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A preliminary study of the site-dependence of the multifractal features of geoelectric measurements

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Abstract
Multifractal analysis was performed to characterize the fluctuations in dynamics of the hourly time variability of self-potential signals measured from January 2001 to September 2002 by three stations installed in the Basilicata region (Southern Italy). Two stations (Giuliano and Tito) are located in a seismic area, and one (Laterza) in an aseismic area. Multifractal formalism leads to the identification of a set of parameters derived from the shape of the multifractal spectrum (the maximum α_0, the asymmetry B and the width W) and measuring the «complexity» of the signals. Furthermore, the multifractal parameters seem to discriminate self-potential signals measured in seismic areas from those recorded in aseismic areas.

Key words *self-potential signals – multifractal formalism*

1. Introduction

The dynamics underlying tectonic processes could be directly revealed by the investigation of the temporal fluctuations of self-potential signals, which may be useful to monitor and understand many seemingly complex phenomena linked to seismic activity (Johnston, 1997; Park, 1997). Self-potential field variability may be induced by stress and fluid flow field variability (Scholz, 1990). Therefore, the analysis of these induced fluctuations could yield information on the geophysical mechanisms governing normal as well as intense seismic activity. In this context, this work investigated the dynamic properties of geoelectrical signals, as they can be detected from observational time series.

Self-potential signals are the result of the interaction among very heterogeneous and not well known mechanisms, which can be influenced by the particular structure of the monitored zone (Patella *et al.*, 1997). This means that local features can be mixed with the general thereby increasing the difficulty of rightly characterizing and interpreting the signal time variations. In addition, as occurs for many environmental signals, observational data are made even more erratic by the presence of anthropic phenomena: electrical signals coming from anthropic sources may be added, *e.g.*, to the natural signal, hindering its dynamical characterization (Cuomo *et al.*, 1997; Pham *et al.*, 1998).

In a previous paper, Cuomo *et al.* (1998) analyzed the geoelectrical daily means to give information on the statistical features of the geoelectrical background noise and the inner dynamics of geophysical processes producing the electrical phenomena observed on Earth surface in seismic areas. They discussed the statistical analysis of dynamic systems based on the estimation of their

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degree of predictability, distinguishing randomness from chaos and providing a parsimonious representation in terms of autoregressive models of observations, by means of the only information coming from the time series itself.

In the study of seemingly complex phenomena, like those generating self-potential signals, methodologies able to capture the dynamic peculiarities in observational time series are particularly useful tools to obtain information on the features and causes of signal time variability. In particular, fractal techniques, developed to extract qualitative and quantitative information from time series, have been applied recently to the study of a large variety of irregular, erratic signals and have proved very useful to reveal deep dynamic features. Cuomo *et al.* (2001) detected scaling behaviour in the power spectra of geoelectrical time series, revealing the antipersistent character of the self-potential fluctuations. Telesca *et al.* (2001) proposed a new approach to investigate correlations between geoelectrical signals and earthquakes, analyzing the time variations of the fractal parameters, characterizing their dynamics. Balasco *et al.* (2002) found that self-potential measurements seem to be featured by long-range correlations with scaling exponents that indicate that the underlying geophysical process is characterized by stabilizing mechanisms.

In all the previous works, monofractal analyses were performed leading to the estimation of only one scaling exponent. Monofractals are homogeneous objects, in the sense that they have the same scaling properties characterized by a single singularity exponent (Stanley *et al.*, 1999; and references therein). The need for more than one scaling exponent to describe the scaling properties of the process uniquely indicates that the process is not a monofractal but could be a multifractal. A multifractal object requires many indices to characterize its scaling properties. Multifractals can be decomposed into many – possibly infinitely many – sub-sets characterized by different scaling exponents. Thus multifractals are intrinsically more complex and inhomogeneous than monofractals (Stanley *et al.*, 1999), and characterize systems featured by very irregular dynamics, with sudden and intense bursts of high frequency fluctuations (Davis *et al.*, 1994). The most adequate manner to investigate multifractals is to analyze their fractality or singularity spectra. The singularity spectrum quantifies the fractal dimension of the sub-set characterized by a particular exponent, that is gives information on the relative dominance of various fractal exponents present in the process. In particular, the maximum of the spectra furnishes the dominant fractal exponent and the width of the spectrum denotes the range of the fractal exponents. Therefore, the multifractal formalism appears to furnish more deep information on the complexity of a time series.

In the present work, we investigate the temporal fluctuations of self-potential data, measured in Southern Italy from January 2001 to September 2002, using multifractal formalism, to disclose typical dynamic features.

