NOISE INFLUENCE ON FRACTAL DIMENSION IN THE PROCESS OF GENERATION OF THE JULIA FRACTALS

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Fractalii pot fi întâlniti în natură într-o mare varietate de domenii. Una din principalele lor caracteristici este mărimea fracţionară a dimensiunii geometrice. Spre deosebire de fractalii determinişti la care auto-replicarea poate fi explorată la orice scară, in sistemele reale aceasta este limitata de prezenta zgomotului, cu influente asupra dimensiunii fractale. Ne propunem să explorăm in această lucrare efectul zgomotului asupra mărimii dimensiunii fractale, calculată prin metoda box-counting. Analiza este efectuată prin introducerea zgomotului în mărimea parametrilor care intervin în procesul de generare a fractalilor. Rezultatele obținute prezintă interes în caracterizarea sistemelor haotice prin intermediul dimensiunii fractale.

Fractals are encountered in nature in many domains. One of their main characteristics is the fractional geometric dimension. As opposed to deterministic fractals in which the self- similarity can be explored at any scale, in real systems this is limited by the presence of noise, that influences the fractal dimensions. Our goal is to explore the noise effect on fractal dimension, using the box-counting method. The analysis is done by randomizing the fractal generation parameters. The obtained results are interesting in the characterization of chaotic systems by fractal dimension.

Keywords: Fractal dimension, noise.

1. Introduction

The dimension problem is essential in the study of all knowledge domains: essentially it deals with the determination and the characterization of the minimum number of quantities (geometrical coordinates, physical parameters, etc.) necessary for the univocal description of the system. Even if intuitively the geometric dimension problem is evident (a one, two or three dimension body has the dimensions 1, 2, or 3 respectively), the algorithm of determination of the number of dimensions of a system and of their respective values is generally an open problem, with some notable exceptions. Thus in geometry the definitions of Borel and Lebesgue have been specified and algorithmized by Hausdorff [1-3] in a rigorous way. Starting from the difficulty of characterizing the complex forms

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and processes in nature (by example the clouds, the new liquid crystal textures, the temporal evolution of the stock market, etc.) using the Euclidian geometrical forms, Mandelbrot [4] introduced the concept of fractal. In an imprecise but suggestive definition [5, 6], the fractal is an object in which any of its parts is a reduced size-copy of the entire. Applying the Hausdorff algorithm for computing the dimension, fractional values are obtained for the fractal dimension. We mention that there are several equivalent variants of the Hausdorff dimension [5-7]: the similar dimension, the box-counting dimension, the correlation dimension, the Lyapunov dimension, etc. They are different in accuracy and computing speed.

In a generalized way [8], the construction of a fractal reproduces (see fig.1) the evolution of the processing unit studied in the presence of a feedback line, with the control parameter, denoted by CU in fig. 1. According to the analysis [8], the evolution of this system can generate complex structures even in the case of a very simple form of the feedback function.

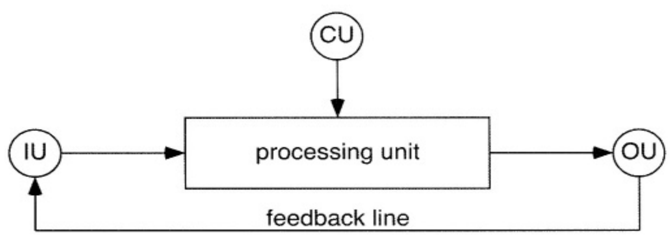
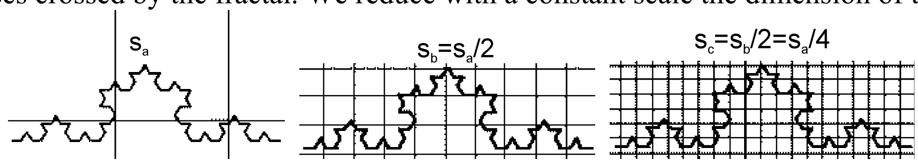


Fig.1 Fractals generation seen as the evolution of a feedback system.

If the parameters of the feed-back function and the value of the control parameter are constant in time, the generated fractal is a regular fractal; if these parameters are random, the fractal will be a random fractal. In this paper, we examine the noise parameters dependency of the fractal dimension of some random fractals. This study is of practical interest when the quantitative differentiation of similar systems is needed. Since the real systems have random evolution parameters, one should know their influence on the fractal dimension, and on the application based on it, respectively.

2. Theoretical considerations

We shell use the box-counting for the determination of the fractal dimension. This method is also known as the Minkowski-Bouligand dimension [7], and it uses a grid devided by evenly-spaced squares, of dimension s (the squares of s side below). The grid is superposed on the figure for which the fractal dimension is to be determined (a Koch fractal in fig. 2). Let N be the number of cases crossed by the fractal. We reduce with a constant scale the dimension of the



(a) (b) (c)

Fig.2. Calculus steps in the box-counting method of the fractal dimension.

squares (the scaling factor is 1/2 in fig. 2: $s_c=s_b/2=s_a/4$) and we obtain the dependency :
$\ln(N)=f(s)$.

