# Parity-energy ATL for Qualitative and Quantitative Reasoning in MAS

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# Outline

1. Introduction and motivations
2. The logic pe-ATL
   - pe-ATL at work
3. Model checking pe-ATL
   - Unbounded $[-\infty, +\infty]$ and bounded $[a, b]$ energy range
   - Left-bounded $[a, +\infty]$ and right-bounded $[-\infty, b]$ energy range
4. Conclusions

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**COALITION** - modeling collective behaviors/strategies

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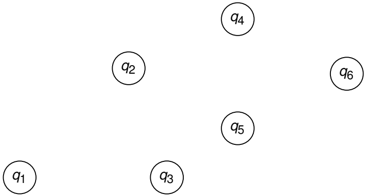


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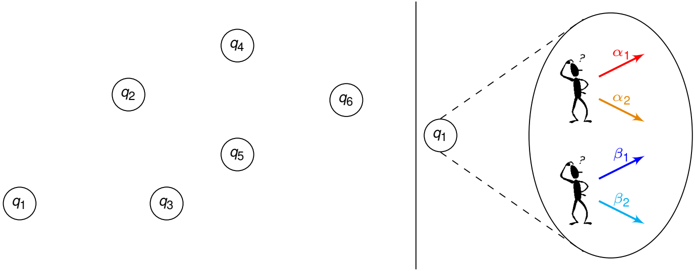


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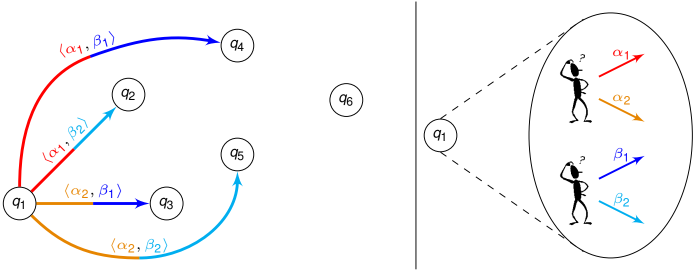


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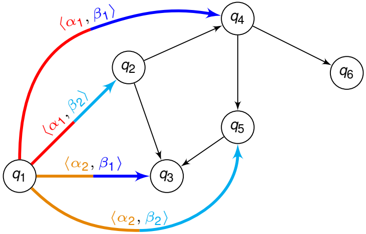


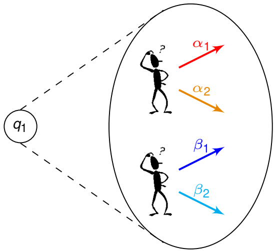


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# ATL: (intuitive) semantics

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Sample scenario:

- printing system: $n$ printers + shared bounded printing queue
- $n + m$ agents ($n$ printers + $m$ users/environment)
- printer actions: $\{$ **n** (*do-nothing*), **p** (*print*) $\}$
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pe-ATL abilities

- avoid errors (*i* printers do *print* and queue only contains $j < i$ jobs) (safety $\mapsto$ coalition+temporal)
- queue is emptied infinitely often (Büchi $\mapsto$ parity)
- users send infinitely many jobs $\Rightarrow$ queue is filled up infinitely often (fairness $\mapsto$ parity)
- devices' turnover (alternation $\mapsto$ energy)

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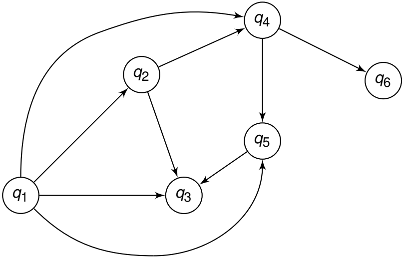


