Long-Term Dependence Characteristics Of European Stock Indices

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Abstract

This paper measures the degrees of persistence of the daily returns of eight European stock market indices, after their lack of ergodicity and stationarity has been established. The proper identification of the nature of the persistence of financial time series forms a crucial step in deciding what kind of diffusion modeling of such series might provide invariant results. Our results indicate that ergodicity and stationarity are very difficult to establish with only daily observations of market indexes and thus various price diffusion models cannot be successfully identified. However, the measured degrees of persistence point to the existence of long-term dependencies, most likely of a nonlinear nature. Global Hurst exponents, computed from wavelet multi-resolution analysis, measure the long-term dependence of the data series. The FTSE turns out to be an ultra-efficient market with abnormally fast mean-reversion, faster than theoretically postulated by a Geometric Brownian Motion. But the various measurement methodologies produce non-unique empirical results and thus it is very difficult to obtain definite conclusions regarding the presence or absence of long-term dependence phenomena based on the global Hurst exponents. Although it is our judgment from these daily data that most stock markets in Europe appear to be anti-persistent, more powerful methods, such as the computation of the multifractal spectra of financial time series may be required. Still, we demonstrate that the visualization of the wavelet resonance coefficients and their power spectra, in the form of localized scalograms and averaged scalegrams, forcefully assists with the detection and measurement of several nonlinear types of market price diffusion.

1. Persistence in Financial Price Series

For a long time, financial researchers have struggled with the identification of properly specified econometric and time series models that can capture the dynamic dependence in financial time series. The most popular models that account for lagged observations are the family of ARIMA models and the family of GARCH models. Some models from these families have become extremely popular among technical analysts due to their ability to capture short-term dependence. Unfortunately, these, often linear, models are criticized for not being able to model long-term dependence too well and for requiring (and often presuming) Gaussian distribution characteristics for the residuals.

Loretan and Phillips (1994) recognize that distribution characteristics of time series often vary over time. Cochrane (1988) already pointed out several weaknesses of the ARIMA models and suggested a measure for long-term dependence, like unit root integration A(1), because of the presence of approximate common factors in the AR and MA polynomials. A very common and easily recognizable weakness of the ARIMA and GARCH models is the requirement for the modeled residual series to be stationary. Although one can achieve apparent wide-sense stationarity of the financial series after several adjusted differencing transformations, it has become clear that such adjusted differencing cannot remove the time dependence in the series between far-distant observations: the remaining auto-covariance functions (ACFs) of the squared errors just don't die out. This feature has also been observed in certain hydrological studies of long-range rivers and has been called the Hurst effect (Mandelbrot and Van Ness, 1968).

It has again come into research focus in the past decade. Recently, there a number of studies have appeared devoted to measuring such persistence phenomena in various financial data series using newer measurement technologies used in signal processing. A short discussion on persistence is provided, for example, in Mills (1999). Mandelbrot (1969, 1972) introduced the concept of the long-term persistence in the study of time series of economic and financial prices. With Fama he researched the resulting non-Gaussian distributions of financial prices. Once the concept of long-term memory in prices was accepted in the late 1970s, financial researchers searched for models that could properly identify such long-term dependence behavior. Hosking (1981) and Granger and Joyeux (1980) built on the prevalence of the well-known ARIMA models and proposed fractionally integrated ARMA models to measure long-term dependence. These models are more recently discussed in greater detail in Beran (1992), Baillie (1996) and Robinson (1994). Empirical studies of long-term dependence often rely on the study of Geweke and Porter-Hudak (1983), who proposed a method for the calculation of Hosking's fractional differencing parameter *d*.

The finding of long-term dependence in financial data might be in contradiction with the Efficient Markets Hypothesis of Fama (1970), which is based on the assumption of martingale behavior of financial market prices. The martingale theory requires an invariant stationarity and independence of the innovations of the historical price information sets, but it is difficult to show that this requirement is met either in weak form or, even less so, in strong form. Peters' work (1994) on the Fractional Market Hypothesis is an application of long-term dependence concept that is broader and encompasses Fama's theory of market efficiency.

The objective of this paper is to identify the dynamic diffusion models of several European equity indexes. This is done primarily to demonstrate that even though in the literature on econometric modeling one can find various models that fit financial data apparently well, one cannot fully rely on the conclusions about the properness of the identification of these models. For example, conventional econometric and time series modeling emphasizes only the measurement of the first two moments of the residuals, but ignores the measurement of the higher moments. Or, in frequency terms, it ignores the whole power spectrum of the innovations. Caution with respect to these "statistically estimated" models is therefore highly recommended, because, when these models are estimated. It is more often than not presumed that the data meet the assumptions of the theoretical models, even though the data show glaring discrepancies from those basic assumptions. Sometimes it is now even admitted that the data do not meet the assumptions of the theoretical models, but despite that admission the models are still being "estimated" and the results used with a confidence that is scientifically unwarranted (Los, 2001).

To pursue the objective of this paper and to shed some light on such analytical inconsistencies, we'll try to answer several questions about the dynamic character of stock market index prices for several European countries. Answering these questions is crucial for performing further econometric and time series analysis of the daily price traces, which are used for the valuation of and hedging by derivatives and, thus, for serious portfolio risk management. These questions are:

1. Are the pricing series or their innovations *ergodic*?
2. Is the pricing series or their innovations *stationary*? And if so, are they *strict* or *wide-sense stationary*?
3. Do the pricing series, after proper Taylor expansion type differencing, exhibit *independence*, *short-term dependence* or *long-term dependence*?
4. If the pricing series exhibit long-term dependence, are they *persistent* or *anti-persistent*?
5. What are the theoretical *benchmark models* for the analyzed pricing series?

6. How far do the *empirically identified* dynamic price diffusion models deviate from theoretical benchmark models?
7. Can the identified pricing models help market traders to earn *abnormal returns*?

In the context of this paper, a more detailed discussion of the approach to question 4 might be worthy some more elaboration, because of its unfamiliarity among financial analysts and econometric researchers. We measure the degree of global persistence by computing the Hurst exponent from the wavelet multi-resolution analysis (MRA) developed by signal processing engineers, such as Mallat (1989). MRA is a powerful technique that allows one to simultaneously analyze time series in both time and frequency domains. This feature is a simple way to identify time series data, since it also allows for the measurement and visualization of nonlinear dependencies, and thus of dependencies other than the usual collinearities between integer lags, which only measure simple linear dependencies.

A *scalogram*, which is a color-coding visualization of the measured wavelet resonance coefficients, *i.e.,* of the squared correlation coefficients between the time series and the chosen wavelet bases, allows one to *immediately* detect shocks in financial markets and their localized frequency strength or power or risk.

The average of such scalograms over time, that can be graphically represented by the logarithm of the average power spectrum of the financial time series, or a *scalegram*, allows one to investigate the autocorrelation function of the financial time series in the conventional Fourier-type frequency dimension and thus to identify possible (certain) periodicities or cyclicities (= uncertain periodicities) in the stock indexes. Such periodicities cannot be easily viewed when the statistical methodologies of classical time series analysis are used.

This paper is organized as follows: section 2 contains a short review of the long-term dependence literature; section 3 presents the details of the stock market index data; and section 4 discusses the methodologies used in this paper, together with the measured empirical results. Finally, section 5 draws some tentative conclusions.

2. Long-Term Dependence

One of the first finance researchers who formally recognized long-term persistence in financial economic data was Mandelbrot (1969, 1972). By doing so, he launched a search for the proper model identification to account for this phenomenon. Granger and Joyeux (1980), and Hosking (1981) developed a method of determining long-term dependence with fractionally integrated ARMA, or ARFIMA models. Geweke and Porter-Hudak (1983) proposed calculating the differencing parameter *d* that allows one to determine the level of long-term dependence. Beran (1992), Baillie (1996) and Robinson (1994) review such models of long-term dependence and their applications. Ding, Granger, and Engle (1993) focus on the detection of the long-term memory process in second moments, which are of importance to financial risk analysis and management. Baillie, Bollerslev, and Mikkelsen (1996) capture long-term dependence with their newly introduced class of fractionally integrated generalized autoregressive conditionally heteroskedastic (FIGARCH) processes by applying it to daily Deutschmark-U.S. dollar exchange rates. Bollerslev and Mikkelsen (1996) use the FIGARCH process to model financial market volatility, in particular in the foreign exchange markets, and assert, not completely convincingly, that a mean-reverting fractionally integrated process is superior in characterizing the volatility than any other model. In a slightly different approach, Crato and de Lima (1994) find long-memory or persistent stochastic volatility in high-frequency stock market data.

Thus far, the identification results, regarding the degree of long-term dependence or memory in the analyzed data, appear to depend very much on the analytic methodology used and therefore calls into question how and if it really can be identified by the existing methodologies and technologies. For example, Green and Fielitz (1977) and Aydogan and Booth (1988) apply in their studies the R/S (range-over-scale) metric of Hurst (1951) to test the long-term dependence in common stock return. But then Lo (1991) uses the modified rescaled range statistic for value and equal weighted CRSP index returns and finds that although the original Hurst rescaled range statistic detects the existence of the long-memory in the data, his modified Hurst statistic rejects such long-term memory. Moreover, Lo also cannot find the long-term dependence in annual returns for a long period from 1872 until 1986. We suspect that Lo focused on only one type of long-term dependence-persistence and could not find it, because these series represent the other type of long-term dependence: anti-persistence. If so, the research question should be reformulated and the technology adjusted to enable the detection of both types of long – term dependence. Also, Lo's modification incorporates only the linear research technology of collinearity analysis, which, per definition, cannot detect nonlinear long-term dependencies. It is of importance to emphasize that collinearity analysis can only detect linear dependencies in the data. It cannot detect nonlinear dependencies. We suspect that nonlinear dependencies are more prevalent in the data than linear dependencies.

The detection of long-term dependence processes has crucial implications for the measurement of the efficiency of financial markets. If long-term dependence is confirmed in asset prices, then one can have a viable suspicion about the existence of even the weakest form of efficiency, not to mention of other forms of financial market efficiency. Los (2000) and by Sadique and Silvapulle (2001) brought this issue into focus. Los (2000) used nonparametric efficiency tests of markets of Hong Kong, Indonesia, Malaysia, Singapore, Taiwan, and Thailand and rejected their efficiency on the basis of lack of stationarity and independence of the time series innovations. Sadique and Silvapulle looked specifically for long memory process in the stock market returns of Japan, Korea, New Zealand, Malaysia, Singapore, the USA and Australia, with the help of classical and modified rescaled range tests, the semi-parametric test proposed by Geweke and Porter-Hudak, the frequency domain score test proposed by Robinson and its time-domain counterpart derived by Silvapulle. Their study finds long-term dependence in stock market returns in Korea, Malaysia, Singapore and New Zealand. The results of Los and of Sadique and Silvapulle are in contradiction with the results of Cheung (1995), who did not find a persuasive support for the stock returns of eighteen countries of Asia, Europe, and North America using the classical techniques.

Given the lack of agreement on the existence of long-term memory process in stock returns, it is important to study this phenomenon further using more powerful methodologies and technologies. A significant contribution to such a more and more influential study of the persistence in financial data is Los (2003), who, in great detail and in language understandable to financial and economic researchers, reviews the currently available time-frequency signal processing methodologies and technologies to detect and measure long-term dependence, including the measurement not only of homogeneous or global Hurst exponents, but also of multifractal spectra of Lipschitz alphas. Recently, Mandelbrot contended that financial time-series probably are multifractal, or more precisely, can be modeled by Geometric Brownian Motion in multifractal time (Mandelbrot, 1997).

