SEGMENTED FRACTAL DIMENSION MEASUREMENT OF 1-D SIGNALS: A WAVELET BASED METHOD

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ABSTRACT

Fractal dimension is an important characteristic of signals that contain information about their structure complexity. Although, fractal dimension estimation of geometric objects has shown very good result, it is a big problem to estimate the dimension of a 1-D signal. As the dynamics of the signal may vary over time, then its fractal dimension also vary over time.

This paper shows our estimations of fractal dimensions of 1-D signal based on wavelet coefficients which are obtain from maxima lines of continues wavelet transform. Instead of their exact estimated values, we more interested to analyze their dimension variation over time.

Our measurement shows that stationer signal has relatively constant dimension, at the other way, non-stationer signal has varying dimension. These measurements lead us to have a different method of signal characterization.

Keyword: *fractal dimension, wavelet, maxima lines, 1-D signal*

1. INTRODUCTION

Fractal models arise frequently in a variety of scientific such as physic, chemistry, and biology. In the field of digital signal processing, fractal model has proven useful for application such as data network analysis, texture analysis or image compression. A very important characteristic of fractals is their dimension, which shows their degree of complexity. Since its concepts came from geometric approach, fractal analysis of geometric object becoming very natural. We can find a number of fractal dimension estimation based on geometric approach [7].

Although the fractal geometry is the most popular, there are numerous 1-D signals with fractal properties such as speech signals and fractional Brownian motion (fBm). The 1-D signals representing these measurements are fractals in the sense that their graph is a fractal set. In this research, we like to measure the complexity of the signal by estimate its fractal dimensions. Measuring fractal dimension of geometric object using the standard methrod like the famous box counting is very obvious. However, measuring fractal dimension of a 1-D signal becoming more complex task.

Petros Maragos[1] has developed an algorithm to measure the fractal dimension for 1-D signal using morphological covering method. In this paper we also deal with the problem of measuring segmented 1-D signal. Based on our previous work[8] , estimation of fractal dimensions using wavelet transform modulus maxima are not accurate, as it follows inequality instead of equality. Thus, we more interested in analyzing their trend of dimension variation over time rather than their exact values.

This paper organizes as follows: Section 2 gives a brief discussion to a number of fractal dimension measurements using geometric approach. Section 3 described a number of 1-D fractal signals. Section 4 focuses on the discussion of wavelet transform modulus maxima approach introduced by Mallat and Wang[6] , which applied to fractal dimension measurement. Our method of estimation described in Section 5 followed in Section 6 for our experiment result and discussion. Section 7 is conclusions of this research.

2. FRACTAL DIMENSION

The concepts and methods used for classic geometric model and analysis are based on regular shape of objects. But, we have known there are some objects that have irregular shape. These objects cannot be analyzing using classical geometric method. The fractal geometry introduced by Mandelbrot[7] gives us concepts and practical technique for irregular objects.

A fractal dimension describes how many pieces of a set are resolved as the resolution scale decreases [3] . It tells us the degree of chaos or dynamic properties of the system.

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The relationship between measurement (of the pieces) and measuring scale r has a power law property with the following curve

$$C(r) = \alpha . r^{\gamma} \quad (1)$$

Here, α is a constant, and $C(r)$ is the number of pieces of a size r covering the signal. The analysis of such a curve is much easier in a log-log domain in which it becomes a straight line

$$\log C(r) = \log \alpha + \gamma \log r \quad (2)$$

The slope of line, γ, is the fractal dimension.

Since a fractal is self-similar, a fractal dimension can be evaluated by comparing a property between any different two scales, i.e.,

$$d_C = \frac{\log(C(r))}{\log(r)} \quad (3)$$

In practice, the correlation dimension d_C is evaluated by fitting $\log(C(r))$ versus $\log(r)$ over a range of r, where the accuracy of the determination of $C(r)$ is affected by the finite number of data.

3. ONE DIMENSIONAL FRACTAL SIGNALS

We have four different signals to evaluate.