$G$

  - **vertices** labeled by **atomic propositions**
  - in vertices agents choose **actions**
  - possible combinations $\rightarrow$ **transitions** (edges of the graph)
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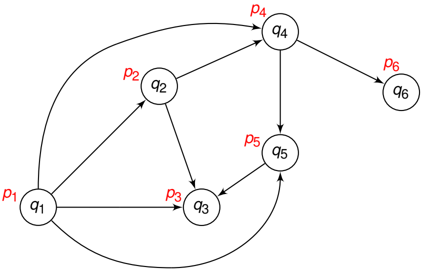


$\langle G, p \quad \rangle$

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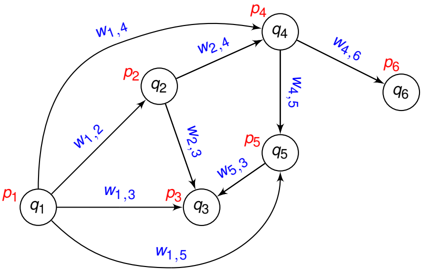


$\langle G, p, e \rangle$

- initial energy level $\mathcal{E}_0$
- energy range $[a, b]$

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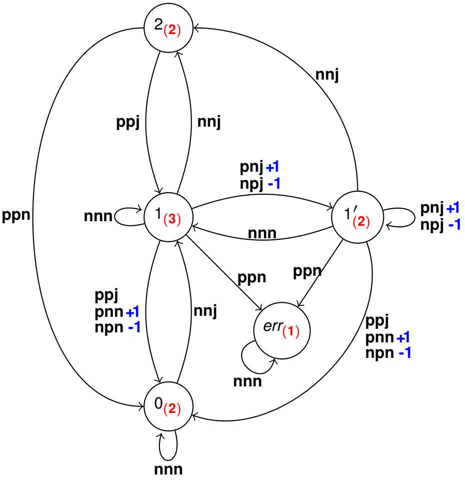
regardless of actions performed by other agents (opponent)

strategies must be $(p, e)$-strategies, i.e.,
they only produce plays satisfying parity and energy conditions

# Outline

# The printing system scenario




*agents* $= \{p_1, p_2, u\}$

*actions*

| | $p_1$ | $p_2$ | $u$ | *joint actions* |
|---|---|---|---|---|
| 0 | **n** | **n** | **nj** | {**nnn**, **nnj**} |
| 1 | **np** | **np** | **nj** | {**nnn**, **nnj**, **npn**, **npj**, **pnn**, **pnj**, **ppn**, **ppj**} |
| 1′ | **np** | **np** | **nj** | {**nnn**, **nnj**, **npn**, **npj**, **pnn**, **pnj**, **ppn**, **ppj**} |
| 2 | **p** | **p** | **nj** | {**ppn**, **ppj**} |
| *err* | **n** | **n** | **n** | {**nnn**} |

**energy weights**

$$w(\mathbf{nn}x) = w(\mathbf{pp}x) = 0$$
$$w(\mathbf{pn}x) = +1$$
$$w(\mathbf{np}x) = -1$$

*energy range = [0,1]*

*initial energy level* $\mathcal{E}_0 = 0$

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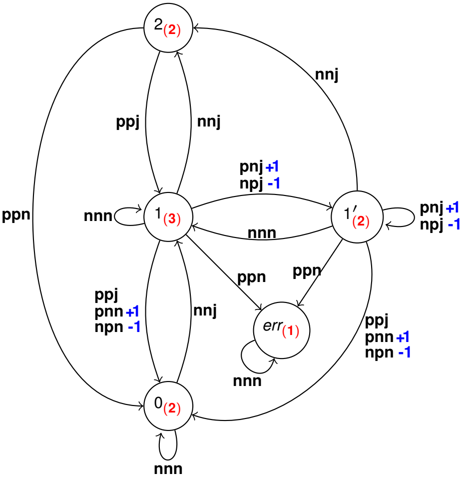


$$\mathcal{G}, 0 \models \langle\langle\{p_1, p_2\}\rangle\rangle \Box \neg err$$

$\exists$ joint strategy for $p_1$ and $p_2$ s.t.:

