Subprime Mortgage Crisis Detection in U.S. Foreign Exchange Rate Market by Multifractal Analysis

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Abstract

We apply multifractal methods to analyze the impact of American subprime mortgage crisis on American foreign exchange market. By analyzing the local Hölder exponent α and the multifractal spectrum of Japanese Yen to U.S. Dollar (USD/JPY) exchange rate and Euro to U.S. Dollar (EUR/USD) exchange rate ticked every hour from late 2006 to early 2008, we find that the periods where critical events took place are characterized by sudden increase in α, which passes above 1, followed by very small values for long period of time. To observe the effect and future trend of the subprime mortgage crisis, we compare the multifractal spectrum before, and during the crisis. The width of f(α) during subprime mortgage crisis is much bigger than the one before the crisis. These results provide solid and important values for further study on the dynamic mechanism of exchange market price fluctuation.

***Keywords**: Subprime mortgage crisis; foreign exchange market; local Hölder exponent; multifractal spectrum*

1. Introduction

In the past, the efficient market hypothesis (EMH) was used to analyze the capital market. However, recent researches have shown that the capital market is not “informationally" efficient and the changes are not strictly random. Instead, the capital market can be regarded as an extreme complex system which is characterized by multifractal properties.

Fractal theory, which was created by Mandelbrot [1], is one of the greatest achievements in the 20th century. Edgar E.Peters[2] first used the concept of Fractal market to describe the economic systems. Since then, many studies have found that there are some nonlinear properties in the time series of financial markets [3, 4]. Among all the models, the multifractal model is regarded as the most comprehensive one to describe the feature of the complexity in economic systems. It can describe the financial series with multifractal spectrum and can be used to analyze and predict the market’s performance. Many studies have applied multifractal method to financial time series. For example, H.Katsuragi[5] analyzed the Japanese stock market and found the evidence of multi-affinity. F.Feng[6] used the multifractal analysis on stock market returns in China. Additionally, Ding-sun Ho [7] and X sun [8,9,10] conducted multifractal analysis on Taiwan Stock index and Hang Seng index, and confirmed the existence of multifractal characteristics. However, these studies mainly focused on verifying the fatal weakness of EMH method and the existence of multifractal properties in finance market. There has been little research about how to build up dynamic multifractal models and how to make full use of the information from these models to analyze and predict the real financial market.

The subprime mortgage crisis, which began to manifest itself in early 2007 since the bursting of the housing bubbles and the high defaults rate on the subprime mortgages, has triggered a financial crisis worldwide in 2007 and 2008. It is still uncertain that how long the crisis will last and when it will come to the end. Since multifractal concept is a well known feature of complex system such as financial system, in this paper, we apply multifractal model to analyze the impact of the subprime mortgage crisis on American foreign exchange market. By calculating the local Hölder exponent and its spectrum, we can have a deep insight into the relation between the crisis and the fluctuation of the foreign exchange rate.

978-0-7695-3398-8/08 $25.00 © 2008 IEEE
DOI 10.1109/ICYCS.2008.454

2. Methodologies

2.1 Local Hölder Exponent

Define the local Hölder exponent α(t) as
$\alpha(t) = \sup\{\beta \geq 0 \mid f(t+\Delta t) - f(t) = O(|\Delta t|^{\beta}), \Delta t \to 0\}$
where f(t) is the function in a neighborhood of t.

The local Hölder exponent has a direct explanation, that is

$$f(t+\Delta t) - f(t) \propto C_t \, |\Delta t|^{\alpha(t)} \qquad (1)$$

where C_t is the pre-coefficient. The value of α(t) describes the singularity of the function or the financial time series at t. If α(t) becomes larger, the function or the time series becomes more regular. So the local Hölder exponent is the most essential characteristic of the multifractal analysis.

One method can be used to estimate α(t) of the stock index series, exchange rate series, future price series and so on. First, the financial price time series p_i is normalized to probability $\Pr_i$, by $\Pr_i = p_i / \sum p_i$ .The normalized series is then divided into N parts with equal length $\varepsilon(\varepsilon < 1)$. Suppose the growing probability of different growing interface areas is $\Pr_i(\varepsilon)$. According to the fractal physics, different areas have different growing probabilities, indicated by the local Hölder exponent α,

$$\Pr_i(\varepsilon) \propto \varepsilon^{\alpha}, i = 1,2,3,...,N \qquad (2)$$

If α are the same in all areas, it is monofractal. Otherwise it is multifractal. Let $N_{\alpha}(\varepsilon)$ be the number of areas with same α, thus

$$N_{\alpha}(\varepsilon) \propto \varepsilon^{-f(\alpha)} (\varepsilon \to 0) \qquad (3)$$

where f(α) is called scale exponent or fractal dimension or singular spectrum .