1. Basic sinusoidal, this is not a fractal signal. We have known that this signal is stationer, thus we expect to have information from our method, which will show us that the analyzed signal is stationer. We use this signal to determine that the method is good enough to describe the signal characteristics.
2. Time varying sinusoidal, a basic self-similar signal, where the frequency is time varying.

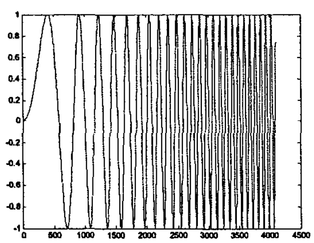


Figure 1: Time varying sinusoidal

3. Weierstrass Cosine Function (WCF) defined as :

$$W_H(t) = \sum \gamma^{kH} \cos(2\pi\gamma^k t), \quad 0 < H < 1 \quad (4)$$

It is continuous but nowhere differentiable. Its fractal dimension is D = 2 – H.

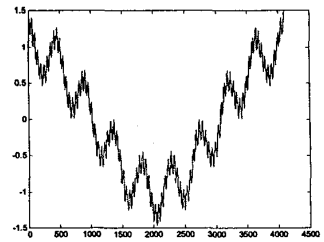


Figure 2: Weierstrass Cosine Function

4. Fractional Brownian motion (fBm), a popular synthesis fractal signal, is a time varying random function with stationary, Gaussian distributed, and statistically self-affined increment. The fractal dimension D of fBm (H,t) is D = 2 – H.

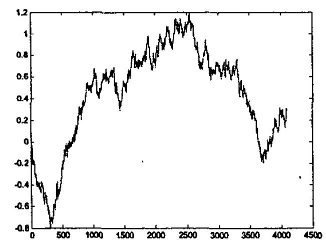


Figure 3: Fractional Brownian motion

The first three signals are generated by equation, the fBm signal generated by **Fraclab**, a fractal toolbox for **Matlab** developed by INRIA Fractal Laboratory[9] .

4. WAVELET TRANSFORM MODULUS MAXIMA

One method to characterize a signal $x(t)$ is to decompose it into elementary building blocks of simple waveforms $x_i(t)$ as $x(t) = \sum_i \alpha_i x_i(t)$, where α_i are weighting scalars. We can define a mother wavelet, ψ, which is a fixed function, as the simple waveform. A mother wavelet can be modified by translations and dilations to have a family of functions with scale a and a position b,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right), \quad a > 0, b \in \Re \quad (5)$$

Goupillaud, Grossman and Morlet[10] define a continuous wavelet transform to be

$$W_f(a,b) = \langle f, \psi_{a,b} \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-a}{b}\right)dt \quad (6)$$

The factor $1/\sqrt{a}$ ensures that the function has constant energy.

Suppose that the wavelet $\psi(t)$ and the function we analyze $f(t)$ are both real, and $W_f(a,b)$ is the wavelet transform of $f(t)$, then we define [6]

- Modulus maxima are points (a_0, b_0) such that $|W_f(a_0,b)| < |W_f(a_0,b_0)|$ when b belongs to either a right or left neighborhood of b_0.
- Maxima lines, are any connected curve in the scale space (a,b) along which all points are modulus maxima

The wavelet transform modulus maxima, WTMM, are defined as the set of $W_f(a,b)$, where (a, b) are points on the real line that are arbitrarily close to some modulus maxima in the scale-space. All singularities of $f(x)$ can be located by following the maxima lines as the scale goes to zero.

Suppose that there exists a scale $a_0 > 0$, and a constant C, such that all the modulus maxima of $W_f(a,b)$ belong to a cone defined by

$$|b - b_0| \le Ca \quad (7)$$

The function $f(x)$ has Lipschitz exponent γ at b_0, if and only if there exists a constant α such that at each modulus maxima (a,b) in the cone defined by (10)[6]

$$|W_f(a,b)| \le \alpha . a^{\gamma+0.5} \quad (8)$$

or equivalently

$$\log|W_f(a,b)| \le \log \alpha + (\gamma+0.5) \log a \quad (9)$$

Using similar approach we can use WTMM to estimate the Lipschitz exponent γ of a function $f(t)$.