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- all jobs are processed (parity)
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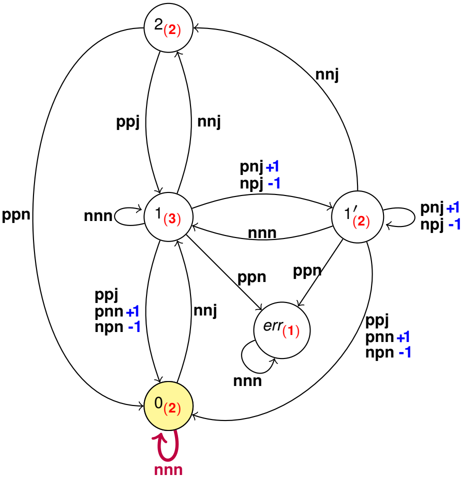


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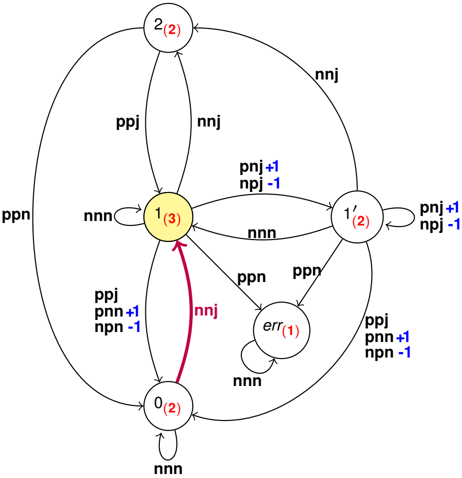


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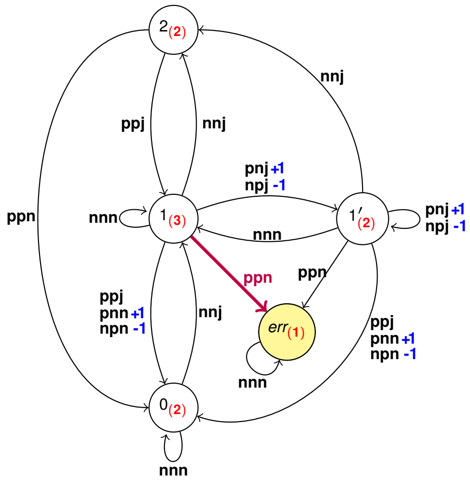


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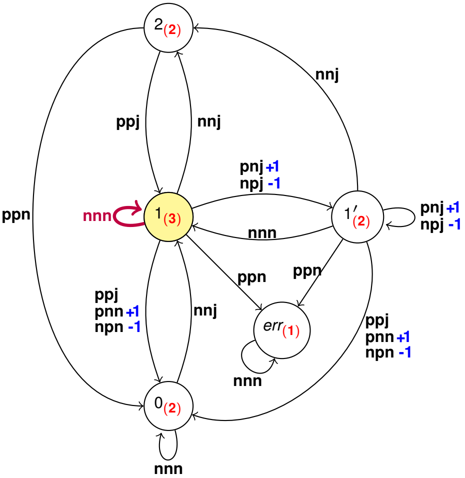


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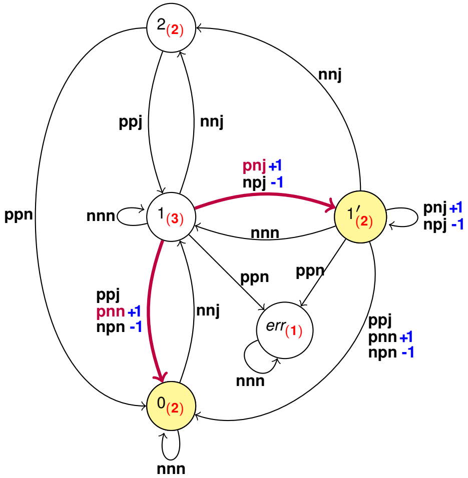