2.2 Multifractal Analysis

The main task of the multifractal analysis is about the distribution of the character of the spectrum function and then to understand the interior subtle structure and information of the system. Let P(t) be the asset price.

Let $X(\Delta t) = \log P(\Delta t) - \log P(0)$,

and $X(i\Delta t, \Delta t) = X((i+1)\Delta t) - X(i\Delta t)$.

Divide [0, T] into N intervals of length Δt and define sample sum:

$$Z_q(T, \Delta t) = \sum_{i=0}^{N-1} | X(i\Delta t, \Delta t)|^q \qquad (4)$$

where q is a real number ranging from $-\infty$ to $+\infty$ and $Z_q(T, \Delta t)$ is called partition function. For multifractally distributed measures, $Z_q(T, \Delta t)$ varies with the time resolution as:

$$Z_q(T, \Delta t) \propto \Delta t^{\tau(q)} \qquad (5)$$

where $\tau(q)$ is called scaling function .It can be obtained by plotting log $Z_q(T, \Delta t)$ ~log Δt .If there's a strong linear relationship between $\log Z_q(T, \Delta t)$ and $\log \Delta t$,the data set is considered as multifractal .The slope of the curve is $\tau(q)$.The multifractal spectrum f(α) can be obtained through q and $\tau(q)$ as:

$$\alpha = \frac{d\tau(q)}{dq} \qquad (6)$$
$$\tau(q) = q\alpha(q) - f(\alpha)$$

From the plot of $\alpha \sim f(\alpha)$, we can easily obtain the width of the multifractal spectrum, $\Delta\alpha, (\Delta\alpha = \alpha_{\max} - \alpha_{\min})$. It can describe the difference of maximum and minimum probability subsets.

3. Data and parameters

3.1 Data

In order to analyze the impact of subprime mortgage crisis on foreign exchange rate which emerged since early 2007 and widely spread till 2008, we choose the closing prices of USD/JPE exchange rate and EUR/USD exchange rate both ticked every hour at the same period, that is, from November 2006 to February 2008, being a total of 13353 samples, are used in our study. See Figure 1, 2.

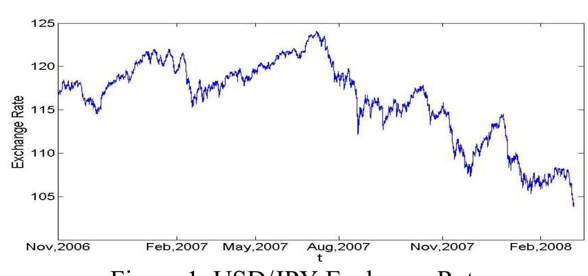

Figure 1. USD/JPY Exchange Rate

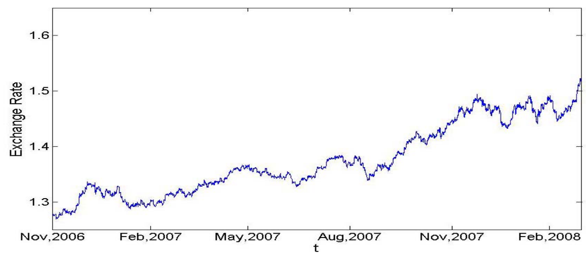

Figure 2. EUR/USD Exchange Rate

3.2 Parameters

To calculate the local Hölder exponent α(t), we directly followed the definition in formula (1). For time t, we measure how the fluctuation of the signal behaves with response to the different sizes of the neighborhood. Nine values of 1, 4, 8, 12, 16, 20, 24, 28 and 32 are used as the size of neighborhood.

For Legendre spectrum f(α), we use the box method with the parameter q, just as mentioned in 2.2, changing from -10 to 10 by increment of 0.1.

And these methods are applied by Fraclab toolbox in Matlab.

4. Result

4.1 Review of economic events

Let us remind the most significant events related to the subprime mortgage crisis in U.S. from November 2006 to February 2008. All of them had different effects on the U.S. economy and brought great impacts to the foreign exchange market.