The box-counting dimension is the exponent $N=ct\cdot s^{-D}$ of the power function. Practically, it is the slope of the dependency $\ln(N)=f(s)$, obtained for different values of the cases (fig. 2a, b, c). For the calculation of the fractal dimension we have used the soft program Fraclab from Matlab. It allows the calculus of the fractal dimension by the box-counting method, on graphic files.

3. Results and discussions

We shall examine the influence of the generation noise on a Julia fractal [5], for which the feed-back (see fig.1) is ensured by the iteration function of complex variable *z=x+iy*:

$$z_{n+1}=z_n^p+c\,. \qquad (1)$$

The coefficient c is the control parameter CU in fig.1, z_{n+1} is the calculated value obtained by the processing unit at the output OU and z_n^p is the input value IU. The values of z_n are computed beginning with a start value conveniently chosen and are plotted in the complex plane x-y. In the iteration process, a noise with an uniform probability distribution was introduced. We denote by a the coefficient of variation of the randomized parameters . The function used from the Mathematica soft program is:

$$f_z = 1+RandomReal[\{-a, a\}] \qquad (2)$$

This function multiplies the parameter considered; consequently, we obtain a random variation of the parameter during the iteration process. For the variation of the control parameter c, we have obtained (see fig. 3 for some examples) a moderate influence of the noise on the frontier of the Julia fractal; for the regular fractal we have considered $p=2$ and $c=-0.5$. As one can see from fig. 3b-d the shape of the fractal with the randomized control parameter remains identical to that of the regular fractal (fig.3a) at large scale. At small scale, the images will no longer be similar, and the differences increase with the increase of the amplitude.

If we randomize the exponent p from the function (1) by mean of the function (2), with the mean value $p = 2$, the noise influence becomes dramatic. As one can see in fig. 4, the presence of a noise that randomly modifies the exponent with a=0.1 (fig. 4d) practically destroys the fractal. This proves how small parameters variations in the iteration function lead to the transition from a fractal with a clearly defined frontier to a fractal with a more and more diffuse frontier.

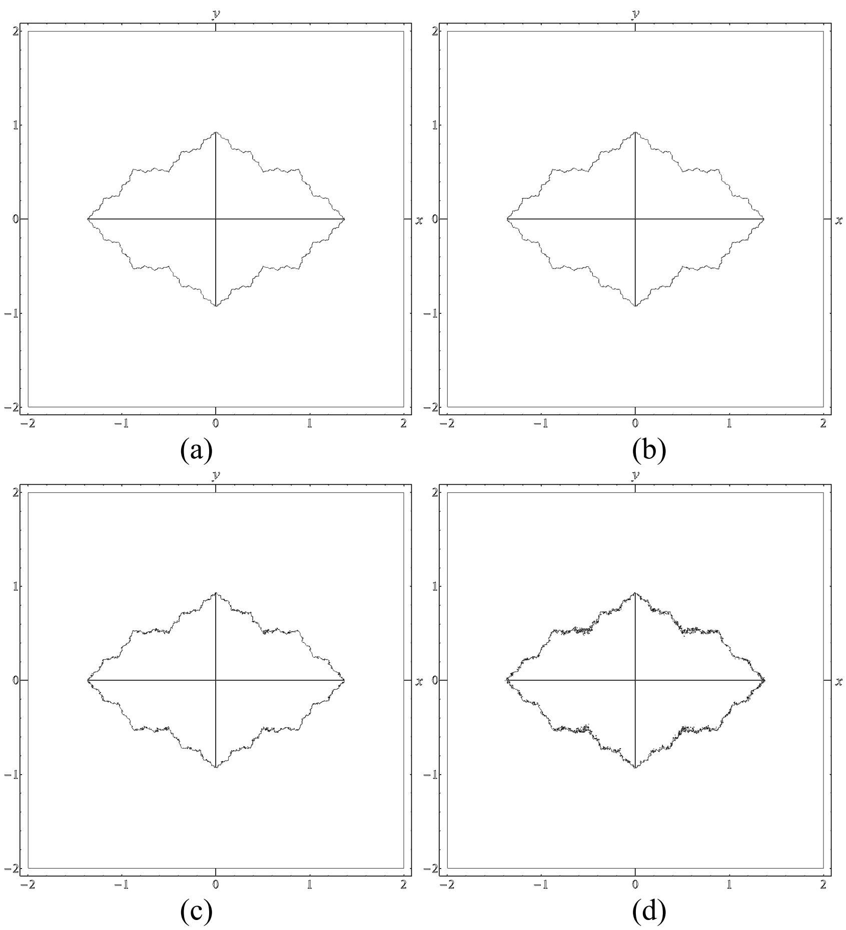


Fig. 3. The Julia fractal: the control parameter p is randomized by choosing the parameter *a* as 0 (fig. 3a); 0.01(fig. 3b); 0.05 (fig. 3c) and a = 0.1(fig. 3d).