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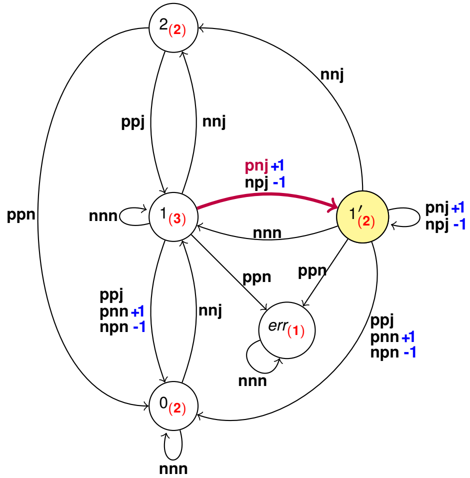


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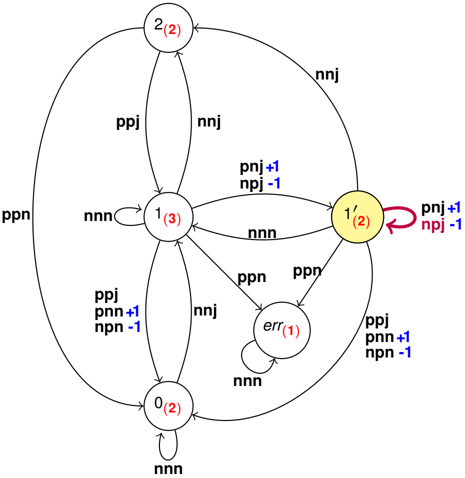


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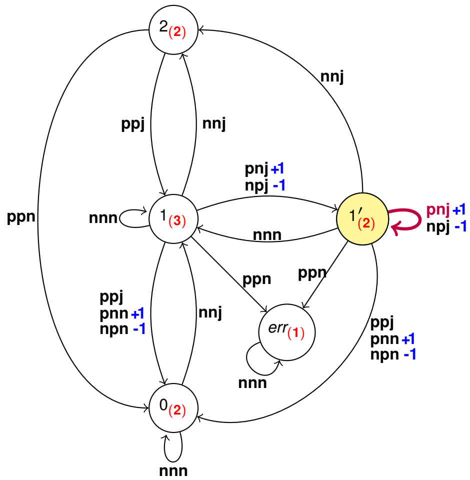


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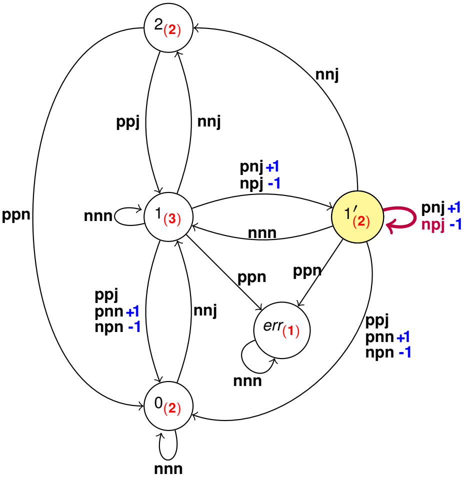


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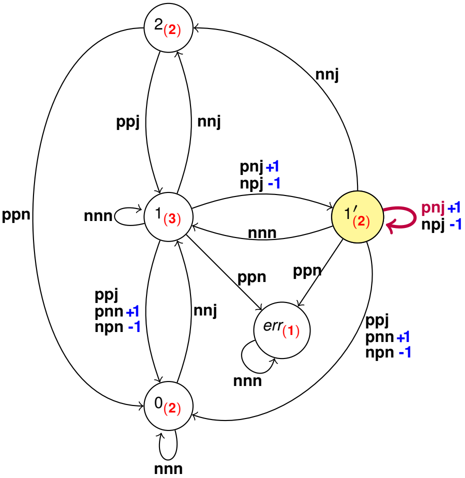


