0^{\frac{\lambda^I}{\lambda}} e^{-\Upsilon_p(\lambda,t)}dt$$
$$= \frac{1}{\lambda^I}\mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I} e^{-\Upsilon_p(\lambda,t/\lambda)}dt,$$

(ii) *for secondary users:*

$$p^{II}_{p,\text{COP}}(\lambda^I,\lambda^{II}) = \frac{\lambda}{\lambda^{II}}\mathcal{L}_W(\mu T r^\alpha)\int_{\frac{\lambda^I}{\lambda}}^1 e^{-\Upsilon_p(\lambda,t)}dt$$
$$= \frac{1}{\lambda^{II}}\mathcal{L}_W(\mu T r^\alpha)\int_{\lambda^I}^{\lambda} e^{-\Upsilon_p(\lambda,t/\lambda)}dt,$$

where:

$$\Upsilon_p(\lambda,t) = \int_{\mathbb{R}^2}\frac{Tr^\alpha}{Tr^\alpha+|y-rl(0)|^\alpha}\lambda h_p(\lambda,y,t)dy.$$

Proof.
We first compute the COP of a typical user conditioned on its timer t. Proceeding as in Proposition 2 and using the fact that the receiver is uniformly distributed on the cicle of radius r centered at o, we have:

$$\mathbf{P}_o(\text{SINR}_0 > T | t_0 = t) \approx \frac{1}{2\pi}\int_0^{2\pi}$$
$$\exp\left\{-\int_{\mathbb{R}^2}\frac{Tr^\alpha}{Tr^\alpha+|y-rl(\theta)|^\alpha}\lambda h(\lambda,y,t)dy\right\}d\theta.$$

Note that $\zeta_a(x) = \zeta(x')$ if $|x| = |x'|$, we have that $h_p(\lambda,x,t) = h_p(\lambda,x',t)$ if $|x| = |x'|$. By a rotation of angle $-\theta$ which is denoted $S_{-\theta}(.)$, we have:

$$\int_{\mathbb{R}^2}\frac{Tr^\alpha}{Tr^\alpha+|y-rl(\theta)|^\alpha}\lambda h(\lambda,y,t)dy$$
$$= \int_{\mathbb{R}^2}\frac{Tr^\alpha}{Tr^\alpha+|S_{-\theta}(y)-rl(0)|^\alpha}\lambda h(\lambda,S_{-\theta}(y),t)dS_{-\theta}(y)$$
$$= \int_{\mathbb{R}^2}\frac{Tr^\alpha}{Tr^\alpha+|y-rl(0)|^\alpha}\lambda h(\lambda,y,t)dy = \Upsilon_p(\lambda,t).$$

We can then compute:

$$p^I_{\text{COP}}(\lambda^I,\lambda^{II}) = \mathbf{P}_o(\text{SINR}_0 > T | t_0 < \frac{\lambda^I}{\lambda})$$
$$= \frac{\lambda}{\lambda^I}\mathcal{L}_W(\mu T r^\alpha)\int_0^{\frac{\lambda^I}{\lambda}} e^{-\Upsilon_p(\lambda,t)}dt.$$

And similarly for secondary users:

$$p^{II}_{\text{COP}}(\lambda^I,\lambda^{II}) = \mathbf{P}_o(\text{SINR}_0 > T | t_0 \geq \frac{\lambda^I}{\lambda})$$
$$= \frac{\lambda}{\lambda^{II}}\mathcal{L}_W(\mu T r^\alpha)\int_{\frac{\lambda^I}{\lambda}}^1 e^{-\Upsilon_p(\lambda,t)}dt.$$

□

One may also be interested in the global performance of each class in the network. This can be quantified using the notion of *Spatial Density of Throughput* (SDT), which generalizes TT to the present stationary setting. This is defined as the mean number of successfully transmitted packets in each class per time slot and per unit surface. It is measured in packets/s/m^2. This tells us how fast the network functions on a global scale.

Corollary 1. *In passive mode, the spatial densities of throughput of primary and secondary users are:*

(i) *For primary users:*

$$S_p^I(\lambda^I,\lambda^{II}) = \lambda\mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I/\lambda} e^{-\lambda t\overline{N_0}}e^{-\Upsilon_p(\lambda,t)}dt = \mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I} e^{-t\overline{N_0}}e^{-\Upsilon_p(\lambda,t/\lambda)}dt.$$

(ii) *For secondary users:*

$$S_p^{II}(\lambda^I,\lambda^{II}) = \lambda\mathcal{L}_W(\mu T r^\alpha)\int_{\lambda^I/\lambda}^1 e^{-\lambda t\overline{N_0}}e^{-\Upsilon_p(\lambda,t)}dt = \mathcal{L}_W(\mu T r^\alpha)\int_{\lambda^I}^{\lambda} e^{-t\overline{N_0}}e^{-\Upsilon_p(\lambda,t/\lambda)}dt.$$

with $\lambda = \lambda^I + \lambda^{II}$.

Proof.
To compute these values, we first fix a 1×1 square C containing the center o of the plane. Since Φ is stationary, the densities of throughput are respectively:

$$S_p^I(\lambda^I,\lambda^{II}) = \mathbf{E}\left[\sum_{T_i\in\Phi}\mathbf{1}_{T_i\in C}\mathbf{1}_{U_i=1}\mathbf{1}_{\mathrm{SINR}_i>T}\mathbf{1}_{t_i<\frac{\lambda^I}{\lambda}}\right] = \lambda\mathbf{E}_o\left[\mathbf{1}_{U_0=1}\mathbf{1}_{\mathrm{SINR}_0>T}\mathbf{1}_{t_0<\frac{\lambda^I}{\lambda}}\right]$$

$$S_p^{II}(\lambda^I,\lambda^{II}) = \mathbf{E}\left[\sum_{T_i\in\Phi}\mathbf{1}_{X_i\in C}\mathbf{1}_{U_i=1}\mathbf{1}_{\mathrm{SINR}_i>T}\mathbf{1}_{t_i\geq\frac{\lambda^I}{\lambda}}\right] = \lambda\mathbf{E}_o\left[\mathbf{1}_{U_0=1}\mathbf{1}_{\mathrm{SINR}_0>T}\mathbf{1}_{t_0\geq\frac{\lambda^I}{\lambda}}\right]. \quad (27)$$