3. Data

The data used in this paper are daily deviations on eight European stock market indices and their various simple transformations. Detailed information for the indices is presented in Table 1 and in Table 2. The time period for the series varies from index to index due to data availability. All the series were taken from the "Yahoo, Finance!" website and therefore are freely available for further inspection and for replication of the results of this paper.

We analyze various transformations of the stock market indices: the index levels, X(t), logarithms of index levels, ln{X(t)}, differenced index levels D[X(t)], differenced logarithm of index levels, D[ln{X(t)}], which are the stock market returns, and differenced returns, D[x(t)]. These transformations of index levels and their returns are analyzed to find out whether the applied transformations allow one to find desirable properties of the data, such as stationarity and ergodicity. We also provide graphs supporting the conclusions of our study. The graphs included in this paper are only for the FTSE series, due to space limitations. The graphs for all other data series studied in this paper are available upon request.

4. Methodology and Empirical Identification Results

A) Ergodicity, Stationarity, and Independence

Ergodicity is defined by Terence C. Mills (1999, p. 9) as follows: "… the process is ergodic, which roughly means that the sample moments for finite stretches of the realization approach their population counterparts as the length of the realization becomes infinite." Mills remarks that it is impossible to test for the ergodicity of time series using only one realization and thus he *assumes* that all time series have this property. That is a very strong but tenuous assumption. Obviously, one cannot have more than one historical realization of any time series.

However, one can use time-ordered sample drawings of data from the available historical time series as substitutes for various length increasing realizations of a given population or infinite sample. Thus, moments for the five time series for each index using time windows of increasing size are computed and then these computed moments are plotted against their window length. One can then visually inspect whether the plots gradually converge to a flat line, which would suggest ergodicity of the time series.

In neither of our data series, we observe such gradual convergence to a flat time line. Several sharp discontinuities and shifts occur, which is an indication of fractality in the time series, and no convergence points appear to exist. Thus, visually it is reasonable to conclude that these series are not ergodic. Of course, it is an empirical scientific question, what realization is long enough to decide that the estimated moments can be relied on to make conclusions about ergodic property of a time series. Therefore several window sizes were tried. None provides results that even remotely could suggest ergodicity of these stock market time series. As an example of the increasing-window methodology, the first four moments of the analyzed FTSE in are plotted in Figure 2. This lack of visual ergodicity suggests that the usual procedure of using time moments as substitutes for ensemble moments is empirically severely flawed (or may we even conclude: it is visually falsified?).

A time series is said to be *strictly stationary* if the joint distribution of any set of n observations $Xt_1, Xt_2, \ldots, Xt_n$ is the same as the joint distribution of $Xt_{1+k}, Xt_{2+k}, \ldots, Xt_{n+k}$ for all n and k. Strict stationarity is difficult to observe in financial time series data, because we would have to compute an infinite set of moments, since for an unknown distribution it is unknown how many moments exist. Thus, the strict stationarity assumption is often relaxed to weak or wide sense stationarity. A time series is said to be *weakly stationary* if its first moment or mean is constant and its second moment or auto-covariance function (ACF) depends only on the time lags. If you normalize the ACF on the time lags you would see one constant standard definition or a horizontal time line over time in the normalized ACF plot.

In order to test for wide sense stationarity with an expansion to third and fourth moments, rolling windows are computed for the first four moments of all stock market indices and their transformations. As a representative example of the plots for these four rolling-window moments, the moments for FTSE are plotted in Figure 3. In neither case constant moments are observed. Again sharp shifts occur in the rolling window moments, an indication of the possible fractality of the time series. Thus, we conclude that our series are neither strict-sense nor wide-sense stationary.

The elements of a time series are independent if the autocorrelation function of this series equals one for the lag equal to zero and zero for any lag different from zero. A time series is long-term dependent if the autocorrelation function for the series decays at some hyperbolic rate. The decay at a hyperbolic rate is much slower than the decay at the geometric rate. To investigate the nature of the dependence of the time series, one can thus visually inspect the autocorrelation function of each of the five series. The autocorrelation functions, or ACFs of the five stock market indices, and their transformations were computed and inspected up to 200 lags.

Table 3 shows that the behavior of the autocorrelation function varies for each of the five series. For prices, logarithms of prices, and differences of returns, one can easily detect short-term dependence, because the autocorrelation function takes significant values for initial lags. For example, for one lag the absolute values of the ACFs vary from 0.441 up to 0.999 for these series. ACFs for the differences of prices and for returns are much smaller than for other series and their absolute values vary between 0.036 and 0.176. For the price levels and for the logarithmic transformations of prices, the autocorrelation function has a clear pattern and slowly dies off, thereby suggesting the existence of long-term dependence.

An empirical question is for how many lags the autocorrelation function should be different from zero in order to undoubtedly admit the long-term dependence. Assuming that 200 days is a long enough period to determine the long-term dependence for our daily data, large values for ACFs for 200 lags for prices and logarithms of prices suggest that one can detect long-term memory in these series. For other transformations of prices, like for the differences of prices, returns, and differences of returns, there are no clear patterns in the autocorrelation functions. The plots oscillate around zero without visible decline in the amplitudes of the ACF.

The nature of the ACF functions for the studied series is indicated in Table 4, which reports maximum, minimum, and the difference for minimum and maximum values for the various ACFs. The minimum and maximum values constitute a bandwidth for the ACF. Because the autocorrelation function is a decreasing function for prices and for logarithms of prices for the first 200 lags, small values of the difference between the maximum and minimum for ACFs suggest a stronger long-term dependence, and large values for the difference between maximum and minimum for ACFs suggest a weaker long-term dependence. Based on this, one can see that the strongest long-term dependence in prices occurs for the FTSE and the weakest long-term dependence for prices and for logarithm of prices occurs for the SMSI. The differences between maximum and minimum for ACFs for the differences of prices, returns, and differences of returns are very small. However, they remain different from zero. Because the plot for these series is rough with visible positive and negative spikes occurring at different lags, this suggests that these series retain weak long-term memory. Examples of plots of the autocorrelation functions are provided in Figure 1 and Figures 4 and 5.

B) Persistence

Based on the visual inspection of the ACF function, the series appear to be long-term dependent, and therefore can be better represented by Fractal Brownian Motion (FBM) then by Geometric Brownian Motion. For the model of Fractal Brownian Motion, the Hurst exponents, H, are computed for all the series, in order to determine the degree if their long-term dependence. The three manifestations of the long-term dependence are anti-persistence, when $0 < H < 0.5$, white (independent) noise when $H = 0.5$, and persistence when $0.5 < H < 1$.

We computed the Hurst exponent by seven different method and summarized the results in Tables 5 and 6: (1) R/S Analysis Method (R/S), (2) Power-Spectral Analysis Method (P-S), (3) Roughness-Length Relationship Method (R-L), and (4) Variogram Method (V), (5) the method proposed by developers of the IDL Wavelet Toolkit software[1], (6) the method developed by Veitch and Arby of the University of Melbourne[2], and (7) the method proposed by developers of FracLab software[3]. Table 7 reports more extensive results obtained from a procedure developed by Veitch and Arby.

Based on the calculated Hurst exponents, we find that stock market index series might be either persistent, P, or anti-persistent, AP, or white noise. It depends on the particular stock market. In order to draw any general conclusion about the data, one might decide that the series has a given property, if the majority of the proposed methods of analysis identify the given property. Thus, based on the results in Table 6, the ATX, CAC 40, DAX, IBEX, SMSI, and FTSE prices appear persistent. In case of KFX and TOTX, however, the results remain

[1] The IDL Wavelet Toolkit software was developed by Research Systems, a Kodak Company, and is available on the http://ion.researchsystems.com/IONScript/wavelet/ website

[2] The code for the procedure and the description of the procedure is available on the following website http://www.cubinlab.ee.mu.oz.au/~darryl/secondorder_code.html

[3] FracLab software is available for free on the following website http://fractales.inria.fr/index.php?page=fraclab

inconclusive. If one requires that all methods allow for the same unique conclusion about the nature of long-term dependence, for stock market in general, then the empirical results remain inconclusive. It would clearly be of no added value to require some sort of "significance" criterion, since each of the methods has different residual noise characteristics, because of the different projections involved. Thus, our conclusion is that the degree of the measured persistence depends on the particular stock market. Some stock markets are anti-persistent and are thus ultra-efficient. Some stock markets show independent innovations, and thus are efficient in the traditional sense. But some stock markets are persistent and thus inefficient and even dangerous: long periods of calmness in pricing may be disrupted by sudden and large discontinuities and drawdowns. Such differences in the degrees of persistence between the various financial markets are probably caused by the differences in their institutional organization.

Because in modeling of financial series the idea has always been accepted to test whether well-established models can fit the data, this paper also examines whether the European indexes can be proxied by some theoretical models available in the theoretical financial literature. The theoretical models that can be used to compare with the empirical time series are the random walk model, the Geometric Brownian Motion and the Fractional Brownian Motion. These models are defined in the following way (Los, 2003):

Definition 1: A random walk model is a particular wide sense Markov or unit root process of the original variables with independent innovations:

$X(t) - X(t\text{-}1) = (1 - L)X(t) = e(t)$, where $e(t) \sim i.i.d.(0, s_e^2)$ and L is the one – period lag operator.

Definition 2: A geometric Brownian motion is a random walk of the natural logarithm of the original process. Thus $\ln X(t) - \ln X(t-1) = x(t)$ are the rates of return and for Brownian motion:

$?x(t) = x(t) - x(t-1) = (1 - L)x(t) = e(t)$, where $e(t) \sim$ i.i.d. $(0, s_e^2)$

Definition 3: Fractional Brownian Motion (FBM) is defined by the fractionally differenced time series $(1 - L)^d x(t) = e(t)$, $d ? (-0.5, 0.5)$ with $e(t) \sim$ i.i.d.$(0, s_e^2)$.

Based on the rolling window test of the first four moments and based on the ACF function, the first differences of prices, returns, which are first differences of logarithms of prices, and the first differences of returns are not identically and independently distributed. Thus, prices, logarithms of prices and returns are clearly not processes integrated from or driven by white noise and the random walk model is immediately falsified.

To compare our series with the geometric Brownian motion, the ACF function for the analyzed series is compared with the correlations of geometric Brownian motion. (ACF comparison for FTSE series is provided in Figure 4). In almost all cases, the correlations of geometric Brownian motion substantially differ from the calculated ACFs for the original series. Thus, we also reject the geometric Brownian motion as a good model to fit the analyzed series.