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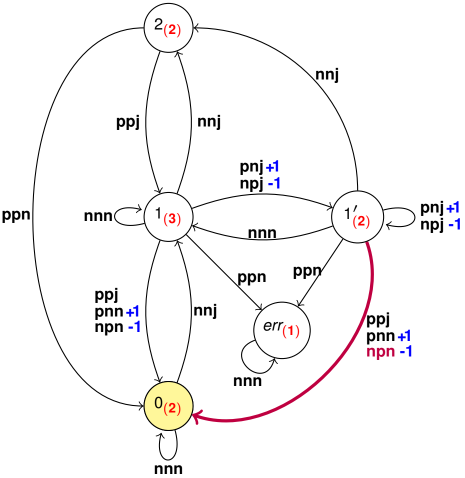


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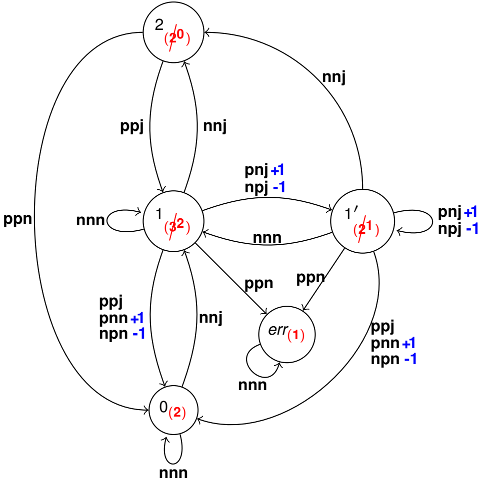


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# The model checking problem

**Definition (pe-ATL model checking problem)**

Given a pe-CGS $\mathcal{G} = \langle G, p, e \rangle$ and a pe-ATL formula $\varphi$, establish whether $\mathcal{G} \models \varphi$

We consider the following cases:

- unbounded energy range $[-\infty, +\infty]$ — NP
- bounded energy range $[a, b] \in \mathbb{Q}$ — NEXPTIME
- left-bounded energy range $[a, +\infty]$ (right-bounded is symmetric) — NP

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# Unbounded energy range $[-\infty, +\infty]$

- Reduction to p-ATL (just ignore the energy condition)

- Reduction to the case of bounded energy range $[a, b]$

$$\text{pe-ATL}_{[-\infty,+\infty]} \longrightarrow \text{pe-ATL}_{[a,b]}$$

weights, initial energy level, and energy bounds are set to 0

# Bounded energy range $[a, b]$

- $a \neq -\infty$, $b \neq +\infty$

## Lemma (normalization)

It is possible to focus on instances where no rationals are involved

- integer energy range ($a, b \in \mathbb{Z}$)
- integer initial energy level ($\mathcal{E}^{init} \in \mathbb{Z}$)
- weights over transitions are integers as well

## Lemma (positional strategies)

- a $(p, e)$-strategy exists iff a uniform one exists (bounded instance)
- a $(p, e)$-strategy exists iff a memoryless one exists (unbounded instance)

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# (Un)Bounded energy range $[a, b]$: Complexity

- uniform strategies are positional in $Q \times [a, b]$
  - exponentially many positions $(q, \mathit{energy\text{-}level})$ when $a$ and $b$ are in binary—thanks to normalization
- memoryless strategies are positional in $Q$
  - polynomially many positions $q$

A non-deterministic algorithm:

- guess the strategy
- return *true* when a loop with even parity is detected while staying within energy range
- stop at the first loop: only one position is visited twice
- bounded case: exponential time
- unbounded case: polynomial time

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# Left-bounded energy range $[a, +\infty]$

*(right-bounded energy range $[-\infty, b]$ is symmetric)*

- Model-theoretic argument (technically quite involved)
- Difficulty: the space of positions ($q$, *energy*-*level*) is infinite
- We define suitable structures (witnesses)
  - compact representations for strategies
  - polynomially bounded size
  - we prove it to be complete for strategies
- A non-deterministic algorithm guesses one such structure and check that it is indeed a witness for the desired strategy