For the primary users:

$$\begin{aligned}&\mathbf{E}_o\left[\mathbf{1}_{U_0=1}\mathbf{1}_{\mathrm{SINR}_0>T}\mathbf{1}_{t_0<\frac{\lambda^I}{\lambda}}\right]\\ &= \int_0^{\lambda^I/\lambda}\mathbf{E}_o\left[\mathbf{1}_{U_0=1}\mathbf{1}_{\mathrm{SINR}_0>T}|t_0=t\right]dt\\ &= \int_0^{\lambda^I/\lambda}\mathbf{P}_o\left(U_i=1|t_0=t\right)\mathbf{P}_o(\mathrm{SINR}_0>T|t_0=t)\,dt\\ &= \mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I/\lambda}e^{-\lambda t\overline{N_0}}e^{-\Upsilon_p(\lambda,t)}dt.\end{aligned}$$

In the third equality we have used the fact that U_0 and SINR_0 are independent conditioned on t_0. The formula for S_p^I follows directly. The case of secondary users is treated in a similar way.

□

4.2.2. Active mode

MAP
The MAP of a typical primary and secondary user in active mode are:

$$p^I_{a,\mathrm{MAP}}(\lambda^I,\lambda^{II}) = \mathbf{P}(U^I_{a,i}=1)$$
$$p^{II}_{a,\mathrm{MAP}}(\lambda^I,\lambda^{II}) = \mathbf{P}(U^{II}_{a,i}=1),$$

respectively.

Proposition 16. *The MAP for a typical user is*

$$p^I_{a,\mathrm{MAP}}(\lambda^I,\lambda^{II})\frac{1-\exp\{-\lambda^I\overline{M}_0\}}{\lambda^I\overline{M}_0}, \quad (28)$$

for primary users and

$$p^{II}_{a,\mathrm{MAP}}(\lambda^I,\lambda^{II}) = \frac{1-e^{-\lambda^{II}\overline{M}_0}}{\lambda^{II}\overline{M}_0}e^{-\lambda^I\overline{N}_0} \quad (29)$$

for secondary users, with

$$\overline{M}_0 = \int_{\mathbb{R}^2}\left(1-(1-e^{-\mu\rho|y-rl(0)|^\alpha})\sigma(0,y)\right)dy, \quad (30)$$

$$\sigma(x,y) = \frac{1}{2\pi}(1-e^{-\mu\rho|y+rl(\theta)-x|^\alpha})d\theta. \quad (31)$$

Proof.
We first compute:

$$\begin{aligned}&\mathbf{P}_o(U_0^I=1|R_0^I,t_0^I) = \mathbf{E}_o[U^I_{p,0}|R_0^I,t_0^I] =\\ &\mathbf{E}_o\left[\prod_{j\neq 0}\left(\mathbf{1}_{t_0^I\leq t_j^I}+\mathbf{1}_{t_0^I>t_j^I}\mathbf{1}_{\frac{F_{0,j}^{I-I}}{|T_0^I-R_j^I|^\alpha}<\rho}\mathbf{1}_{\frac{F_{j,0}^{I-I}}{|T_j^I-R_0^I|^\alpha}<\rho}\right)\right]\\ &= \mathbf{E}_o\left[\prod_{j\neq 0}\left(1-t_0^I+t_0^I(1-e^{-\mu\rho|T_j^I-R_0^I|^\alpha})\sigma(0,T_j^I)\right)\right]\\ &= \exp\left\{-\lambda^I t_0^I\int_{\mathbb{R}^2}\left(1-(1-e^{-\mu\rho|y-R_0^I|^\alpha})\sigma(0,y)\right)dy\right\}.\end{aligned}$$

Since R_0^I is uniformly distributed on the circle of radius centered at o. One can express $R_0^I = rl(\theta_0^I)$, with θ_0 uniformly distributed on $[0,2\pi]$ and with $l(.)$ introduced in Subsection 2.1. Note that $\sigma(0,y)=\sigma(0,y')$ for any y' such that $|y'|=|y|$. Thus, by a rotation $S_{-\theta_0^I}$, one can prove that:

$$\int_{\mathbb{R}^2}\left(1-(1-e^{-\mu\rho|y-rl(\theta_0^I)|^\alpha})\sigma(0,y)\right)dy = \int_{\mathbb{R}^2}\left(1-(1-e^{-\mu\rho|y-rl(0)|^\alpha})\sigma(0,y)\right)dy = \overline{M_0}.$$

Hence:

$$p^I_{a,\mathrm{MAP}}(\lambda^I,\lambda^{II}) = \mathbf{P}_o(U_0^I=1|R_0^I,t_0^I) = \int_0^1\exp\{-\lambda^I t\overline{M_0}\}dt = \frac{1-\exp\{-\lambda^I\overline{M}_0\}}{\lambda^I\overline{M}_0}.$$

For secondary users, using the same kind of arguments as in the proof of Proposition 13 and in the first part of this proof gives us the wanted result. The proof is long but contains no important ideas and is hence omitted.

□

COP and throughput
For analyzing the COP and throughput under the active mode, we consider again the modified model with

virtual timers. The notation in this modified model is as follows:

$$U_{a,i} = \prod_{j\neq i}(\mathbf{1}_{t_i<t_j} + \mathbf{1}_{t_i\geq t_j}(1-\mathbf{1}_{\mathcal{C}(T_i,T_j)}))) = \prod_{j\neq i}(1-\mathbf{1}_{t_i\geq t_j}\mathbf{1}_{\mathcal{C}(T_i,T_j)}),$$

$$\mathcal{C}(T_i,T_j) := (\frac{F_{i,j}}{|T_i-R_j|^\alpha} > \rho) \textbf{ OR } (\frac{F_{j,i}}{|T_j-T_i|^\alpha} > \rho),$$

$$\mathrm{SINR}_i = \frac{F_{i,i}/r^\alpha}{W(R_i)+\Gamma_a(R_i)},$$

$$\Gamma_a(R_i) = \sum_{j\neq i} U_{a,j}F_{j,i}/|T_j-R_i|^\alpha.$$

In the active mode, we also have the following proposition regarding the second moment of the process of retained users:

Proposition 17. *Conditioned on the fact that the network has two users i and j such that $T_i - T_j = x$, $R_i = T_i + rl(\theta)$, $R_j = T_j + rl(\theta')$, and the fact that user i with virtual timer $t_i = t$ is retained by the Cognitive-CSMA protocol, the probability $h_a(\lambda,x,t,\theta,\theta')$ that the protocol also retains user j is:*

$$\left(\frac{1-e^{-t\lambda\zeta_a(x,\theta,\theta')}}{\lambda\zeta_a(x,\theta,\theta')} + e^{-t\lambda\zeta_a(x,\theta,\theta')}\frac{1-e^{(1-t)\lambda\overline{M_0}}}{\lambda\overline{M_0}}\right) \frac{(1-e^{-\mu\rho|x-rl(\theta)|^\alpha})(1-e^{-\mu\rho|x+rl(\theta')|^\alpha})}{(1-t+t(1-e^{-\mu\rho|x-rl(\theta)|^\alpha})(1-e^{-\mu\rho|x+rl(\theta')|^\alpha}))}, \quad (32)$$

where $\zeta_a(x,\theta,\theta')$ is defined as:

$$\zeta_a(x,\theta,\theta') = -\overline{M_0} + \int_{\mathbb{R}^2}(1-\kappa(0,x,y) (1-e^{-\mu\rho|y-rl(\theta)|^\alpha})(1-e^{-\mu\rho|y-x-rl(\theta')|^\alpha}))dy, \quad (33)$$

$$\kappa(x,y,z) = \frac{1}{2\pi}\int_0^{2\pi}(1-e^{-\mu\rho|x-z-rl(\theta)|^\alpha}) (1-e^{-\mu\rho|y-z-rl(\theta)|^\alpha})d\theta. \quad (34)$$

Proof.
The proof can be found in Appendix A.2.

□

The above result suggests that, conditioned on a typical user T_0 at the center of the plane which has an intended receiver at $R_0 = rl(\theta)$ and a virtual timer $t_0 = t$, we can approximate the process of remaining retained users as an inhomogeneous PPP of intensity measure $\lambda\frac{1}{2\pi}\int_0^{2\pi}h_a(\lambda,x,t,\theta,\theta')d\theta'$. Then, using Theorem 2.1, we have

Proposition 18. *Conditioned on the fact that a typical user T_0 at the center of the plane which has an intended receiver at $R_0 = rl(\theta)$ and a virtual timer $t_0 = t$, when approximating the process of remaining retained points by an inhomogeneous PPP of intensity measure $\lambda\frac{1}{2\pi}\int_0^{2\pi}h_a(\lambda,x,t,\theta,\theta')d\theta'$, the COP of a typical primary and secondary user is:*

(i) *For primary users:*

$$p^I_{a,\mathrm{COP}}(\lambda^I,\lambda^{II}) = \frac{\lambda}{\lambda^I}\int_0^{\frac{\lambda^I}{\lambda}}\mathcal{L}_W(\mu Tr^\alpha)e^{-\Upsilon_a(\lambda,t)}dt = \frac{1}{\lambda^I}\int_0^{\lambda^I}\mathcal{L}_W(\mu Tr^\alpha)e^{-\Upsilon_a(\lambda,t/\lambda)}dt.$$

(ii) *For secondary users:*

$$p^I_{a,\mathrm{COP}}(\lambda^I,\lambda^{II}) = \frac{\lambda}{\lambda^{II}}\int_{\frac{\lambda^I}{\lambda}}^1\mathcal{L}_W(\mu Tr^\alpha)e^{-\Upsilon_a(\lambda,t)}dt = \frac{1}{\lambda^{II}}\int_{\lambda^I}^{\lambda}\mathcal{L}_W(\mu Tr^\alpha)e^{-\Upsilon_a(\lambda,t/\lambda)}dt,$$

with:

$$\Upsilon_a(\lambda,t) = \lambda\int_{\mathbb{R}^2}g(y-rl(\theta),r)\frac{\int_0^{2\pi}h_a(\lambda,x,t,\theta,\theta')d\theta'}{2\pi(1-e^{-\mu\rho|y-rl(\theta)|^\alpha})}dy = \lambda\int_{\mathbb{R}^2}g(y-rl(0),r)\frac{\int_0^{2\pi}h_a(\lambda,x,t,0,\theta')d\theta'}{2\pi(1-e^{-\mu\rho|y-rl(0)|^\alpha})}dy.$$

Proof. This proof is similar to that of Proposition 15.

□

For the SDT, we have:

Corollary 2. *In active mode, the SDT of the two classes are:*

(i) *For primary users:*

$$S^I_a(\lambda^I,\lambda^{II}) = \mathcal{L}_W(\mu Tr^\alpha)\lambda\int_0^{\lambda^I/\lambda}e^{-\lambda t\overline{M_0}}e^{-\Upsilon_a(\lambda,t)}dt = \mathcal{L}_W(\mu Tr^\alpha)\int_0^{\lambda^I}e^{-t\overline{M_0}}e^{-\Upsilon_a(\lambda,t/\lambda)}dt.$$

(ii) *For secondary users:*

$$S^{II}_a(\lambda^I,\lambda^{II}) = \mathcal{L}_W(\mu Tr^\alpha)\lambda\int_{\lambda^I/\lambda}^1 e^{-\lambda t\overline{M_0}}e^{-\Upsilon_a(\lambda,t)}dt = \mathcal{L}_W(\mu Tr^\alpha)\int_{\lambda^I}^{\lambda}e^{-t\overline{M_0}}e^{-\Upsilon_a(\lambda,t/\lambda)}dt.$$

Proof.
This proof is similar to that of Corollary 1.

□.

4.3. Cognitive guarantee

A quick review of the formulas of the three performance metrics shows that they depend on the two key system parameters (λ^I and λ^{II}) in a same way for both the passive and the active mode. Thus, we only discuss cognitive guarantee policies for the active mode.