To compare the empirical series with the theoretical FBM, one can compare the autocorrelation function of the series with the FBM based autocorrelation function that is given by the formula: $?(t) = t^{?} G(t)$ for $? ? [-1, 0)$, or $?(t) = -t^{?} G(t)$ for $? ? [-2, -1)$. In this formula t is time lag, G(t) is a slowly varying function at infinity (like a constant or a proportion of the time lag t, and the exponent ? is related to the Hurst exponent by the following relationship $? = 2H - 2$. Because we obtain different Hurst exponents with different estimation methods, one needs to compute different ACFs functions with different ?s and then to compare the obtained ACFs functions with ACFs of the original series. We suspected that FBM based ACFs are the closest to the original ACFs. Thus we computed ACFs for the theoretical FBMs for all series using the empirical ?s, but even these ACFs still do not approximate well the original ACFs. (The ACF comparison for FTSE series is provided in Figure 5)

The models that most likely can be used to identify abnormal stock market returns are those models which represent the persistence of the time series, that is models for which $0.5 < H < 1.0$. The series that appear to be anti-persistent with $0 < H < 0.5$ are abnormally fast mean – reverting and will not generate abnormally high returns, since those markets are ultra-efficient.

C) Persistence and Wavelet MRA Plots

Examples of the scalogram and scalegram results of the wavelet MRA are plotted in Figure 6. A scalogram measures all power spectra localized in time and frequency (=1/scale) domains at various scales and for various times. The wavelet resonance coefficients are computed by Mallat's (1989) wavelet MRA with the use of Morlet-6 wavelet[4]. A scalogram, which is a visualization of the colorized wavelet resonance coefficients, allows one to identify the precise timing and power of the innovations or shocks occurring in the markets. Scalegrams are averaged based on wavelet bases scalograms and thus comparable to Fourier spectra based on trigonometric bases. They help to detect the institutional periodicities or, more precisely, the aperiodic cyclicities (= uncertain "periodicities") of the financial markets, which cannot be easily identified by the static ergodicity-based methodologies. Scalegrams also assist with the identification of the global or homogeneous Hurst exponent for each time series and can

[4] Often wavelets with six or more non-vanishing moments produce similar results. Less non-vanishing moments tend to obfuscate the details of the analysis because the wavelet basis is too regular. The more non-vanishing moments, the more irregular the wavelet is. The less non-vanishing moments, the shorter a wavelet is. E.g., the Gaussian wavelet has only two non-vanishing moments.

determine if the residuals are, indeed, white noise. The discussed scalograms and scalegrams in this paper are computed with the help of software available on the following website: http://ion.researchsystems.com/IONScript/wavelet.

There are three parts in each plot in Figure 3. Part (a) is the plot of original time series and the type of wavelet used to analyze the time series, *c.q.* the Morlet-6 wavelet, often used for the analysis of meteorological and environmental time series, such as the El Niño effect, or the level of CO_2. Part (b) is the scalogram, which is the color-coded plot of the magnitude of the wavelet resonance coefficients. Finally, part (c) is the scalegram, which is the logarithm of the power spectrum or Fourier transform of the series' autocorrelation function (ACF).

On the basis of the price and return time series of FTSE index, one can see in Figure 7 that there are numerous spikes in the processes, which are consistent with sudden changes in the stock market prices. Figure 7 shows that the most significant price changes in the FTSE have occurred in October 1987, October 1989, April 1992, September 1992, October 1998, January 2000, and September 2001. For example, the sudden decline in the FTSE stock index in October, 1987 followed the crash in the US stock markets (black Monday), caused by rapidly rising of short term US interest rates, followed by rapidly rising long-term US interest rates, a weakening US dollar, deteriorating US current account deficit, unjustifiably high domestic price-earnings-ratios, very low dividend yields, and, most likely, too optimistic investor sentiment.

In terms of the wavelet analysis, stock market crashes can be easily detected by sudden spikes in power, or singularities, indicated in the scalogram by a steep upward migration of blue, green to red color. In October 1987, on the scalogram, one sees the burst of higher power through all frequencies for both stock market prices and returns, spreading from the high frequencies (at the top) to the low frequencies (at the bottom). The scalegram makes it easy to calculate the Hurst exponent from slope of the line fitted to the scalegram, which is 2H+2 for the price indices. The Hurst exponents calculated from the slope of the line fitted to the scalegram are reported in Table 5 under the title IDL Wavelet Toolkit. In case of the FTSE, the Hurst exponent is 0.33, indicating anti-persistence in the FTSE stock market returns data and definitely not consistent with a long memory or persistent process of $H > 0.5$. It indicates that the FTSE is an ultra-efficient market with abnormally fast mean – reversion, faster than theoretically postulated by a Geometric Brownian Motion (which has $H = 0.5$).

V. Conclusions

This paper attempts to identify the ergodicity, stationarity, independence, and persistence of the eight European index prices and their transforms, or the lack thereof. We find that the analyzed data are far from being either ergodic, or stationary or independent. Thus, such series cannot be modeled with ARIMA or GARCH family models that assume stationarity of the final residual series. The stock market prices and their returns and their various transformations are then compared with theoretical benchmark models, which are white (independent) noise (which integrates to Brown noise), Geometric Brownian Motion, and Fractional Brownian Motions. Even though some series appear to be fitted quite well by the white noise residual model (based on the computed global Hurst exponent), the estimated ACFs contradict often this finding. This demonstrates that the indiscriminate use of the global, homogeneous Hurst exponent computed from the average power spectrum (or Fourier transform of the ACF) is also not completely substantiated.

It remains an empirical scientific question which theoretical model is better for modeling of the original financial market series. The Fractional Brownian Motion is more general and encompasses the Geometric Brownian Motion. But also the Fractional Brownian Motion cannot capture all the empirically observed intricacies, such as “cyclicities” or “uncertain and time-varying periodicities” and the extremely valued power spikes observable in the power spectra of the stock market returns, as was originally suggested by Mandelbrot. Finally, the question should be raised whether such models can be used to earn abnormal stock market returns, in particular when persistence is observed. The methods thus far suggested in the literature appear not to lead to unique scientific conclusions regarding stock market returns in general. For example, not all European stock markets are conventionally efficient, but some appear to be anti-persistent or ultra- efficient, such as the FTSE, and some are persistent and inefficient.

The more important question for regulators and risk managers is thus which of the other European stock market indices are persistent and thus inefficient and which can therefore produce abnormal returns? By strictly focusing on long-term memory, i.e., persistence, and by not allowing for the possibility of anti-persistence, many research analysts have been guided themselves into blind alleys, since most of the European stock market indices appear to exhibit anti-persistent behavior. But the methods currently suggested in the literature lead to non-unique overall results. There exists no general stock market model. The various stock markets clearly differ in their degrees of persistence. It is most disturbing is that the various research methodologies do not yet lead to unique model identification results even for the same market. However, this paper does find that visualization of the time-frequency spectra by wavelet scalograms is a useful way to visualize the important localized characteristics of the financial time series. Of course, scalegrams and spectrograms are also based on computed averages, be it based on wavelets or Fourier transforms in the scale, respectively frequency domains and thus on the ergodicity in the frequency domain. Accordingly they also tend to obscure the important and not easily modeled, localized risk and time-variant higher moment phenomena, which are clearly observable in scalograms. This suggests that researchers must pay more attention to the changes in frequencies of the time series over time

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Table 1 Index prices analyzed in this study.

Country	Index	Index Yahoo Symbol	Symbol Used in the Project	Daily Data Range	Number of Observations
Austria	ATX-Index (Vienna)	ATX	ATX	11 Nov 92 – 23 Oct 00	2235
Denmark	KFX- Index (Copenhagen)	KFX	KFX	26 Jan 93– 23 Oct 00	2194
France	CAC 40 Index(Paris)	FCHI	FCHI or CAC or CAC 40	1 Mar 90– 23 Oct 00	2918
Germany	XETRA DAX Index	GDAXI	GDAXI or DAX	26 Nov 90– 23 Oct 00	2737
Norway	Oslo Total Index	NTOT	NTOT or TOTX	1 July 97– 23 Oct 00	1065
Spain	IBEX 35 Index (Barcelona)	IBEX	IBEX	9 Sept 97– 23 Oct 00	435
Spain	Madrid GEN Index	SMSI	SMSI	29 Apr 99 – 23 Oct 00	550
United Kingdom	FTSE 100 Index (London)	FTSE	FTSE	2 Apr 84– 23 Oct 00	4437

Table 2 Description of the indexes analyzed in the study. (Information presented in this table comes from http://www.finix.at/).

ATX (Austria)	
Long name	Austrian Traded Index
Owner/publisher/sponsor	Wiener Börse AG (Vienna Stock Exchange)
Constituents	22 Austrian companies continuously traded on the Vienna Stock Exchange
Construction principle	Capitalization-weighted value ratio
Base date	January 2, 1991
Base value	1,000.00
Interval of calculation	Real time

KFX (Denmark)	
Long name	Københavns Fondsbørs Index (Copenhagen Stock Exchange Index)
Owner/publisher/sponsor	Københavns Fondsbørs AS (Copenhagen Stock Exchange)
Constituents	21 Danish companies
Construction principle	Capitalization-weighted value ratio
Base date	July 3, 1989
Base value	100.00
Interval of calculation	1 minute

Table 2 Continued

CAC-40 (France)	
Long name	Compagnie des Agents de Change 40 Index
Owner/publisher/sponsor	Société des Bourses Françaises (SBF)-Bourse de Paris (Association of French Stock Exchanges-Paris Stock Exchange)
Constituents	40 French companies listed on the Paris Stock Exchange that are also traded on the options market
Construction principle	Capitalization-weighted value ratio
Base date	December 31, 1987
Base value	1,000.00
Interval of calculation	30 seconds

DAX (Germany)	
Long name	Deutscher Aktienindex DAX
Owner/publisher/sponsor	Deutsche Börse Group (German Stock Exchange)
Constituents	30 German companies
Construction principle	Capitalization-weighted total return Laspeyres index
Base date	December 30, 1987
Base value	1,000.00
Interval of calculation	1 minute

Total Index (Norway)	
Long name	Oslo Bors Total Index
Owner/publisher/sponsor	Oslo Bors
Number of constituents	All stocks registered on the Main List of the Oslo Stock Exchange
Construction principle	Capitalization-weighted total return value ratio
Base date/base value	January 1, 1983 / 100.00
Interval of calculation	1 minute

IBEX 35 (Spain)	
Long name	IBEX 35
Owner/publisher/sponsor	Association of Stock Exchanges (Sociedad de Bolsas S.A.)
Constituents	35 Spanish companies
Construction principle	Capitalization-weighted value ratio
Base date	December 29, 1989
Base value	3000.00
Interval of calculation	Real time

FT-SE 100 (UK)	
Long name	Financial Times Stock Exchange 100 Index
Owner/publisher/sponsor	FT-SE International Limited
Constituents	Shares of the top 100 UK companies ranked by market capitalization
Construction principle	Capitalization-weighted value ratio
Base date	December 31, 1983
Base value	1,000.00
Interval of calculation	1 minute

Table 3 Autocorrelation function values for one lag and for two hundred lags.