The fractal dimension was measured as a function of the amplitude a of the noise (1) and it is represented in fig 5. As one can see, the fractal dimension increases with the noise amplitude. The increase presents a saturation process, which is normal, because when the surface is uniformly covered with iteration points, the dimension tends towards the value 2, specific to a bi-dimensional surface. The relatively slow variation of the dimension lead to the conclusion that the analysis of the fractal dimension permits however a reliable pattern recognition in the case of the variation of the control parameter. Exception makes the variation of the

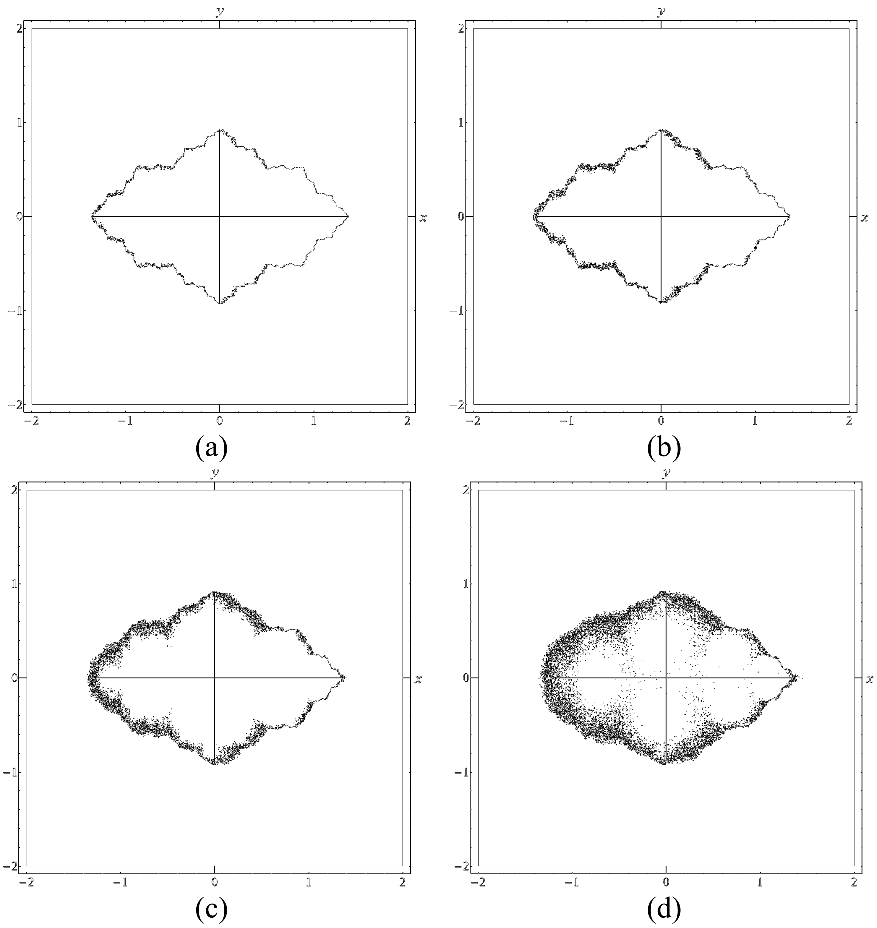


Fig.4 Julia fractal with the p exponent randomized by choosing the parameter a = 0.01; 0.02; 0.05; 0.1; in figures a, b, c and d respectively.

exponent in the iteration function, in this situation the system behavior back away from the fractal one. It is interesting to study the noise influence for various values of the power p in the reaction function. In fig. 6 is represented the noise influence on the fractal generation, for the case of randomizing the power p with a noise amplitude $a = 0.1$ [see equation (2)]. As one can see in fig. 6, a variation of the exponent numerically equal to the noise amplitude (p varies from p = 2 in fig 5d to p = 1.9 in fig 6a, and the noise amplitude is constant a = 0.1), conserves the general shape of the fractal. Comparing the figures, one observes however the sensible change of shape and the change of the fractal dimension from $D=1.63$ in case of fig. 5d, to $D=1.72$ in fig. 6a. At the increase of the exponent value, the