(i) MAP: The MAP of primary users is not affected by the secondary users. Cognitive -CSMA offers a perfect guarantee for primary users in term of medium access in the sense that if a primary users is selected by the CSMA protocol without secondary user, it is still selected by cognitive-CSMA in the presence of secondary users. As for secondary users, their MAP is $e^{-\lambda^I \overline{M_0}}$ times smaller than their MAP in the case where there is no primary user.

(ii) COP: Since $\Upsilon_a(\lambda, t/\lambda)$ is decreasing in λ, $p^I_{a,\mathrm{COP}}(\lambda^I, \lambda^{II})$ and $p^{II}_{a,\mathrm{COP}}(\lambda^I, \lambda^{II})$ are decreasing functions of λ^{II}. Thus, increasing the secondary users population always has a negative effect on the COP of both primary and secondary user. Moreover:

$$\lim_{\lambda\to\infty} \Upsilon_a(\lambda, t/\lambda) = \Upsilon_a(t) = \int_{\mathbb{R}^2}\int_0^{2\pi} \frac{g(y-rl(0),r)}{2\pi} \frac{1-e^{-\mu\rho|y+rl(\theta')|^{-\alpha}}}{1-t+t(1-e^{-\mu\rho|y+rl(\theta')|^{-\alpha}})(1-e^{-\mu\rho|y-rl(0)|^{-\alpha}})} \left(\frac{1-e^{-t\zeta_a(x,0,\theta')}}{\zeta_a(x,0,\theta')} + \frac{e^{-t\zeta_a(x,0,\theta')}}{\overline{M_0}}\right) d\theta' dy,$$

$$\lim_{\lambda^{II}\to\infty} p^I_{a,\mathrm{COP}}(\lambda^I, \lambda^{II}) = \frac{\mathcal{L}_W(\mu T r^\alpha)}{\lambda^I}\int_0^{\lambda^I} e^{-\Upsilon_a(t)}dt,$$

$$\lim_{\lambda^{II}\to\infty} p^{II}_{a,\mathrm{COP}}(\lambda^I, \lambda^{II}) = 0.$$

The corresponding limits for passive mode are:

$$\lim_{\lambda\to\infty} \Upsilon_p(\lambda, t/\lambda) = \Upsilon_p(t) = \int_{\mathbb{R}^2}\left(\frac{1-e^{-t\zeta_p(x)}}{\zeta_p(x)} + \frac{e^{-t\zeta_p(x)}}{\overline{M_0}}\right) \frac{Tr^\alpha(1-e^{-\mu\rho|y|^\alpha})}{(1-te^{-\mu\rho|y|^\alpha})(Tr^\alpha+|y-rl(0)|^\alpha)}dy,$$

$$\lim_{\lambda^{II}\to\infty} p^I_{p,\mathrm{COP}}(\lambda^I, \lambda^{II}) = \frac{\mathcal{L}_W(\mu T r^\alpha)}{\lambda^I}\int_0^{\lambda^I} e^{-\Upsilon_p(t)}dt,$$

$$\lim_{\lambda^{II}\to\infty} p^{II}_{p,\mathrm{COP}}(\lambda^I, \lambda^{II}) = 0.$$

For any L such that

$$p^I_{a,\mathrm{COP}}(\lambda^I, 0) > L > \frac{\mathcal{L}_W(\mu T r^\alpha)}{\lambda^I}\int_0^{\lambda^I} e^{-\Upsilon_a(t)}dt,$$

there exists a unique $\lambda^*_{a,\mathrm{COP}}$ such that $p^I_{a,\mathrm{COP}}(\lambda^I, \lambda^*_{a,\mathrm{COP}}) = L$. The secondary users must use an ALOHA like MAC protocol to guarantee that their intensity does not exceed $\lambda^*_{a,\mathrm{COP}}$.

(iii) SDT: Since $S^I_a(\lambda^I, \lambda^{II})$ is decreasing in λ^{II} and:

$$\lim_{\lambda^{II}\to\infty} S^I_a(\lambda^I, \lambda^{II}) = \mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I} e^{-t\lambda\overline{M_0}} e^{-\Upsilon_a(t)}dt,$$

we have that for any L such that

$$S^I_a(\lambda^I, 0) > L > \mathcal{L}_W(\mu T r^\alpha)\int_0^{\lambda^I} e^{-t\lambda\overline{M_0}} e^{-\Upsilon_a(t)}dt,$$

there is a unique $\lambda^*_{a,S}$ such that: $S^I_a(\lambda^I, \lambda^{II}) = L$. To guarantee that the SDT of the primary users is at least L, the intensity of secondary users must not exceed $\lambda^*_{a,S}$. Given this constraint, the secondary users optimize their performance by maximizing SDT $S^{II}_a(\lambda^I, \lambda^{II})$.

4.4. Some comparisons

As stated at the beginning of the section, the aim of the active mode is to better control interference. It achieves this aim by requiring both the transmitter and receiver to sense the network. Thus, in active mode, users are more conservative and cause much less interference to other users. The aim of this subsection is to compare the active and passive modes using the analytical results obtained above.

Let us begin with the MAP. In active mode, both transmitter and receiver sense the network to guarantee (a) that no user can harm the transmission (receiver sensing) and (b) that the transmission does not interfere with others (transmitter sensing). As a consequence, the MAP of a user is smaller in active mode than in passive mode. This can be quantified by comparing $\overline{N}_0$ and $\overline{M}_0$. Note that:

$$\overline{M_0} = \int_{\mathbb{R}^2} 1-\sigma(0,x)(1-e^{-\mu\rho|x-rl(0)|^\alpha})dx > \int_{\mathbb{R}^2} 1-(1-e^{-\mu\rho|x-rl(0)|^\alpha})dx = \overline{N}_0.$$

In the first inequality we have used the fact that $\sigma(0,x) < 1$. In the last equality, we used a change of variable from x to $x-l(0)$. The formulas for the MAP of active and passive modes have the same structure with respect to $\overline{N}_0$ and $\overline{M}_0$, which are decreasing functions. So for both primary and secondary users, the MAP is always larger in passive mode than in active mode.