ACF for 1 lag	X(t)	ln{X(t)}	D[X(t)]	D[ln{X(t)}]	D[x(t)]
ATX	0.996	0.996	0.076	0.084	-0.441
KFX	0.999	0.999	-0.176	-0.157	-0.590
CAC	0.999	0.999	0.039	0.040	-0.466
DAX	0.999	0.999	0.036	0.040	-0.471
TOTX	0.993	0.993	0.049	0.047	-0.464
IBEX	0.999	0.973	0.079	0.097	-0.477
SMSI	0.984	0.984	-0.079	-0.086	-0.442
FTSE	0.999	0.999	0.060	0.060	-0.462

ACF for 200 lags	X(t)	ln{X(t)}	D[X(t)]	D[ln{X(t)}]	D[x(t)]
ATX	0.328	0.290	0.010	0.004	0.024
KFX	0.711	0.735	-0.006	0.001	0.005
CAC	0.796	0.818	0.020	0.014	0.014
DAX	0.804	0.826	-0.004	0.006	-0.010
TOTX	0.002	-0.008	-0.036	-0.032	-0.048
IBEX	0.879	-0.125	-0.067	-0.067	-0.053
SMSI	-0.187	-0.166	0.003	0.002	-0.014
FTSE	0.879	0.846	0.033	0.018	0.034

Table 4 Maximum, minimum, and difference between minimum and maximum for ACFs for the analyzed series.

	X(t) MAX	X(t) MIN	X(t) MAX-MIN	ln{X(t)} MAX	ln{X(t)} MIN	ln{X(t)} MAX-MIN	D[X(t)] MAX	D[X(t)] MIN	D[X(t)] MAX-MIN
ATX	0.996	0.328	0.668	0.996	0.290	0.706	0.097	-0.067	0.164
KFX	0.999	0.711	0.288	0.999	0.735	0.264	0.082	-0.176	0.258
CAC	0.999	0.796	0.203	0.999	0.818	0.181	0.076	-0.080	0.156
DAX	0.999	0.804	0.195	0.999	0.826	0.173	0.093	-0.068	0.161
TOTX	0.993	0.002	0.991	0.993	-0.008	1.001	0.083	-0.075	0.158
IBEX	0.999	0.879	0.120	0.973	-0.125	1.098	0.088	-0.277	0.365
SMSI	0.984	-0.187	1.171	0.984	-0.166	1.150	0.105	-0.079	0.184
FTSE	0.999	0.879	0.120	0.999	0.846	0.153	0.078	-0.070	0.148

Table 4 Continued

	D[ln{X(t)}] MAX	D[ln{X(t)}] MIN	D[ln{X(t)}] MAX-MIN	D[x(t)] MAX	D[x(t)] MIN	D[x(t)] MAX-MIN
ATX	0.087	-0.059	0.146	0.090	-0.441	0.531
KFX	0.084	-0.157	0.241	0.109	-0.590	0.699
CAC	0.057	-0.050	0.107	0.066	-0.466	0.532
DAX	0.059	-0.054	0.113	0.070	-0.471	0.541
TOTX	0.085	-0.085	0.170	0.090	-0.464	0.554
IBEX	0.097	-0.237	0.335	0.120	-0.477	0.597
SMSI	0.089	-0.086	0.174	0.111	-0.442	0.553
FTSE	0.060	-0.045	0.105	0.075	-0.462	0.537

Table 5 Hurst exponent for the analyzed series.

	IDL Wavelet Toolkit	D. Veitch and P. Abry procedure	FracLab software	R/S (Benoit software)	P-S (Benoit software)	R-L (Benoit software)	V (Benoit software)
Index							
Austria: ATX	0.42	0.48	0.47	0.55	0.50	0.55	0.47
Denmark: KFX	0.55	0.28	0.40	0.50	0.51	0.41	0.52
France: CAC 40	0.46	0.41	0.46	0.51	0.50	0.44	0.56
Germany: DAX	0.47	0.43	0.43	0.51	0.54	0.44	0.52
Norway: TOTX	0.45	0.49	0.50	0.53	0.52	0.49	0.53
Spain: IBEX	0.46	0.46	0.39	0.46	0.51	0.51	0.40
Spain: SMSI	0.48	0.46	0.23	0.41	0.47	0.36	0.46
UK: FTSE	0.33	0.41	0.44	0.51	0.53	0.45	0.49

Table 6 Long-term dependence of the series: P – persistence; AP – anti-persistence, WN – white noise.

	IDL Wavelet Toolkit	D. Veitch and P. Abry procedure	FracLab software	R/S (Benoit software)	P-S (Benoit software)	R-L (Benoit software)	V (Benoit software)
Index							
Austria: ATX	AP	AP	AP	P	WN	P	AP
Denmark: KFX	P	AP	AP	WN	P	AP	P
France: CAC 40	AP	AP	AP	P	WN	AP	P
Germany: DAX	AP	AP	AP	P	P	AP	P
Norway: TOTX	AP	AP	WN	P	P	AP	P
Spain: IBEX	AP	AP	AP	AP	P	P	AP
Spain: SMSI	AP	AP	AP	AP	AP	AP	AP
UK: FTSE	AP	AP	AP	P	P	AP	AP

Table 7 This table reports the identified homogeneous Hurst exponents of the stock indices. The parameters were obtained with the LDestimate function developed by D. Veitch and P. Abry of The University of Melbourne. The LDestimate function estimates two parameter of long-range dependent process (LRD), alpha using the wavelet based joint estimator of Abry and Veitch. CI's are confidence intervals. The relationship between the slope of the power spectrum alpha and the Hurst exponent H is as follows:

alpha = (2H+1), so that $H = \frac{alpha - 1}{2}$. A Hurst exponent of 0.50 indicates that market prices follow a Geometric Brownian motion, while a Hurst exponent between 1 and 0.50 means the market prices are persistent, and a Hurst exponent between 0 and 0.50 means the market prices are anti-persistent.

	Goodness of fit (Probability of data assuming linear regression)	Scaling parameter alpha (LRD) (slope of log-log plot)	Scaling parameter H	Scaling parameter D (fractal dimension, if alpha in (1,3))
Austria: ATX	0.00314	1.950	0.475	1.525
CI's:		[1.880,2.019]	[0.440,0.510]	[1.490,1.560]
Denmark: KFX	0.01618	1.567	0.283	1.717
CI's:		[1.496,1.637]	[0.248,0.319]	[1.681,1.752]
France: CAC 40	0.99767	1.814	0.407	1.593
CI's:		[1.755,1.874]	[0.377,0.437]	[1.563,1.623]
Germany: DAX	0.25144	1.853	0.427	1.573
CI's:		[1.791, 1.915]	[0.396, 0.457]	[1.543, 1.604]
Norway: TOTX	0.12103	1.985	0.493	1.507
CI's:		[1.877, 2.094]	[0.438, 0.547]	[1.453, 1.562]
Spain: IBEX	0.52905	1.92	0.46	1.54
CI's:		[1.715, 2.124]	[0.358, 0.562]	[1.438, 1.642]
Spain: SMSI	0.04523	1.927	0.464	1.536
CI's:		[1.758, 2.097]	[0.379, 0.548]	[1.452, 1.621]
U1UK: FTSE	0.00567	1.821	0.411	1.589
CI's:		[1.774, 1.868]	[0.387, 0.434]	[1.566, 1.613]

Figure 1 Plots of the index level and its transformation, autocorrelations up to 200 lags end empirical distributions for the analyzed FTSE index level and its transformations.

Plots of index level and its transformations

Plot of the autocorrelation function up to 200 lags for the index level and its transformation

Empirical distributions of the index level and its transformations

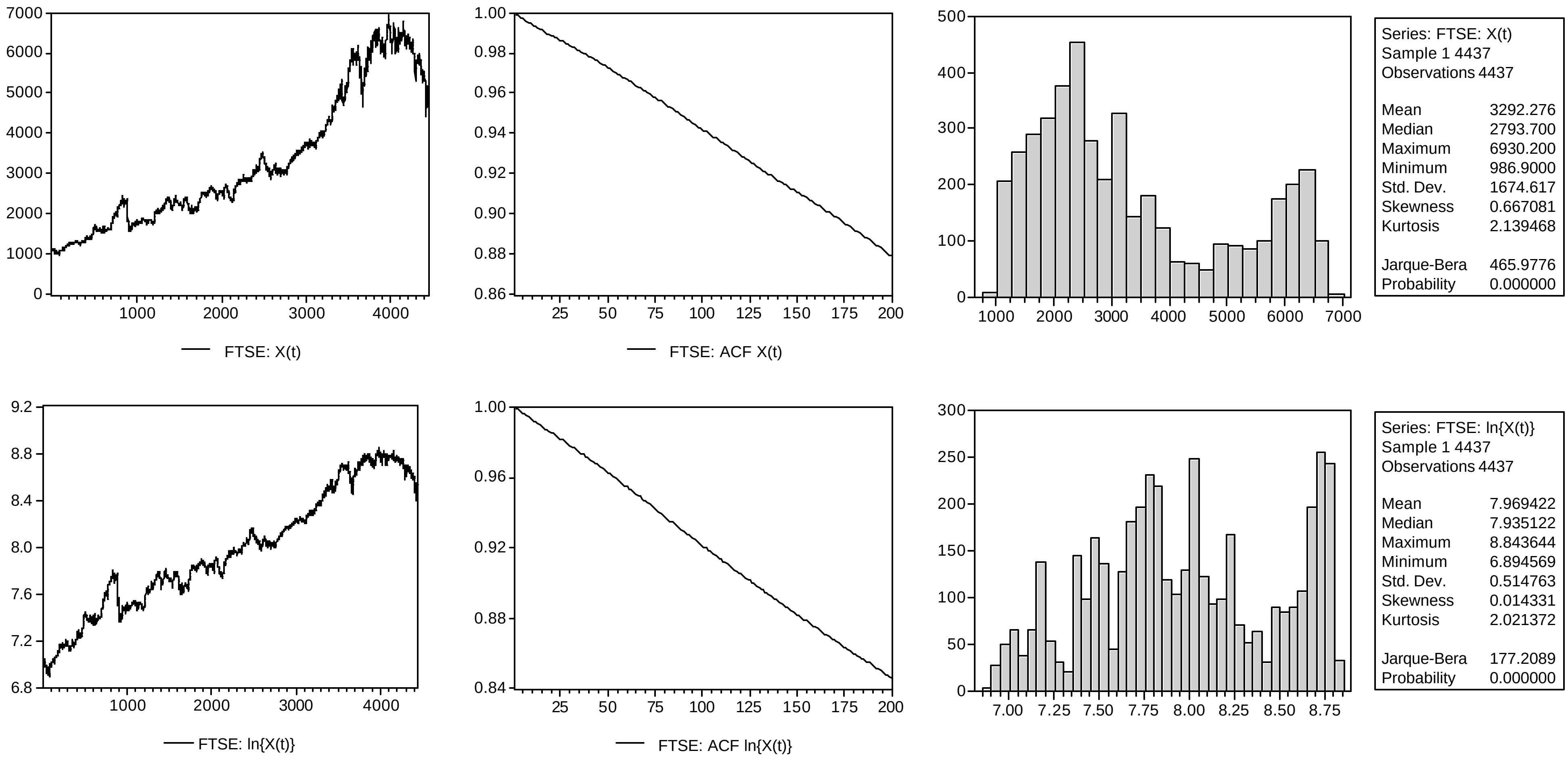


Figure 1 Continued

Plots of index level and its transformations

Plot of the autocorrelation function up to 200 lags for the index level and its transformation