February - March 2007: Subprime industry collapse; more than 25 subprime lenders declaring bankruptcy, announcing significant losses or putting themselves up for sale.

July - August 2007: Worldwide "credit crunch" as subprime mortgage backed securities are discovered in portfolios of banks and hedge funds around the world, from BNP Paribas to Bank of China. Many lenders stop offering home equity loans and "stated income" loans. Federal Reserve injects about $100B into the money supply for banks to borrow at a low rate.

Late October - Early November 2007: Federal Reserve injects $41B into the money supply for banks to borrow at a low rate. It had been the largest single expansion by the Fed since $50.35B on September 19, 2001.

Late December 2007: President Bush announced a plan to voluntarily and temporarily freeze the mortgages of a limited number of mortgage debtors holding adjustable rate mortgages.

4.2 USD/JPY Exchange Rate

4.2.1 α(t) and logarithm return

In Figure 3 we can see a picture with logarithm return of USD/JPY exchange rate from November 2006 to February 2008. Meanwhile, as shown in Figure 4, we calculated the values of the local Hölder exponents α(t). Both time series are at the same time-scale. Hence, their values correspond with each other.

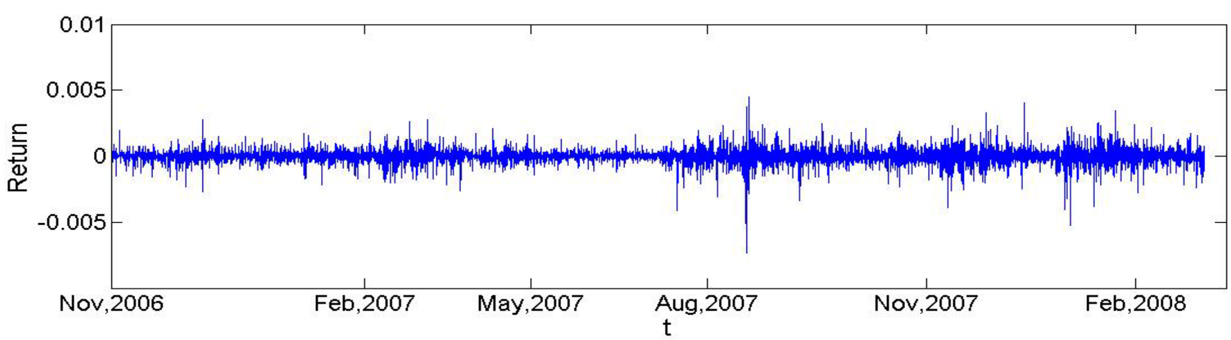

Figure 3. Return of USD/JPY Exchange Rate

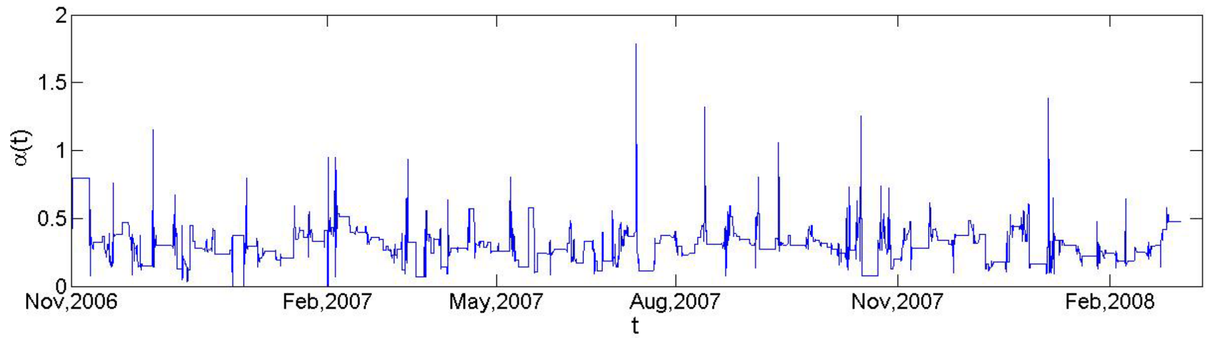

Figure 4. α (t)-t of USD/JPY Exchange Rate

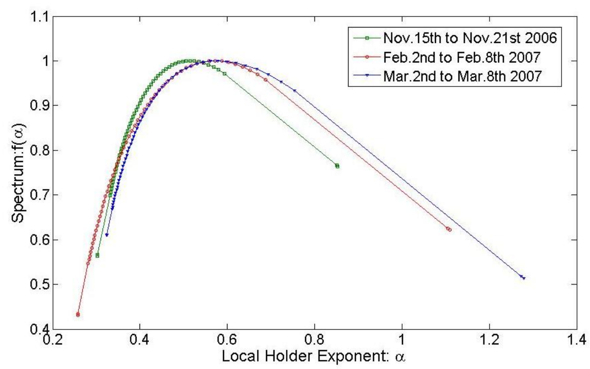

Figure 5. f(α)- α of USD/JPY Exchange Rate From Nov.2006 to March 2007

Extreme cases of Local Hölder exponents are known as follows. As α=1, it coincide with class of differentiable functions. As α=0, it coincide with class of function with irremovable discontinuity. As α is between 0 and 1, it means that the function is continuous but not differentiable at the considered point. Thus, the Hölder exponent is the measure of function irregularity.

In Figure 4 there are more high values of local Hölder exponent from June 2007 than before. This demonstrates that the USD/JPY exchange rate fluctuated significantly. Correspondingly in Figure 3, it appears that the logarithm return deviates largely from 0 at the same period, that is, from June 2007. This can be a strong support to the great fluctuation of the USD/JPY exchange rate since that time.