We now compare the COP and SDT of a typical user in both modes. The noise terms are the same in both formulas. So, for the sake of simplicity, we just consider the case without noise. For this comparison, due to the complexity of the formulas, we have to rely on numerical methods. For the active mode, we can assume that r is very small compared to the network scale (i.e. $r \approx 0$). One can then assume that the position of the transmitter and receiver are identical. The formulas for

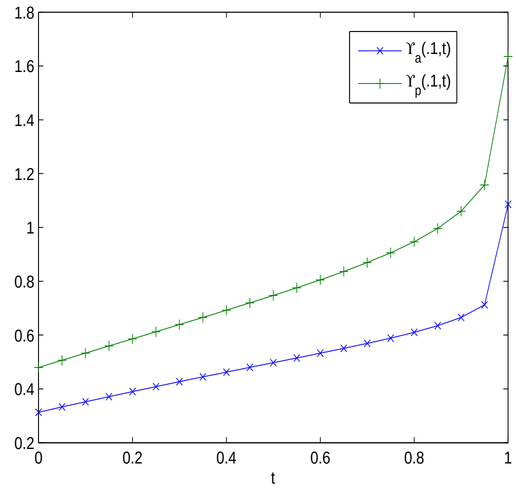


FIGURE 1. Plot of $\Upsilon_p(.1,t)$ and $\Upsilon_a(.1,t)$, the other parameters are $\mu = 10, \alpha = 3, r = 1, T = 15$. One can see that $\Upsilon_p(.1,t)$ is around 50% larger than $\Upsilon_a(.1,t)$.

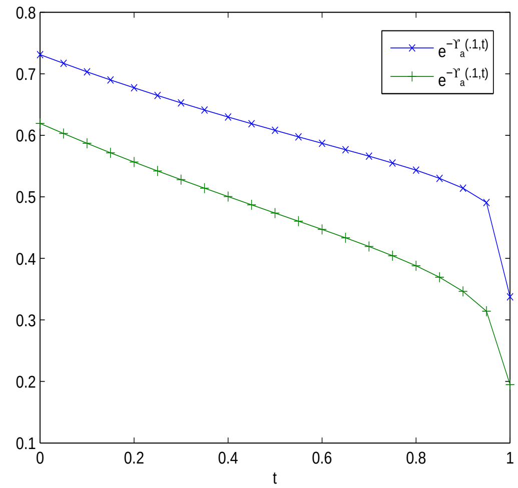


FIGURE 2. Plot of $e^{-\Upsilon_p(.1,t)}$ and $e^{-\Upsilon_p(.1,t)}$. The set of parameter is the same as that of fig. 1.

active mode can be simplified to

$$\overline{M_0} = \int_{\mathbb{R}^2} 1 - (1 - e^{-\mu\rho|x|^\alpha})^2 dx,$$

$$\zeta_a(x) = \int_{\mathbb{R}^2} (1 - e^{-\mu\rho|y|^\alpha})^2 (1 - (1 - e^{-\mu\rho|y-x|^\alpha})^2) dy,$$

$$h_a(x,\lambda,t) = (\frac{1 - e^{-\lambda t \zeta_a(x)}}{\lambda t \zeta_a(x)} + e^{-\lambda t \zeta_a(x)} \frac{1 - e^{-\lambda t \overline{M_0}}}{\lambda t \overline{M_0}}) \frac{(1 - e^{-\mu\rho|x|^\alpha})^2}{1 - t + t(1 - e^{-\mu\rho|x|^\alpha})^2},$$

$$\Upsilon_a(\lambda,t) = \int_{\mathbb{R}^2} \frac{g(x,r)}{1 - e^{-\mu\rho|x|^\alpha}} h_a(x,\lambda,t) dx.$$

Apart from these changes, the formulas of COP and SDT are kept unchanged. For the following set of parameters: $\lambda = .1, \mu = 10, \alpha = 3, r = 1, T = 15$, $\Upsilon_p(\lambda,t)$ and $\Upsilon_a(\lambda,t)$ are plotted versus t in Figure 1.

From this plot one can see that $\Upsilon_p(.1,t)$ is approximately 50% larger than $\Upsilon_a(.1,t)$. To see how this difference impacts COP and SDT, we make a plot of $e^{-\Upsilon_p(.1,t)}$ and $e^{-\Upsilon_p(.1,t)}$, which are essential in the computing of COP and SDT in Figure 2. Then we can consider a cognitive-CSMA network with total user intensity $\lambda = \lambda^I + \lambda^{II} = .1$ and let λ^I/λ be the proportion of primary users in this network. Then $p^I_{p,\mathrm{COP}}(\lambda^I,\lambda^{II})$, $p^I_{a,\mathrm{COP}}(\lambda^I,\lambda^{II})$, $p^{II}_{p,\mathrm{COP}}(\lambda^I,\lambda^{II})$, $p^{II}_{a,\mathrm{COP}}(\lambda^I,\lambda^{II})$ are plotted versus λ^I/λ in Figure 3, and $S^I_p(\lambda^I,\lambda^{II})$, $S^I_a(\lambda^I,\lambda^{II})$, $S^{II}_p(\lambda^I,\lambda^{II})$, $S^{II}_a(\lambda^I,\lambda^{II})$ are plotted versus λ^I/λ in Figure 4. From these plot we get that, with the chosen set of parameters:

(i) For both primary and secondary users, the COP in active mode is larger than that in passive mode (the increase is appr. .1).

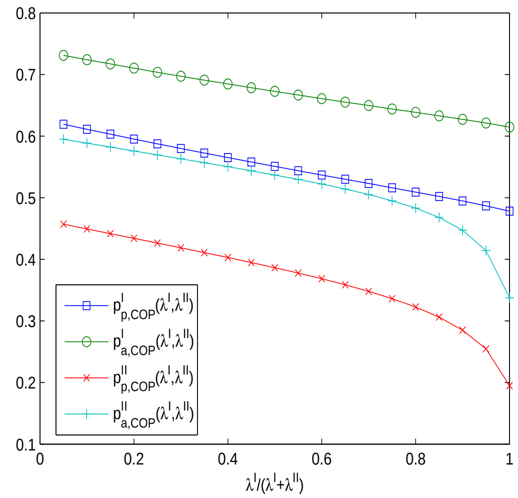


FIGURE 3. Plot of COP of primary and secondary users in active and passive mode versus λ^I/λ. The set of parameter is the same as that of fig. 1.

