We deal with the coupled problem describing the interaction between a viscous incompressible fluid and a thin-walled elastic structure. For this kind of problem, significant progress has been achieved in the development and analysis of stable and accurate explicit coupling schemes when fitted meshes are used. For much applications, it is well known however that this assumption rapidly becomes cumbersome (e.g., large interface deflections, contacting structures, etc.).

Within the unfitted mesh framework splitting schemes of explicit nature are reported and analyzed in [2] using the finite element immersed boundary method, and in [3] for an unfitted Nitsche method. A major drawback of these approaches is that either stability or accuracy demands severe time-step restrictions (e.g., parabolic-CFL) or correction iterations.

Here we present two new numerical methods (semi-implicit and explicit) which bypass the above stability and accuracy issues. Their semi-implicit or explicit nature depends on the order in which the spatial and temporal discretizations are performed. These methods generalize the Robin-Neumann splitting paradigm of [3] to the unfitted mesh framework. A priori error and stability estimates are stated for some of the variants. Their performance is illustrated via numerical experiments in a benchmark.

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**THE LINEAR COUPLED PROBLEM**

We consider the following linear coupled problem:

- Find the fluid velocity \( \mathbf{u} = (u, v, w)^T \in \mathbb{R}^3 \) and pressure \( p \in \mathbb{R} \) in \( \Omega^f \) such that

  \[
  \begin{aligned}
  \nabla \cdot \mathbf{u} &= 0, & \text{in } \Omega^f, \\
  \nabla p &\cdot \mathbf{n} = 0, & \text{on } \Gamma^D, \\
  p &= p^0, & \text{on } \Gamma^P.
  \end{aligned}
  \]

  \[
  \begin{aligned}
  \nabla \cdot \mathbf{u} &= 0, & \text{in } \Omega^f, \\
  \nabla p &\cdot \mathbf{n} = 0, & \text{on } \Gamma^D, \\
  p &= p^0, & \text{on } \Gamma^P.
  \end{aligned}
  \]

**FIRST DISCRETIZE IN SPACE AND THEN IN TIME: SEMI-IMPPLICIT SCHEMES**

We consider a quasi-uniform family of meshes \( \{ T^h \}_{h \in (0,1]} \) which cover the whole fluid domain. Each \( T^h \) is fitted to the boundary \( \Gamma^b \) but not to \( \Sigma \).

Problem (1)-(2) is discretized in space as follows (See [2]): for \( t > 0 \), find \( (u_h, p_h) \) in \( V_h^f \times V_h^p \) such that

\[
\begin{aligned}
\left\{ \begin{array}{l}
\rho \frac{d}{dt} {u_h} + \nabla p_h = - \nabla \cdot \mathbf{f} & \text{in } \Omega^f, \\
nabla \cdot {u_h} = 0 & \text{in } \Omega^f, \\
\mathbf{u}_h &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h & = p^0 & \text{on } \Gamma^P.
\end{array} \right. \\
\end{aligned}
\]

**ALGORITHM 2: EXPLICIT COUPLING SCHEME**

1. Fluid sub-step: find \( (u_h^f, p_h^f) \in V_h^f \times V_h^p \) such that

   \[
   \begin{aligned}
   \rho \frac{d}{dt} u_h^f + \nabla p_h^f = - \nabla \cdot \mathbf{f}, \\
   \mathbf{u}_h^f &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h^f &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h^f & = p^0 & \text{on } \Gamma^P.
   \end{aligned}
   \]

2. Solid sub-step: find \( (u_h^s, p_h^s) \in V_h^s \times V_h^p \) such that

   \[
   \begin{aligned}
   \frac{\partial}{\partial t} u_h^s + a\mathbf{f}(u_h^s) \cdot \mathbf{n} + \frac{\gamma}{2} (u_h^s - u_h^s(0)) \cdot \mathbf{n} &+ K_h u_h^s - \frac{\gamma}{2} (u_h^s - u_h^s(0)) = 0, \\
   \mathbf{u}_h^s &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h^s &\cdot \mathbf{n} = 0 & \text{on } \Gamma^D, \\
p_h^s & = p^0 & \text{on } \Gamma^P.
   \end{aligned}
   \]

**NUMERICAL RESULTS**

We consider the benchmark example of the propagation of a pressure-wave within an elastic tube (2D Stokes equations with a 2D generalized string model).

We compare the results obtained with Algorithms 1 and 2 with those obtained with a first-order fully implicit scheme using fitted meshes. In order to highlight the uniformity of the convergence in \( h \), we have refined both in time and space at the same rate, \( r = O(h) \). In spite of their different semi-implicit and explicit nature, Algorithms 1 and 2 deliver practically the same behavior: stability is obtained with all the variants; optimal first-order convergence is obtained with the extrapolated variants \( r = 1 \), and the implicit (fitted) scheme while a sub-optimal convergence rate is exhibited by the non-extrapolated ones \( r = 0 \).

**REFERENCES**