The important events related to the subprime mortgage crisis in U.S. have a specific signature in values of the local Hölder exponent. We can see that α reached its highest value at the end of July and also significantly in August, October of 2007 and at the beginning of 2008. Comparing to the events listed in 4.1-Review of economic events, the results really fit very well with reality. This indicates that the periods where critical events took place are characterized by sudden increase of the local Hölder exponent in regularity, which passes above 1 and followed by very small values of α for long period of time.

4.2.2 Multifractal Spectrum Analysis

Just as defined in section 2.2-Multifractal Analysis, the width of the multifractal spectrum $\Delta\alpha$ can describe the difference between maximum and minimum probability subsets. Hence, it indicates the unevenness of probability distribution and the strength of multifractal. The larger the value of $\Delta\alpha$,the stronger the strength of multifractal. It reflects the range of the variation.

Figure 5 shows the multifractal spectrum based on three different periods: Nov. 15^{th} – 21^{st}, 2006, February 2^{nd} – 8^{th} and March 2^{nd} – 8^{th}, 2007. Though all of them analyzed the data in seven days, they have different $\Delta\alpha$. We can see the multifractal spectrum become wider and wider from late 2006 to early 2007. That means the effect of subprime mortgage crisis become more and more significant on international current market. USD/JPY exchange rate is one specific reflection, which shows us the great influence.

In Figure 6, another three weeks' multifractal spectrums are shown separately. It is obvious that the $\Delta\alpha$ value based on August 8nd – August 14th data is larger than that based on March 30th – April 5th data in 2007, while the spectrum is even wider in October. So it will be reasonable to say that the impact brought by subprime mortgage crisis still continues.

According to the definition of f(α), the $f(\alpha_{max})$ and $f(\alpha_{min})$ reflect the number of the boxes with maximum probability and minimum probability, respectively. So a positive Δ f represents that the chance of the index at the highest site is larger than that at the lowest site and vice versa. Both in figure 5 and 6, we can see that Δ f decreases from a positive value to a negative one, which is consistent with the tendency that the exchange rate of USD/JPY falls, see figure 1.

The USD/JPY exchange rate witnessed the fallout from the contraction of the subprime mortgage lending market in the United States.

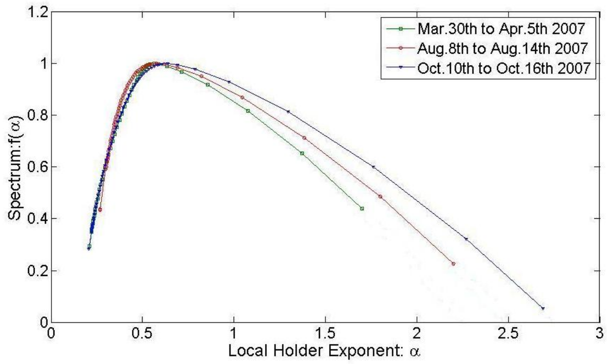

Figure 6. f(α)- α of USD/JPY Exchange Rate From May to Oct. 2007

4.3 EUR/USD Exchange Rate

Correspondingly, we analyze the performance of the EUR/USD exchange rate from November 2006 to February 2008.