(ii) For both primary and secondary users, the SDT in active mode is around 40% larger than that in passive mode. So, in spite of offering less MAP, the active mode eventually pays off in term of SDT by guaranteeing better protection against interference.

5. CONCLUSION

The main contribution of this paper is the outline of a generic probabilistic framework based on stochastic geometry for the analysis of distributed MAC protocols used in cognitive radio networks. As shown in this paper, this framework has the potential of allowing the

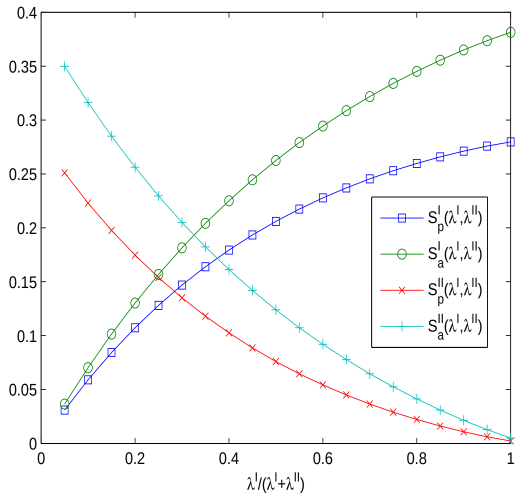


FIGURE 4. Plot of SDT of primary and secondary users in active and passive mode versus λ^I/λ. The set of parameter is the same as that of fig. 1.

modeling and the optimization of a large variety of other network architectures and MAC protocols. Another contribution of the present paper is the tuning of a fully distributed CSMA-based MAC protocol allowing secondary users of a large network to exploit unused spectrum and to comply with predefined requirements on the acceptable impact on primary users. The active version of this protocol using the RTS-CTS handshake technique provides the best interference control and the best utilization of the network resources (time, power, frequency).

Future work will bear upon:

1) Power control. It might be useful to control the transmission power and/or the sensing threshold of secondary users in order to further limit the interference they cause to primary users. A simple scheme would simply make secondary users transmit at a power level lower than that of primary users. More sophisticated schemes would make secondary users decide on their transmission power depending on the primary signal strength.

2) Refined schemes. The schemes proposed and analyzed in the present paper are rather conservative: for instance, in the presence of a primary user, even when the latter is silenced by another primary user, a secondary user backs off. A refined scheme would consist in using this momentary time hole for the secondary user. It ought to be clear that there are several interesting variants along these lines, which offer different compromises between channel access and coverage. Their analysis and comparison should allow one to determine which one is actually the best in terms of guarantees to primary users and throughput for secondary users.

3) Adaptive carrier sensing. In this paper we use a very simple rule for users to detect contenders. This often leads to waste of spatial resource when the user channel to its receiver is strong and offers little protection for users whose channel to its receiver is weak. One could consider a more adaptive carrier sensing strategy, which makes the contention radius of stronger users smaller and that of weaker users larger.

4) General priority laws. Here is an illustration of more general priority schemes. Consider a population of users belonging to n classes. Each class forms an independent PPP The intensity of users of class i is λ_i. With class i, one associates a distribution function with density h_i with support on $[0,1]$. The users of all classes compete for space using CSMA. This competition is represented by a Matérn interaction based on timers with distribution h_i for class i. Users with a density h_i more centered on the left of the $[0,1]$ interval have an advantage compared to those with density more centered on the the right. A special case is that with h_i having its support on $[(i-1)/n, i/n]$ which gives full priority of class i on class $i+1$ for all $i = 1,\ldots,n-1$. Let $\lambda = \sum_i \lambda_i$. Let p_i denote the probability that a typical user of class i and $M(x)$ denote the probability that a typical user retained by the Matérn scheme (or more precisely the extension with fading) based on a PPP of intensity x.

We have the following conservation rule, which can be seen as an analogue of Kleinrock's conservation law, [22] for priority queues:

$$M(\lambda) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i. \quad (35)$$

As a direct corollary, in the full priority case, one can determine the p_i recursively through the equations

$$\begin{aligned} M(\lambda) &= \frac{\lambda-\lambda_n}{\lambda} M(\lambda-\lambda_n) + \frac{\lambda_n}{\lambda} p_n \\ &\ldots \\ M(\lambda_1+\lambda_2) &= \frac{\lambda_1}{\lambda_1+\lambda_2} M(\lambda_1) + \frac{\lambda_2}{\lambda_1+\lambda_2} p_2. \end{aligned}$$

A.1. PROOF OF PROPOSITION 14

W.l.o.g. we can assume that $i = 0$, $j = 1$, $T_0 = o$, $T_1 = x$ (o is the center of the plane). The quantity that we need to compute is:

$$\frac{\mathbf{P}_{o,x}(U_0U_1 = 1|t_0 = t)}{\mathbf{P}_{o,x}(U_0 = 1|t_0 = t)} = \frac{\mathbf{E}_{o,x}[U_0U_1|t_0 = t]}{\mathbf{E}_{o,x}[U_0|t_0 = t]}.$$

First, note that for any i, j

$$\mathbf{1}_{t_i<t_j} + \mathbf{1}_{t_j\le t_i}\mathbf{1}_{\frac{G_{i,j}}{|T_i-T_j|^\alpha}<\rho} = 1 - \mathbf{1}_{t_j\le t_i}\mathbf{1}_{\frac{G_{i,j}}{|T_i-T_j|^\alpha}\ge\rho}.$$

We can then compute the denominator as:

$$\mathbf{E}_{o,x}[U_0|t_0=t]$$
$$= \mathbf{E}_{o,x}\left[(1-\mathbf{1}_{t_1\le t_0}\mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}\ge\rho})\prod_{j\neq 0,1}\left(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}\right)\right.$$
$$\left.\Big| t_0=t\right]$$
$$= \mathbf{E}_{o,x}\left[(1-te^{-\mu\rho|x|^\alpha})\prod_{j\neq 0,1}(1-te^{-\mu\rho|T_j|^\alpha})\right]$$
$$= (1-te^{-\mu\rho|x|^\alpha})\exp\{-\lambda t\overline{N_0}\}.$$