# Key ideas

- A witness (for a $\langle\langle A\rangle\rangle\Box\psi$ formula) is a pair of graphs

$$(S_1, S_2)$$

- $S_1$ represents the strategy for parity
  $S_2$ contains increasing loops to increase the energy levels
- Elements of such graphs are positions $(q, \mathit{energy\text{-}level})$

$(q, \mathit{energy\text{-}level}) \in S$ iff there is a *winning* strategy for $A$, i.e.,
a $(p, e)$-strategy that guarantees the invariant $\psi$

- Left-bounded range ensures monotonicity

a strategy exists from $(q, \mathit{energy\text{-}level})$ iff a strategy exists from $(q, E)$ for all $E \geq \mathit{energy\text{-}level}$

- Thus, only the smallest energy level appears in $S_1$ and $S_2$ for each $q$

$$|S_1| \leq |Q|, \qquad |S_2| \leq |Q|$$

# Key ideas

- A witness (for a $\langle\langle A\rangle\rangle\Box\psi$ formula) is a pair of graphs

$$(S_1, S_2)$$

- $S_1$ represents the strategy for parity
  $S_2$ contains increasing loops to increase the energy levels
- Elements of such graphs are positions $(q, \mathit{energy\text{-}level})$

$(q, \mathit{energy\text{-}level}) \in S$ iff there is a *winning* strategy for $A$, i.e., a $(p, e)$-strategy that guarantees the invariant $\psi$

- Left-bounded range ensures monotonicity

a strategy exists from $(q, \mathit{energy\text{-}level})$ iff a strategy exists from $(q, E)$ for all $E \geq \mathit{energy\text{-}level}$

- Thus, only the smallest energy level appears in $S_1$ and $S_2$ for each $q$

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# From witnesses to strategies

- *internal constraints*
  - e.g., elements of $S_1$ and $S_2$ satisfy the invariant $\psi$ in a formula $\langle\langle A\rangle\rangle\Box\psi$
- *diagonal constraints*
  - e.g., elements of $S_1$ with low energy level also occur as (and can be merged with) elements of $S_2$
- the unfolding/merging of $S_1$ and $S_2$ corresponds to the outcome of a winning strategy for $A$

# From strategies to witnesses

Witness construction
(from the tree $\mathcal{T}$ of outcomes of a winning strategy for $A$)

- $q$ appears in the witness iff it appears in the tree $\mathcal{T}$
- suitably cut tree $\mathcal{T}$ into a finite (not bounded) prefix
- for every $q$, a representative node in the cut of $\mathcal{T}$ is chosen
  - based on their topological order and their energy level in the tree
- energy level and outgoing transition for $q$ in the witness are determined by its representative in the cut of $\mathcal{T}$

# Outline

# Conclusions

- pe-ATL: coalitional abilities to pursue temporal goals while satisfying qualitative (parity) and quantitative (energy) conditions
- pe-ATL model checking problem

**Theorem**

The model checking problem for pe-ATL is:

- in NEXPTIME if the energy range is bounded ($[a, b]$)
- in NPTIME if the energy range is unbounded ($[-\infty, +\infty]$)
- in NPTIME if the energy range is left- or right-unbounded ($[a, +\infty]$ or $[-\infty, b]$)

Notice that ATL$^*$ is 2EXPTIME-complete

# Future work

Open theoretical issues

- to establish thigh complexity bounds (parity game complexity)
- to synthesize parity and energy conditions to express desirable properties of a system
- expressiveness issues
  - comparison with other logics, e.g., $\mathrm{ATL}^*$, Strategy Logic (SL)

Possible variations/extension of the multi-agent scenario

- energy level evolves along the entire game
- limit opponent power with parity and energy conditions as well
- multiple quantitative dimension (several resources besides energy)

# The end

Thank you!