4.3.1 α(t) and logarithm return

As shown in Figure 7, we can see a picture with logarithm return of EUR/USD exchange rate. Meanwhile, in Figure 8, we calculated the values of the local Hölder exponents α(t) at the same time-scale. Hence, their values correspond with each other.

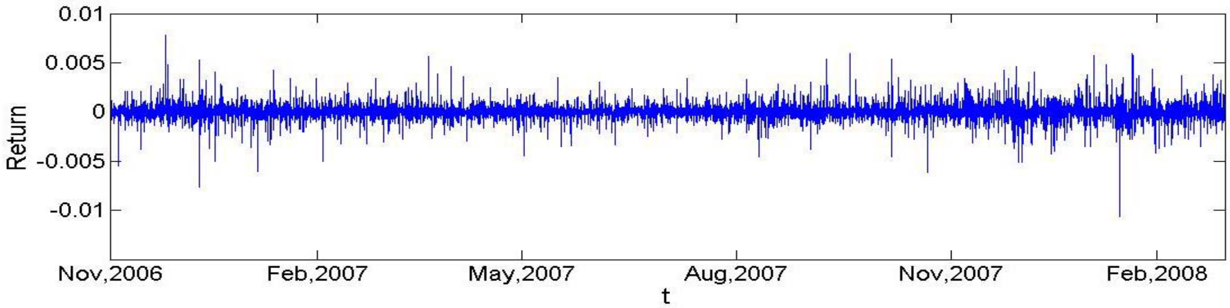

Figure 7. Return of EUR/USD Exchange Rate

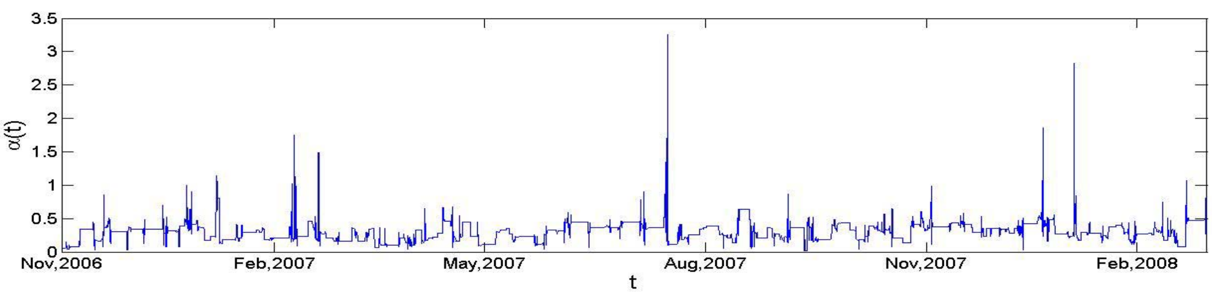

Figure 8. α(t)-t of EUR/USD Exchange Rate

We can see obviously in Figure8 that some values of the local Hölder exponent are extrusive. The one in July 2007 should be mentioned. Worldwide "credit crunch" as subprime mortgage backed securities are discovered in portfolios of banks and hedge funds around the world. The subprime mortgage crisis was taken on board formally. Also the great singularity should not be ignored in early 2008. This is the reflection to President Bush's announcement of a plan to voluntarily and temporarily freeze the mortgages of a limited number of mortgage debtors holding adjustable rate mortgages. This again demonstrates that periods where critical events took place are characterized by sudden increase of the local Hölder exponent in regularity.

4.3.2 Multifractal Spectrum Analysis

Figure 9 shows the multifractal spectrum based on three different time periods: January 10^{th} – 16^{th}, February 14^{th} – 20^{th} and March 21^{st} – 27^{th}. Though all of them analyzed the data in seven days, the multifractal spectrums become wider and wider from January to March in 2007. That means the effect of subprime mortgage crisis becomes more and more significant on international current market.