Now we move on to the numerator:

$$\mathbf{E}_{o,x}[U_0U_1|t_0=t]$$
$$= \mathbf{E}_{o,x}\left[(1-\mathbf{1}_{t_1\le t_0}\mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}\ge\rho})\prod_{j\neq 0,1}\left(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}\right)\right.$$
$$(1-\mathbf{1}_{t_0\le t_1}\mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}\ge\rho})\prod_{j\neq 0,1}\left(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\frac{G_{1,j}}{|T_j-x|^\alpha}\ge\rho}\right)$$
$$\left.\Big| t_0=t\right].$$

Note that:

$$(1-\mathbf{1}_{t_1\le t_0}\mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}\ge\rho})(1-\mathbf{1}_{t_0\le t_1}\mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}\ge\rho}) = \mathbf{1}_{\frac{G_{0,1}}{|x|^\alpha}<\rho}.$$

We get:

$$\mathbf{E}_{o,x}[U_0U_1|t_0=t]$$
$$= \mathbf{E}_{0,x}\left[(1-e^{-\mu\rho|x|^\alpha})\prod_{j\neq 0,1}\left(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}\right)\right.$$
$$\left.\left(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\frac{G_{1,j}}{|T_j-x|^\alpha}\ge\rho}\right)\Big| t_0=t\right].$$

Since:

$$\left(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}\right)\left(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\frac{G_{1,j}}{|T_j-x|^\alpha}\ge\rho}\right)$$
$$= \left(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\frac{G_{1,j}}{|T_j-x|^\alpha}\ge\rho}\right.$$
$$\left.+\mathbf{1}_{t_j\le\min\{t_0,t_1\}}\mathbf{1}_{\frac{G_{0,j}}{|T_j|^\alpha}\ge\rho}\mathbf{1}_{\frac{G_{1,j}}{|T_j-x|^\alpha}\ge\rho}\right),$$

we have:

$$\mathbf{E}_{o,x}[U_0U_1|t_0=t,t_1=\tau]=$$
$$(1-e^{-\mu\rho|x|^\alpha})\mathbf{E}_{o,x}\left[\prod_{j\neq 0,1}\left(1-te^{-\mu\rho|T_j|^\alpha}-\tau e^{-\mu\rho|T_j-x|^\alpha}\right.\right.$$
$$\left.\left.+\min\{t,\tau\}e^{-\mu\rho(|T_j-x|^\alpha+|T_j|^\alpha)}\right)\right]$$
$$= (1-e^{-\mu\rho|x|^\alpha})\exp\left\{-\lambda\int_{\mathbb{R}^2}(te^{-\mu\rho|y|^\alpha}+\tau e^{-\mu\rho|y-x|^\alpha}\right.$$
$$\left.-\min\{t,\tau\}e^{-\mu\rho(|y-x|^\alpha+|y|^\alpha)})dy\right\}.$$

Using the following equalities:

$$\int_{\mathbb{R}^2}e^{-\mu\rho|y|^\alpha}dy=\int_{\mathbb{R}^2}e^{-\mu\rho|y-x|^\alpha}dy=\overline{N_0},$$
$$\int_{\mathbb{R}^2}e^{-\mu\rho(|y|^\alpha+|y-x|^\alpha)}dy=\overline{N_0}-\zeta_p(x),$$

we get:

$$\mathbf{E}_{o,x}[U_0U_1|t_0=t,t_1=\tau]$$
$$=(1-e^{-\mu\rho|x|^\alpha})e^{-\lambda((t+\tau)\overline{N_0}-\min\{t,\tau\}(\overline{N_0}-\zeta_p(x)))}.$$

Now deconditioning on t_1 gives us:

$$\mathbf{E}_{o,x}[U_0U_1|t_0=t]=(1-e^{-\mu\rho|x|^\alpha})$$
$$\left(\int_0^t e^{-\lambda(t\overline{N_0}+\tau g(x))}d\tau+\int_t^1 e^{-\lambda(\tau\overline{N_0}+tg(x))}d\tau\right)$$
$$=e^{-t\lambda\overline{N_0}}\left(\frac{1-e^{-t\lambda g(x)}}{\lambda g(x)}+e^{-t\lambda g(x)}\frac{1-e^{(1-t)\lambda\overline{N_0}}}{\lambda\overline{N_0}}\right)$$
$$(1-e^{-\mu\rho|x|^\alpha}).$$

Dividing the formula for the numerator by that of the denominator gives us the wanted result.

A.2. PROOF OF PROPOSITION 17

W.l.o.g, we can assume that $i=0$, $j=1$ and $T_0=o, T_1=x$, we then want to compute the following conditional probability:

$$\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1|U_{a,0}=1,R_0=rl(\theta),t_0=t)$$
$$=\frac{\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1|R_0=rl(\theta),t_0=t)}{\mathbf{P}_{x,o}(U_{a,0}=1|R_0=rl(\theta),t_0=t)}.$$

We first compute the denominator:

$$\mathbf{P}_{x,o}(U_{a,0}=1|R_0=rl(\theta),t_0=t)$$
$$=\mathbf{E}_{x,o}[U_{a,0}|R_0=rl(\theta),t_0=t]$$
$$=\mathbf{E}_{x,o}\left[(1-\mathbf{1}_{t_1\le t_0}\mathbf{1}_{\mathcal{C}(T_0,T_1)})\right.$$
$$\left.\prod_{j\neq 0,1}(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\mathcal{C}(T_j,T_0)})\Big| R_0=rl(\theta),t_0=t\right].$$

Since

$$\mathbf{P}(C(T_i,T_j)|R_i) = 1-\sigma(T_i,T_j)(1-e^{-\mu\rho|T_j-R_i|^\alpha}),$$

and

$$\mathbf{P}(C(T_i,T_j)|R_i,R_j) = 1-(1-e^{-\mu\rho|T_i-R_j|^\alpha})(1-e^{-\mu\rho|T_j-R_i|^\alpha}),$$

we have:

$$\mathbf{P}_{x,o}(U_{a,0}|R_0 = rl(\theta), t_0 = t) = \mathbf{E}_{x,o}\Big[\Big(1-t\big(1-(1-e^{-\mu\rho|x+rl(\theta')|^\alpha})(1-e^{-\mu\rho|x-R_0|^\alpha})\big)\Big)\prod_{j\neq 0,1}\Big(1-t(1-\sigma(0,T_j)(1-e^{-\mu\rho|T_j-R_0|^\alpha}))\Big)\Big]$$
$$= \exp\left\{-\lambda t\int_{R^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-R_0|^\alpha})dy\right\}(1-t(1-(1-e^{-\mu\rho|x+rl(\theta')|^\alpha})(1-e^{-\mu\rho|x-R_0|^\alpha}))).$$