Empirical distributions of the index level and its transformations

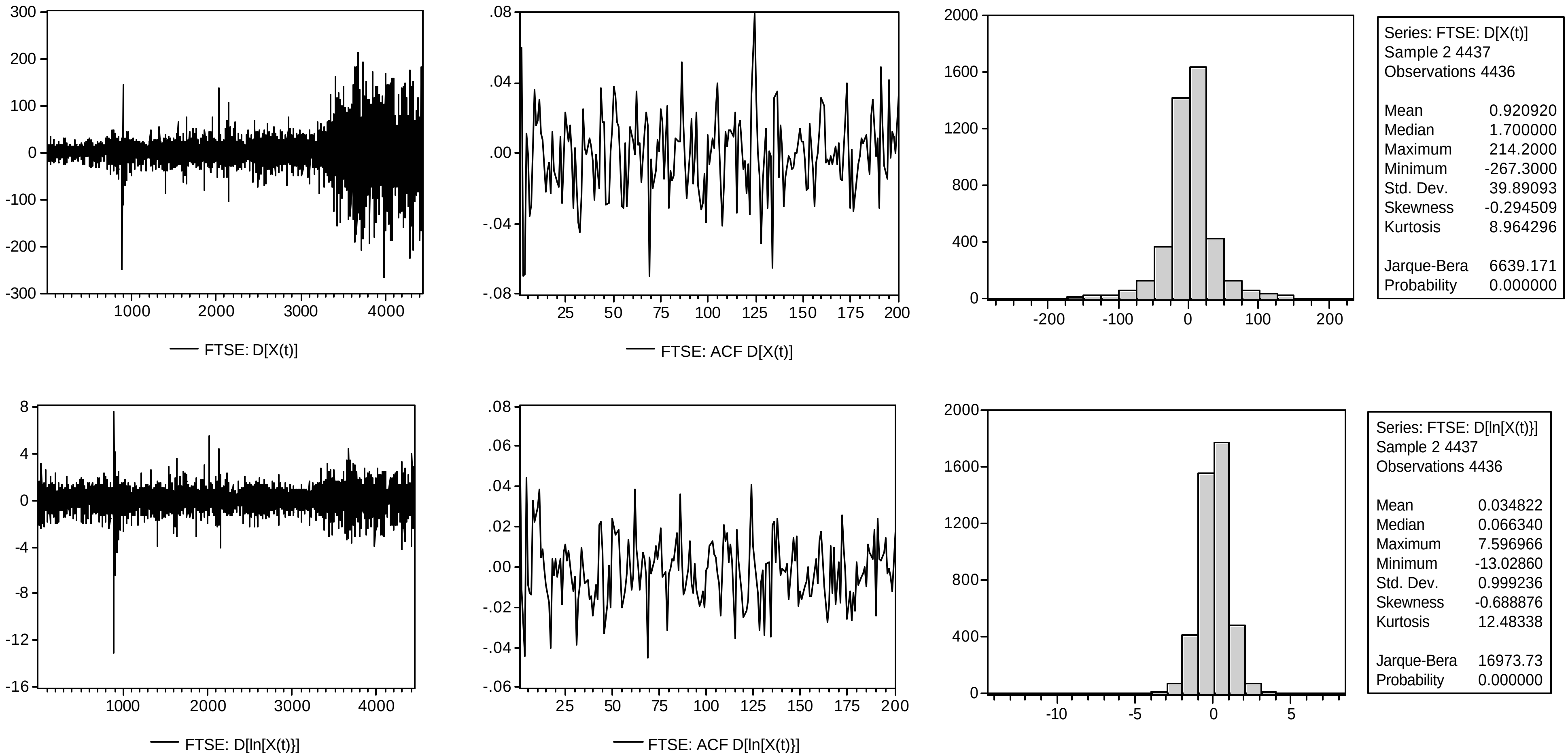
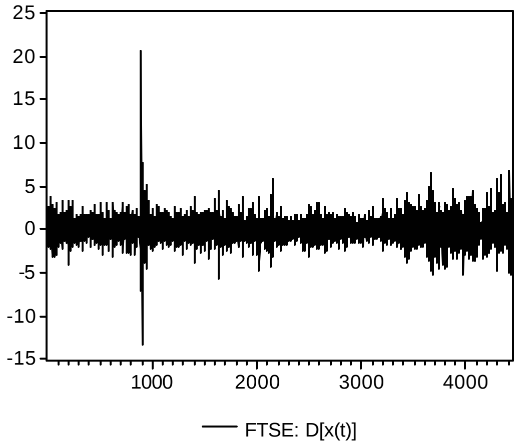
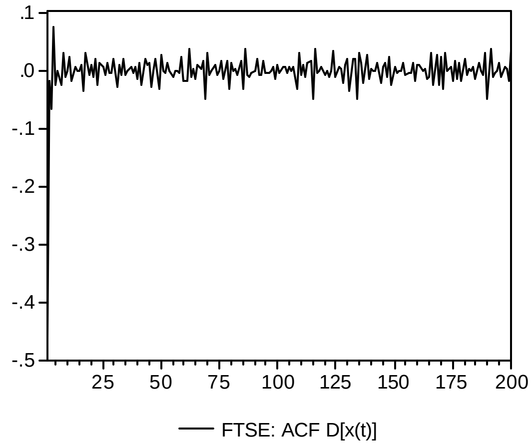


Figure 1 Continued

Plots of index level and its transformations	**Plot of the autocorrelation function up to 200 lags for the index level and its transformation**	**Empirical distributions of the index level and its transformations**





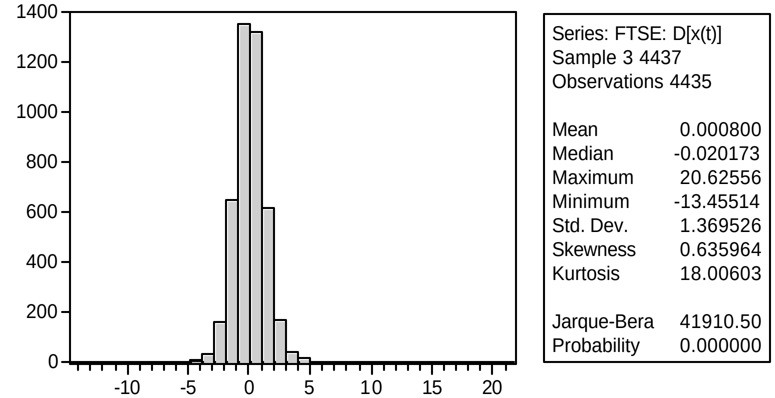


Figure 2 Increasing window moments for FTSE index level.

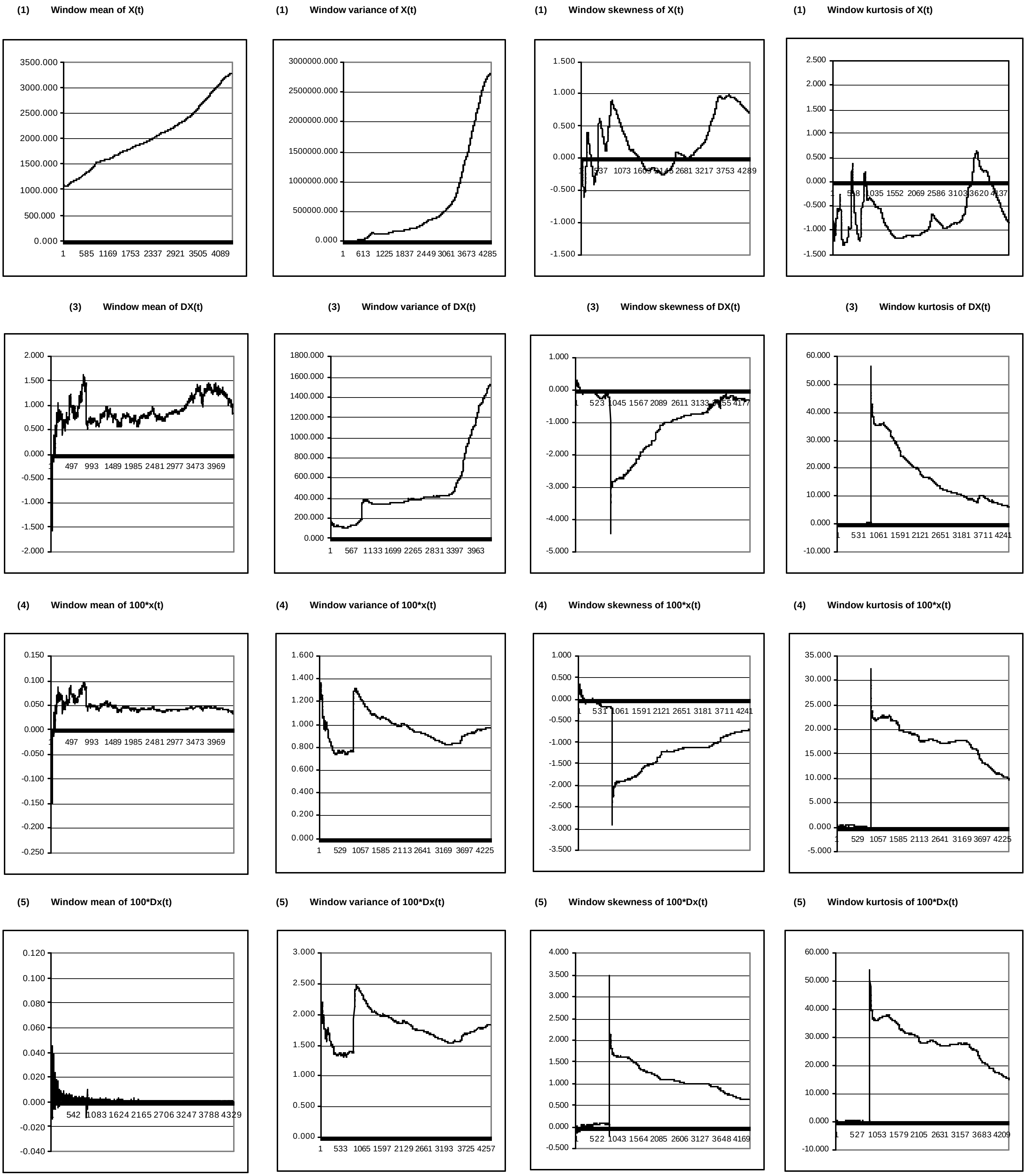
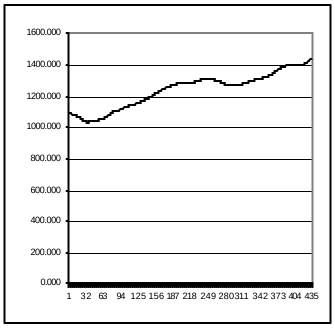
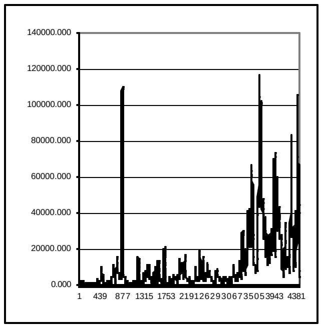


Figure 3 Plots of moving moments (50 observations window) for FTSE index level and its transformations.