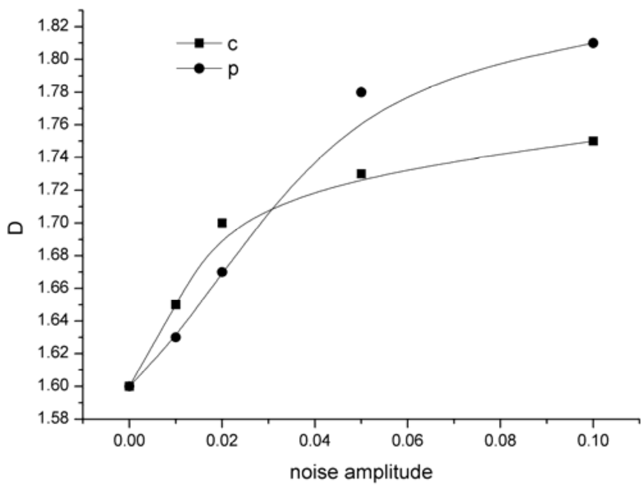
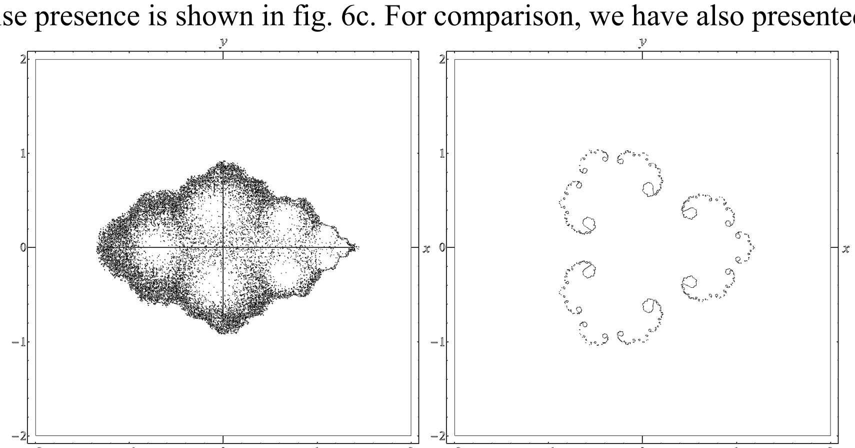
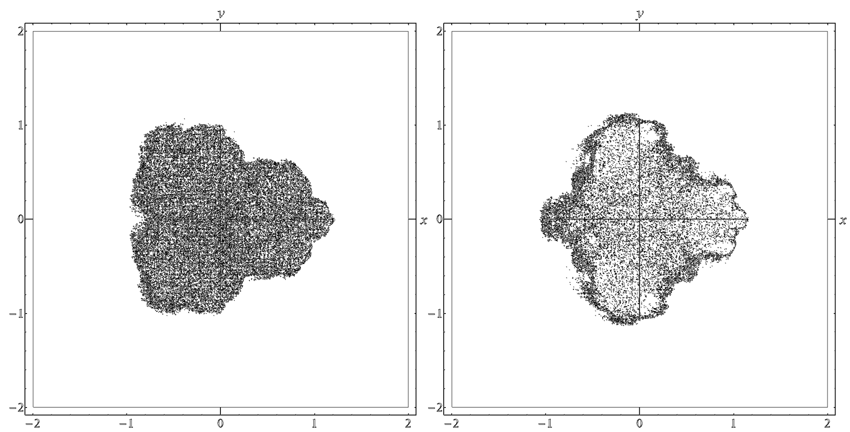


Fig. 5. Variation of the fractal dimension versus noise amplitude 'a' in the case of randomizing the control parameter c (squares) and the exponent p (circle) respectively.

generated fractal was null for $p \in (2, 3)$. For $p = 3.0$, the regulated generated fractal (without noise) was shown in fig. 6b; the corresponding fractal generated in noise presence is shown in fig. 6c. For comparison, we have also presented the



(a) (b)



(c) (d)

Fig.6. Julia fractal generated in the presence of noise: a) p = 1.9, a = 0.1; b) p = 3, a = 0; c) p = 3, a = 0.1; d) p = 3.5, a = 0.1

case of a random fractal with $p = 3.5$ in fig. 6d. The images show the strong influence of the feedback function in the fractal generation. One observes that the topological entropy of the random fractal [7] increases qualitatively with the value of the exponent p; we intend to establish quantitatively the correlation between the box-counting dimension, the topological dimension and the parameters of the feed-back function for the random fractals in a future work. Also, we intend to study in the future the correlation between the liquid crystal texture shape, the phase transitions and the fractal dimension.

4. Conclusions

In this work we have analyzed the noise influence on the fractal parameters on the fractal generation process. We have reached the conclusion that the randomizing of the iteration function has much higher effects than the control parameter. A large modification of the fractal dimension, in the case of noise systems (noise pictures, temporal serials of Brownian type) is an indication of the statistical character of the phenomena governing the system evolution. When increasing the value of the exponent p in the iteration function (1), the influence of the noise increases in the sense that the fractal frontier becomes more and more diffuse.

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