In the proof of Proposition 16, we have proved that

$$\int_{R^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-R_0|^\alpha})dy = \overline{M_0}.$$

Thus,

$$\mathbf{P}_{x,o}(U_{a,0}|R_0 = rl(\theta), t_0 = t) = \exp\{-\lambda t\overline{M_0}\}(1-t(1-(1-e^{-\mu\rho|x+rl(\theta')|^\alpha})(1-e^{-\mu\rho|x-rl(\theta)|^\alpha}))).$$

We now move to the numerator:

$$\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1|R_0 = rl(\theta), t_0 = t) = \mathbf{E}_{x,o}[U_{a,0}U_{a,1}=1|R_0 = rl(\theta), t_0 = t]$$
$$= \mathbf{E}_{x,o}\Bigg[(1-\mathbf{1}_{t_1\le t_0}\mathbf{1}_{\mathcal{C}(T_0,T_1)})(1-\mathbf{1}_{t_0\le t_1}\mathbf{1}_{\mathcal{C}(T_0,T_1)})\prod_{j\neq 0,1}(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\mathcal{C}(T_j,T_0)})\prod_{j\neq 0,1}(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\mathcal{C}(T_j,T_1)})\Bigg|R_0 = rl(\theta), t_0 = t, R_1 = x+rl(\theta')\Bigg]$$
$$= \mathbf{E}_{x,o}\Bigg[\prod_{j\neq 0,1}(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\mathcal{C}(T_j,T_0)})(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\mathcal{C}(T_j,T_1)})\mathbf{1}_{\mathcal{C}(T_0,T_1)}\Bigg|R_0 = rl(\theta), t_0 = t, R_1 = x+rl(\theta')\Bigg].$$

Note that:

$$\mathbf{E}\Big[(1-\mathbf{1}_{t_j\le t_0}\mathbf{1}_{\mathcal{C}(T_j,T_0)})(1-\mathbf{1}_{t_j\le t_1}\mathbf{1}_{\mathcal{C}(T_j,T_1)})\Big|R_0 = rl(\theta), t_0 = t, R_1 = x+rl(\theta'), T_j\Big]$$
$$= 1-t_0(1-\sigma(0,T_j)(1-e^{-\mu\rho|T_j-rl(\theta)|^\alpha})) - t_1(1-\sigma(0,T_j)(1-e^{-\mu\rho|T_j-x-rl(\theta')|^\alpha})) + \min\{t_0,t_1\}(1-\sigma(0,T_j)(1-e^{-\mu\rho|T_j-rl(\theta)|^\alpha}) - \sigma(x,T_j)(1-e^{-\mu\rho|T_j-x-rl(\theta')|^\alpha}) + \kappa(0,x,T_j)(1-e^{-\mu\rho|T_j-rl(\theta)|^\alpha})(1-e^{-\mu\rho|T_j-x-rl(\theta')|^\alpha})).$$

Thus:

$$\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1 \mid R_0 = rl(\theta), R_1 = x+rl(\theta'), t_0 = t, t_1)$$
$$= \exp\left\{-\lambda t\int_{\mathbb{R}^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-rl(\theta)|^\alpha}))dy\right\}\exp\left\{-\lambda t_1\int_{\mathbb{R}^2}(1-\sigma(x,y)(1-e^{-\mu\rho|y-x-rl(\theta')|^\alpha}))dy\right\}$$
$$\exp\Bigg\{-\lambda\min\{t,t_1\}\int_{\mathbb{R}^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-rl(\theta)|^\alpha}) - \sigma(x,y)(1-e^{-\mu\rho|y-x-rl(\theta')|^\alpha}) + \kappa(0,x,T_j)(1-e^{-\mu\rho|T_j-rl(\theta)|^\alpha})(1-e^{-\mu\rho|T_j-x-rl(\theta')|^\alpha}))dy\Bigg\}.$$

Now, using the following equalities:

$$\int_{\mathbb{R}^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-rl(\theta)|^\alpha}))dy = \int_{\mathbb{R}^2}(1-\sigma(x,y)(1-e^{-\mu\rho|y-x-rl(\theta')|^\alpha}))dy = \overline{M_0}$$

$$\int_{\mathbb{R}^2}(1-\sigma(0,y)(1-e^{-\mu\rho|y-rl(\theta)|^\alpha}) - \sigma(x,y)(1-e^{-\mu\rho|y-x-rl(\theta')|^\alpha}) + \kappa(0,x,T_j)(1-e^{-\mu\rho|T_j-rl(\theta)|^\alpha})(1-e^{-\mu\rho|T_j-x-rl(\theta')|^\alpha}))dy = \overline{M_0}-\zeta_a(x,\theta,\theta'),$$

we have:

$$\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1|R_0 = rl(\theta), R_1 = x+rl(\theta'), t_0 = t, t_1) = e^{-\lambda(t+t_1)\overline{M_0}}e^{\lambda\min\{t,t_1\}(\overline{M_0}-\zeta_a(x,\theta,\theta'))}.$$

Deconditioning on t_1 gives us:

$$\mathbf{P}_{x,o}(U_{a,0}U_{a,1}=1|R_0 = rl(\theta), R_1 = x+rl(\theta'), t_0 = t, t_1) = \left(\frac{1-e^{-t\lambda\zeta_a(x,\theta,\theta')}}{\lambda\zeta_a(x,\theta,\theta')} + e^{-t\lambda\zeta_a(x,\theta,\theta')}\frac{1-e^{(1-t)\lambda\overline{M_0}}}{\lambda\overline{M_0}}\right)(1-e^{-\mu\rho|x-rl(\theta)|^\alpha})(1-e^{-\mu\rho|x+rl(\theta')|^\alpha}).$$

The proposition is derived directly from this.

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