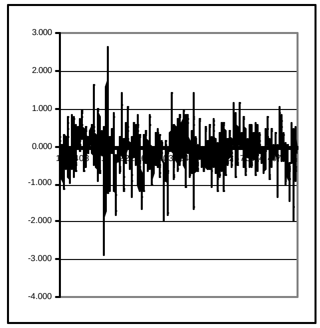
(1) Moving mean of X(t)



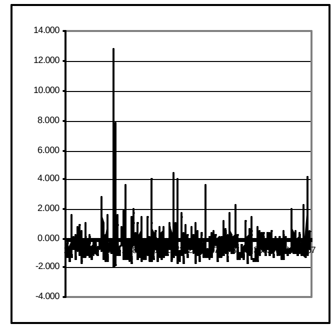
(1) Moving variance of X(t)



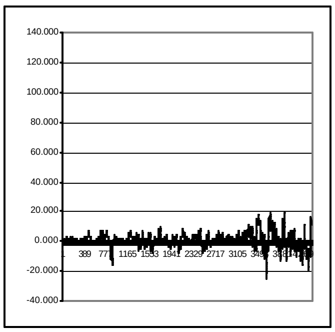
(1) Moving skewness of X(t)



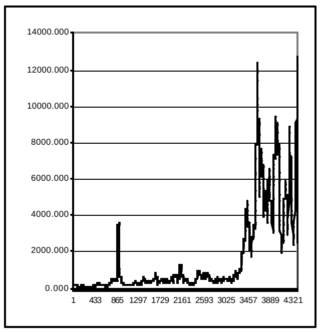
(1) Moving kurtosis of X(t)



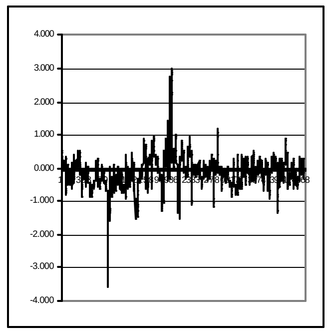
(3) Moving mean of ΔX(t)



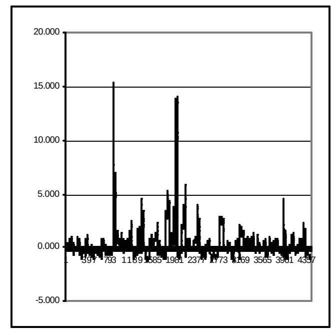
(3) Moving variance of ΔX(t)



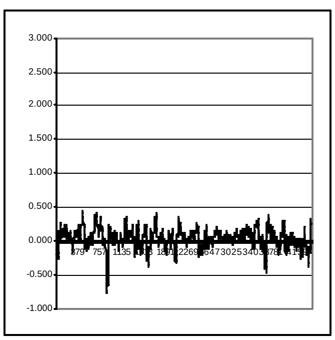
(3) Moving skewness of ΔX(t)



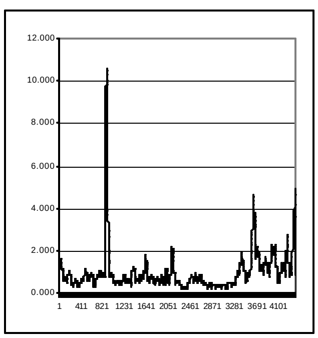
(3) Moving kurtosis of ΔX(t)



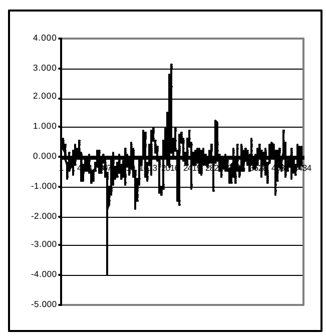
(4) Moving mean of 100*x(t)



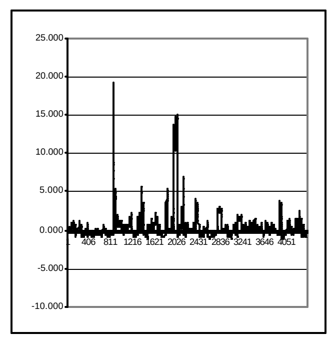
(4) Moving variance of 100*x(t)



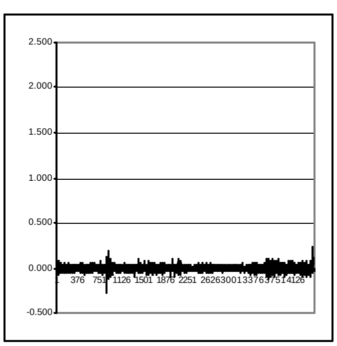
(4) Moving skewness of 100*x(t)



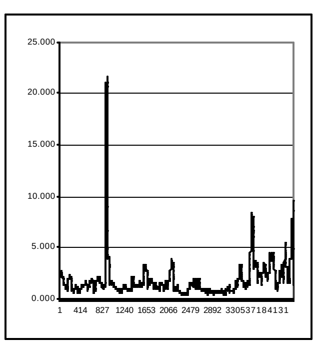
(4) Moving kurtosis of 100*x(t)



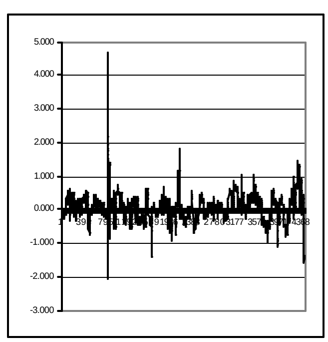
(5) Moving mean of 100*Δx(t)



(5) Moving variance of 100*Δx(t)



(5) Moving skewness of 100*Δx(t)



(5) Moving kurtosis of 100*Δx(t)

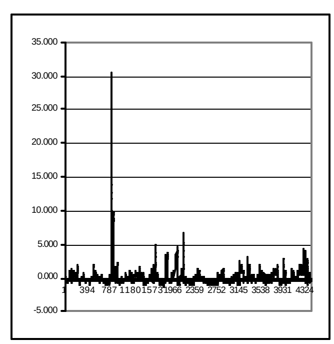


Figure 4. ACF functions for five series, X(t), ln{X(t)}, D[X(t)], D[ln{X(t)}], and D[x(t)], for FTSE index and ACF functions for geometric Brownian motion. (In the legends provided under the figures ACF stands for empirical correlation and FBM stands for autocorrelation that would exist if the data would be geometric Brownian motion.)

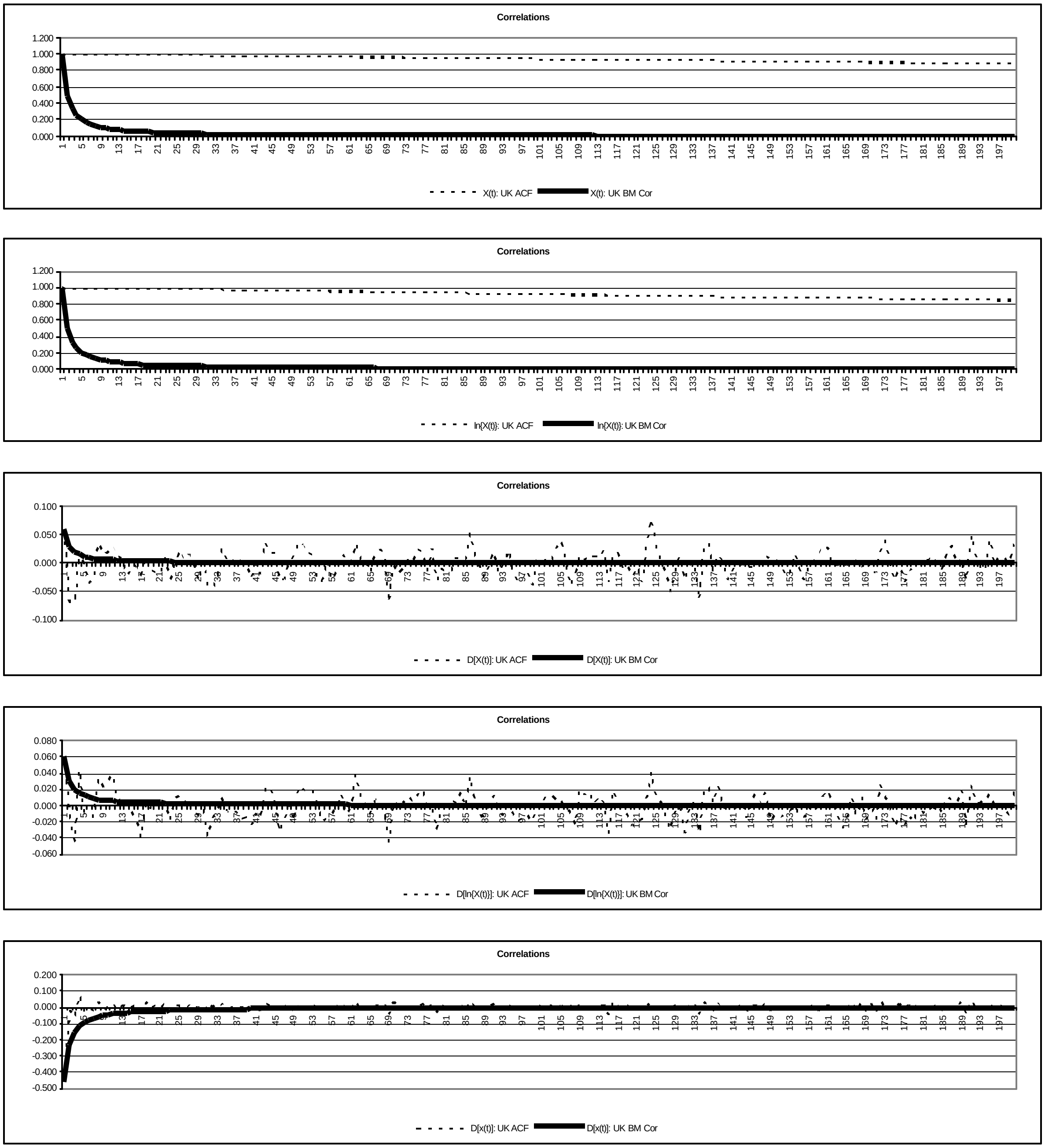


Figure 5 ACF functions for five series, X(t), ln{X(t)}, D[X(t)], D[ln{X(t)}], and D[x(t)], for FTSE index and ACF functions for Fractal Brownian Motion. (In the legends provided under the figures ACF stands for empirical correlation and FBM stands for autocorrelation that would exist if the data would be fractal Brownian motion.)