5. FRACTAL DIMENSION MEASUREMENT

In this research, we estimate the fractal dimension of signal using wavelet scale, a, as substitution to the box in the box-counting method. This method gives us two measures, fractal dimension for every maxima lines, D_l, and fractal dimension for the whole segmented signal, D_s.

We measure the dimension with the following steps:

i. Calculate the continuous wavelet transform (CWT) of the signal.
ii. Find maxima points of the CWT signal. Maxima points are all points, which their absolute values are greater than its adjacent points.
iii. Find the skeletons of maxima points. Skeletons are lines connecting maxima point at a specific scale to another nearest maxima point at different scales.
iv. Find the dimension of the specific skeleton, D_l, by taking the slope of maxima values and scale, a, in logarithmic scale.

6. RESULT AND DISCUSSION

We have analyzed four synthetic signals and calculate their fractal dimension. These figures below show fractal dimension evaluate at every maxima lines.

1. Sinusoidal signal

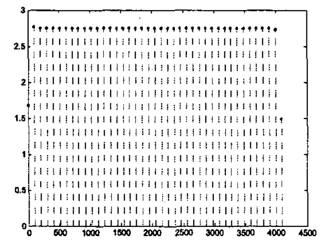
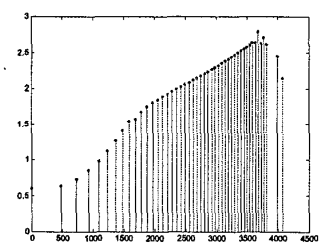


Figure 4: Fractal dimension of sinusoidal

As shown at Figure 4 above, fractal dimension of sinusoidal signal remain constant over time. We can see that the signal is stationer.

2. Time varying sinusoidal



The dimension varies over time as the signal's frequency varies over time. We can see the increasing frequency by maxima lines at which became narrower. We can see anomaly at left and right boundary at which the trend of fractal dimension differ from the other location. We should mention that there is some errors in wavelet transform at the left and right boundaries of the segmented signal, which lead to dimension anomalies at the boundaries.

3. Weierstrass Cosine Function

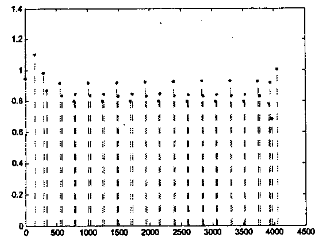


Figure 5: Fractal dimensions of WCF

We can see that the fractal dimension of WCF has regularity. The dimension tends to remain constant. Pairs of maxima lines are spaced at their locations with relatively constant period. From its function at Figure 2 we can see that this function has a constant period. The dimension affected mainly by Holder exponent, H, and its period affected by γ.

4. Fractional Brownian Motion

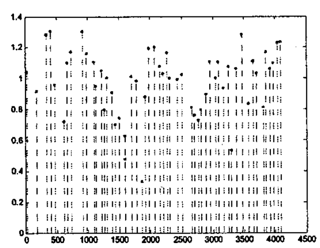


Figure 6: Fractal dimension of fBm

The dimension varies over time with irregular period of maxima lines. For this signal, we are not interested to analyze dimension for instantaneous time. This will lead us to analyze the whole fractal dimension over the segmented signal.

7. CONCLUSIONS

We have estimated fractal dimension of some 1-D signals by estimating fractal dimension of the maxima lines of wavelet transform modulus maxima. Using this method we can evaluate signal for stationer or non-stationer of the signal for a segmented time frame. Instead of evaluate the exact values of the fractal dimension estimation; we prefer to analyze the trend of fractal dimensions to time.

A stationer signal shows as a set of relatively constant of fractal dimension and maxima lines over time. A non-stationer signal shows as varying fractal dimension and maxima lines over time. Estimation of fractal dimension for an instantaneous time gives us an opportunity to develop an algorithm to do a real-time evaluation the dynamics of signal.

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