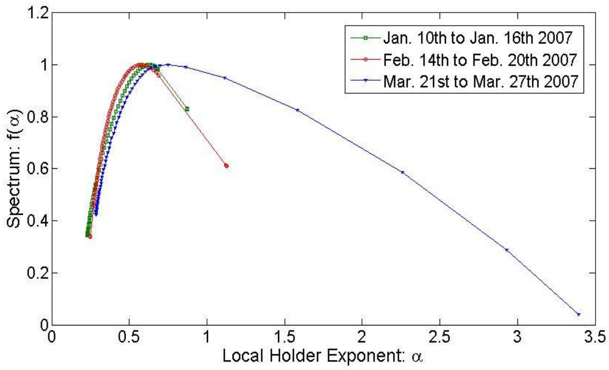

Figure 9. f(α)- α of EUR/USD Exchange Rate From Jan. to March 2007

Besides Japanese Yen, EUR/USD exchange rate is another specific reflection, which shows us the great influence.

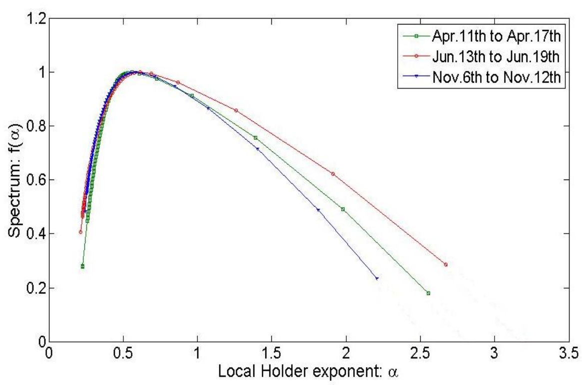

Figure 10. f(α)- α of EUR/USD Exchange Rate From April to Nov. 2007

In Figure 10, it is really obvious that the $\Delta\alpha$ value based on November 6th – 12th data is much larger than that in April and June. So it will be a good support of our conclusion made in the analysis of USD/JPY exchange rate, that is, the impact brought by subprime mortgage crisis still continues this year.

From figure 2, we can see that the exchange rate of EUR/USD goes much more smoothly than USD/JPY. This is why Δ f changes little in figure 10.

5. Conclusion

(1) Since f(α) versus α in Figure 5, Figure 6, Figure 9, Figure 10 contains multiple points rather than a single point, it confirms the existence of multifractal properties in U.S. foreign exchange market.

(2) From the comparison of logarithm return and local Hölder exponent α in Figure3 versus Figure 4 and Figure 7 versus Figure 8, we find that the periods where critical events took place are characterized by sudden increase in α, which passes above 1, followed by very small values for a long period of time.

(3) The width of f(α) after subprime mortgage crisis is much bigger than the one before the crisis. And Δ f can help us to have a better understanding of the tendency of the exchange market.

(4) The impact brought by subprime mortgage crisis still continues.

Acknowledgement

Thank Ron for providing us useful materials and encouragement despite his busy schedule.

Thank IWCFTA committee.

Thank all the members in our team for brainstorming, teamwork, highly-motivation, determination and hardworking.

References

[1] B.B. Mandelbrot, "The variation of certain speculative prices", *J. Bus. 36*, 394-419, 1963

[2] E.E Peters, *Fractal Market Analysis - Applying Chaos Theory to Investment and Economics*, Economic Science Press, July, 2002

[3] J.F.Muzy et al, Eur. Phys. J.B. 17, 537(2000); Z.Eisler et al, Physica A 343, 603(2004); L.Calvet et al., *J. of Econometrics 105*, 27(2001).

[4] J.Kwapien et al., *Physica A 350, 466(2005)*.

[5] H.Katsuragi, "Evidence of multi-affinity in the Japanese stock market", *Physica A 278(2000)*, 275-281

[6] F.Feng, "The multifractal analysis on stock market returns in China", *J.Syst Eng.Theory Practice 6(2004)*, 50-55

[7] Ding-Shun Ho, Chung-Kung Lee, "Scaling characteristics in the Taiwan stock market", *Physica A 332(2004)*, 448-460

[8] X.Sun, H.Chen, Z.Wu, Y.Yuan, "Multifractal analysis of Hang Seng index in Hong Kong stock market", *Physica A, 291(2001)*, 553-562

[9] X.Sun, H.Chen, Z.Wu, Y.Yuan, "Predictability of multifractal analysis of Hang Seng stock index in HongKong", *Physica A, 301(2001)*, 473-482

[10] Xuefeng Song, Liexun Yang, Qingren Cao, *Management Science Series*, Science Press, 305-320, 2004