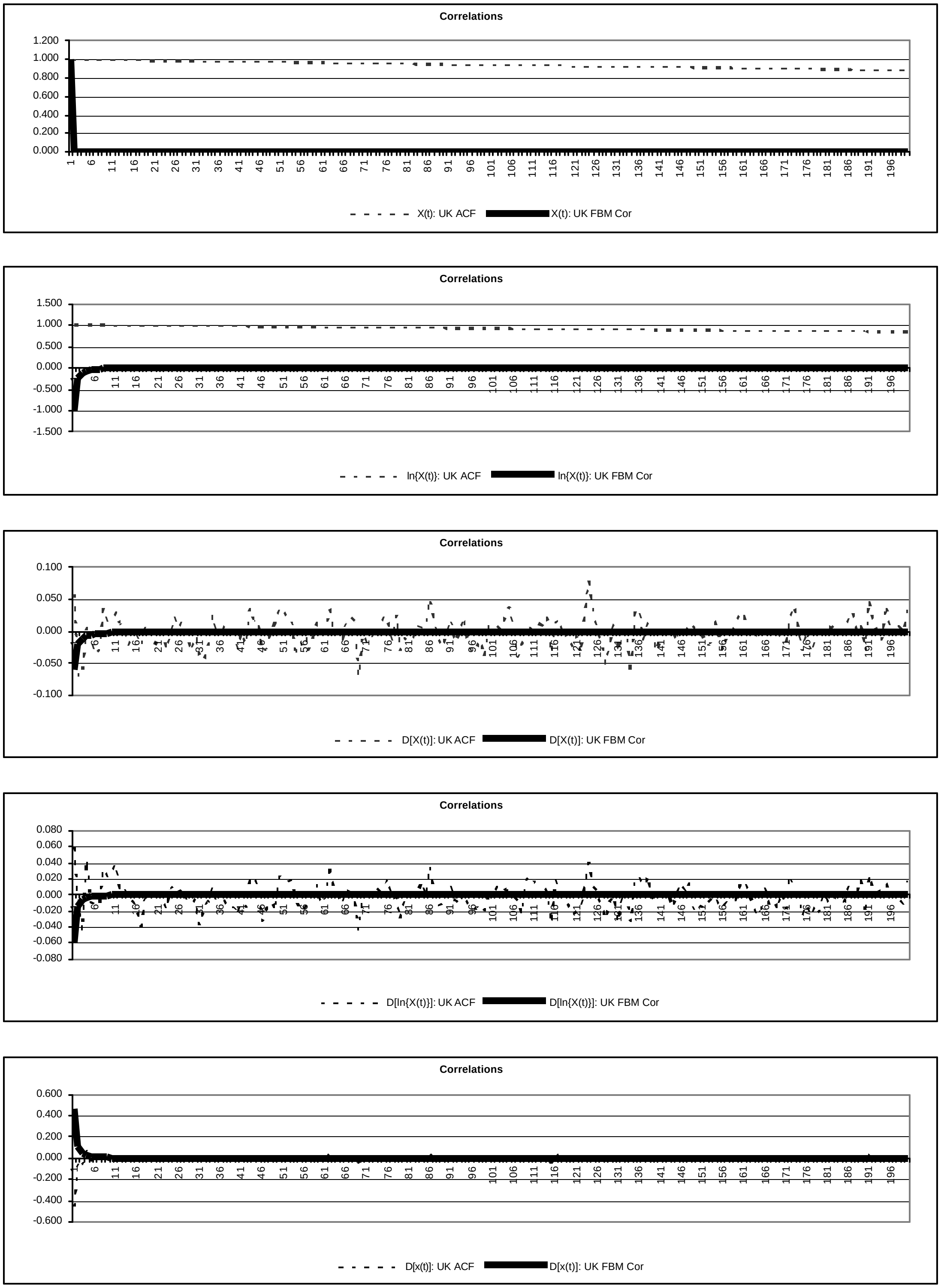


Figure 6. Scalogram and Scalegram from Wavelet Analysis

I. FTSE Index Level (Observations for April 2, 1984 – February 12, 1996)

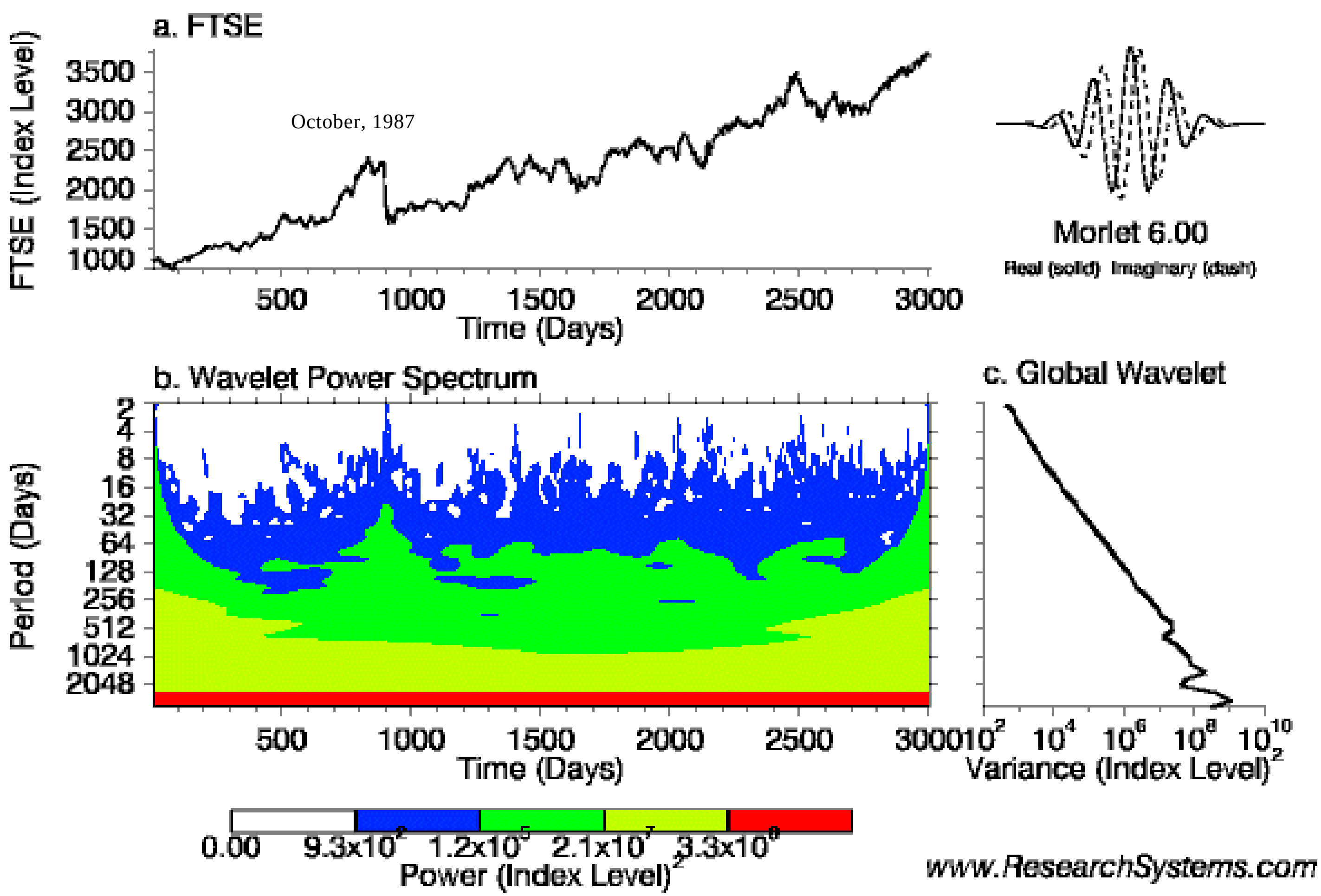


Figure 6 Continued

II. FTSE Index Level (Observations for January 2, 1990 – October 23, 2001)

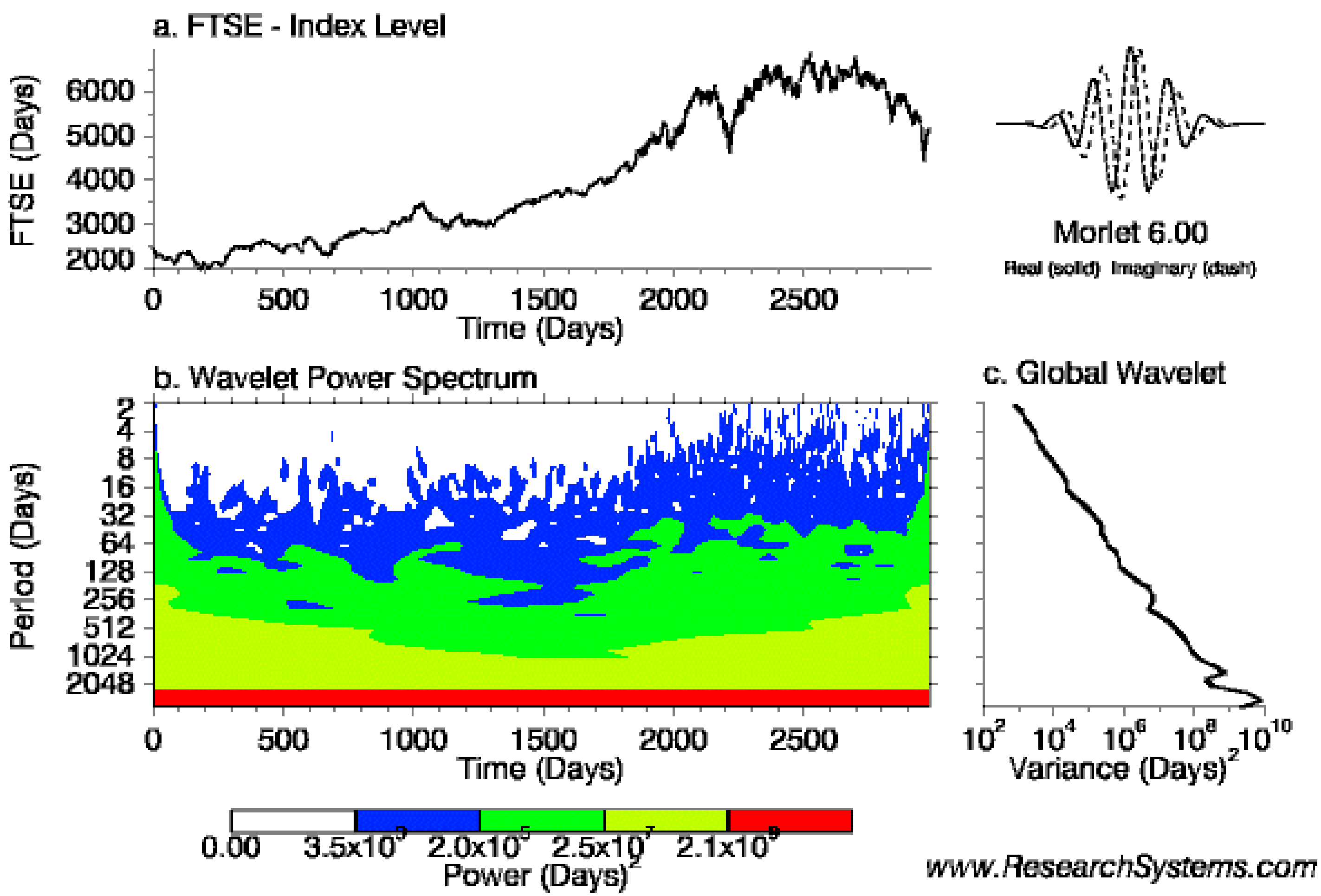


Figure 6 Continued

III. FTSE Index – Returns (Observations for April 2, 1984 – February 25, 1992)

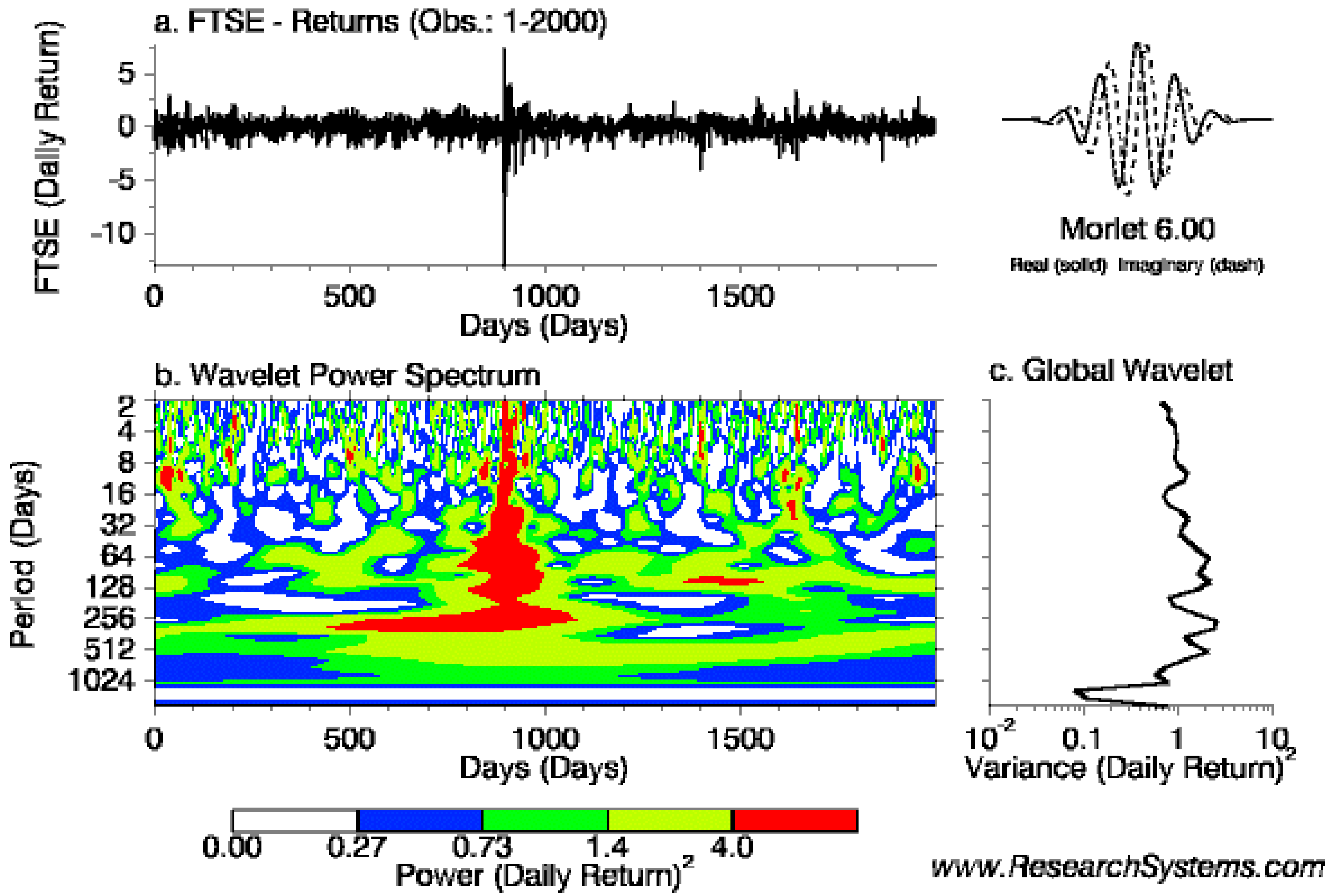


Figure 6 Continued

III. FTSE Index – Returns (Observations for February 26, 1992 – October 23, 2001)

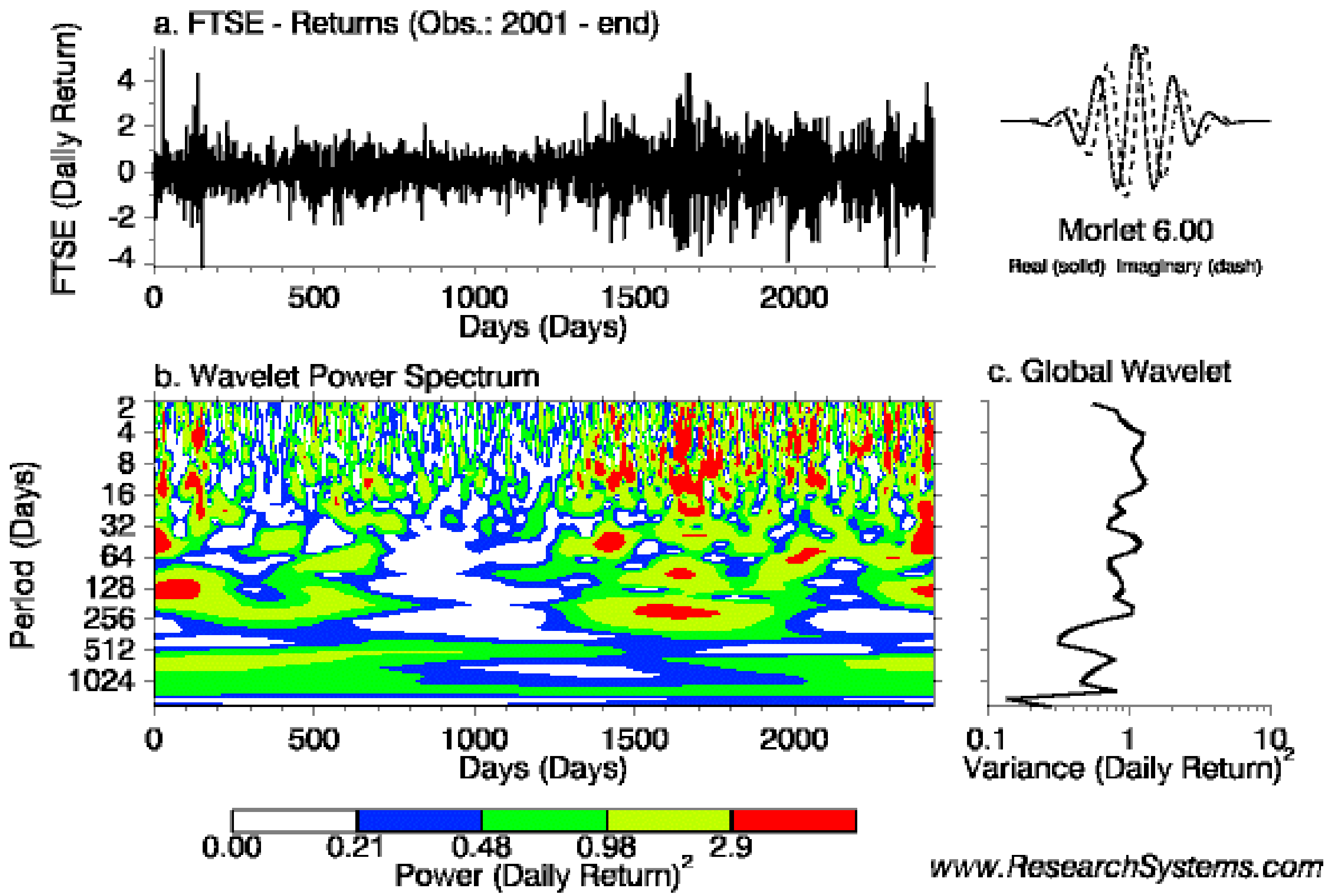
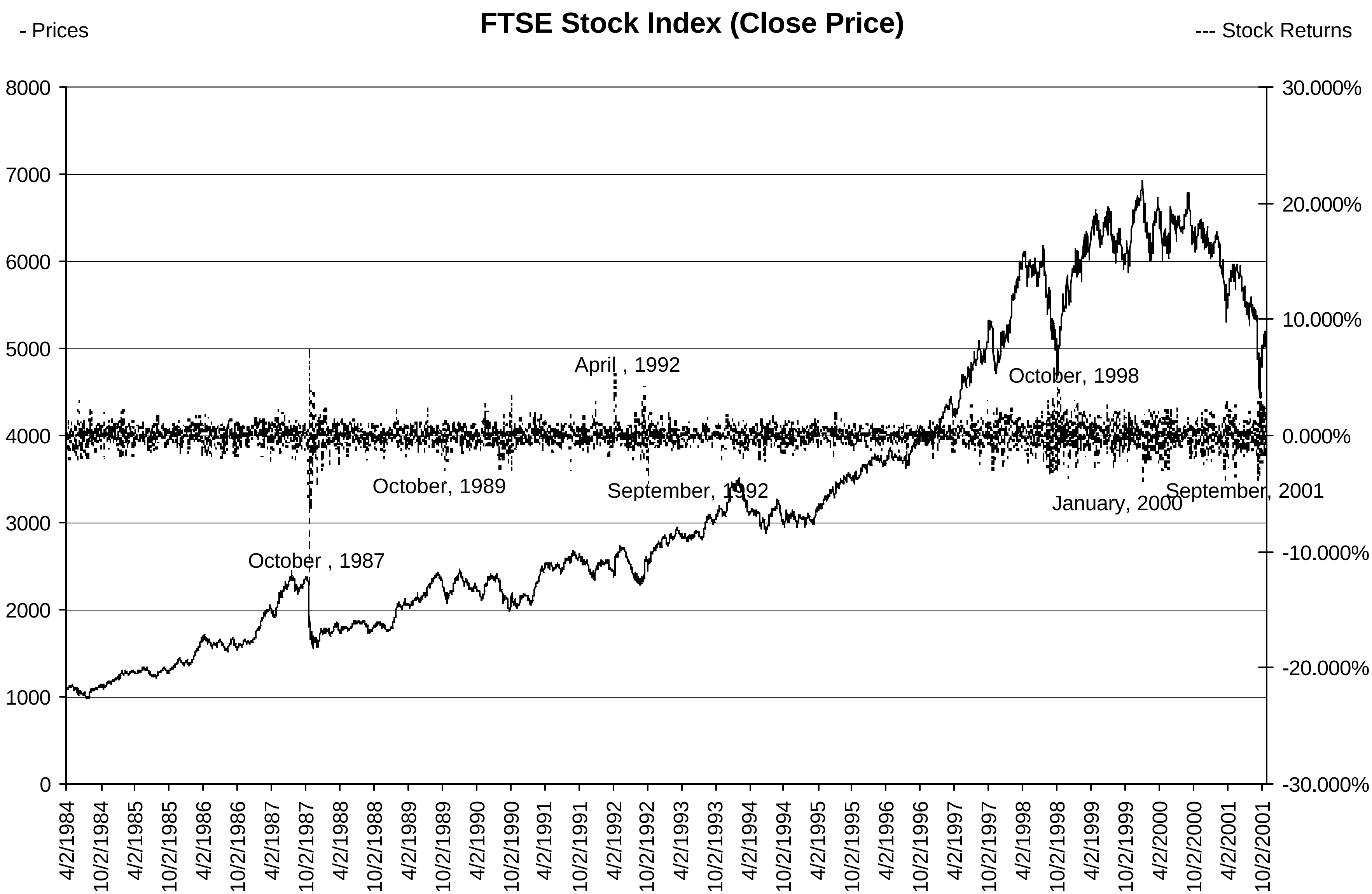


Figure 7


FTSE Stock Index (Close Price)
- Prices
--- Stock Returns
8000
7000
6000
5000
4000
3000
2000
1000
0
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
October , 1987
October, 1989
April , 1992
September, 1992
October, 1998
January, 2000
September, 2001
4/2/1984
10/2/1984
4/2/1985
10/2/1985
4/2/1986
10/2/1986
4/2/1987
10/2/1987
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