2. Data

Our data consist in eight geoelectrical time series recorded at three monitoring stations: Giuliano (Giul1 and Giul2), Tito (Tito1, Tito2, Tito3 and Tito4) and Laterza (Lat1 and Lat2). The first two stations are located in seismic sites, the third in an aseismic one. Figure 1 shows the locations of the monitoring stations. As far as the technical features of the experimental equipment are concerned, we refer the reader to Cuomo *et al.* (1997) and for the results of mono- and multiparametric preliminary statistical analysis of the monitored variables to Di Bello *et al.* (1994). Figure 2 shows the time variations of the self-potential signals measured.

3. Methods

The concept of multifractal object was developed by Mandelbrot (1974) to investigate several features in the intermittency of turbulence (Meneveau and Sreenivasan, 1991). Many authors have applied multifractality to several fields of scientific research.

Multifractal formalism is based on the definition of the so-called partition function $Z(q, \varepsilon)$

$$Z(q,\varepsilon) = \sum_{i=1}^{N_{\text{boxes}}(\varepsilon)} \left[\mu_i(\varepsilon)\right]^q. \qquad (3.1)$$

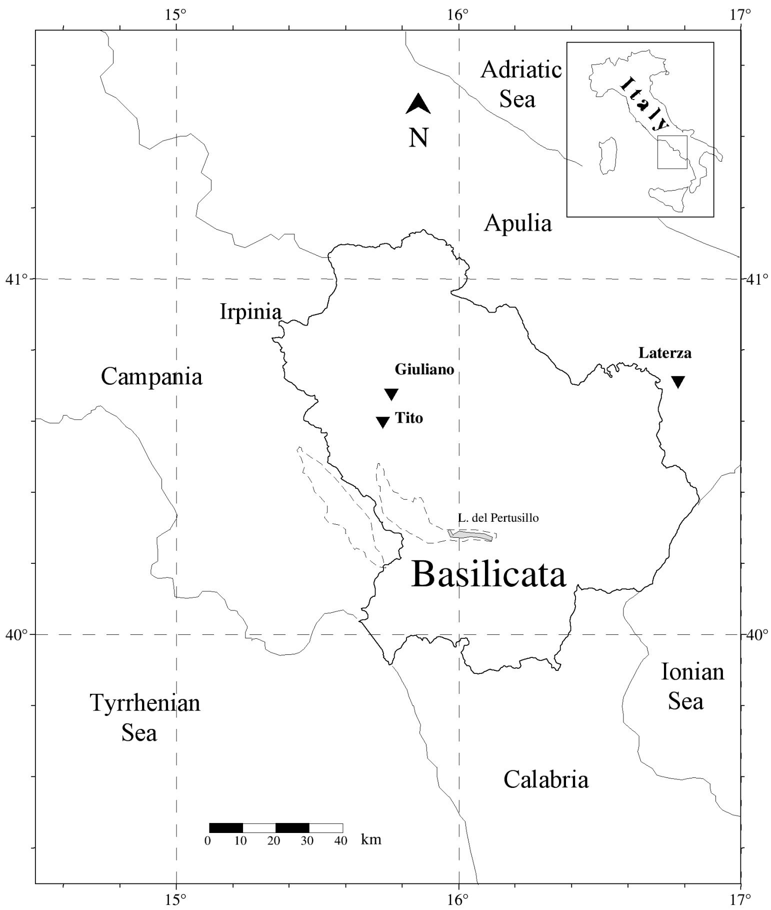

Fig. 1. Location of the geoelectrical monitoring stations Giuliano, Laterza and Tito in Southern Italy.

The quantity $\mu_i(\varepsilon)$ is a measure and it depends on ε, the size or scale of the boxes used to cover the sample. The boxes are labelled by the index i and $N_{\text{boxes}}(\varepsilon)$ indicates the number of boxes of size ε needed to cover the sample. The exponent q is a real parameter, giving the order of the moment of the measure. The choice of the functional form of the measure $\mu_i(\varepsilon)$ is arbitrary, provided that the most restrictive condition $\mu_i(\varepsilon) \geq 0$ is satisfied.

The parameter q can be considered a powerful microscope, able to enhance the smallest differences of two very similar maps (Diego *et al.*, 1999). Furthermore, q represents a selective parameter: high values of q enhance boxes with relatively high values for $\mu_i(\varepsilon)$; while low values of q favour boxes with relatively low values of $\mu_i(\varepsilon)$. The box size ε can be considered a filter, so that large values of the size are equivalent to apply a large scale filter to the map. Changing the size ε, one explores the sample at different scales. Therefore, the partition function $Z(q, \varepsilon)$ furnishes information at different scales and moments.

The generalized dimensions are defined by the following equation:

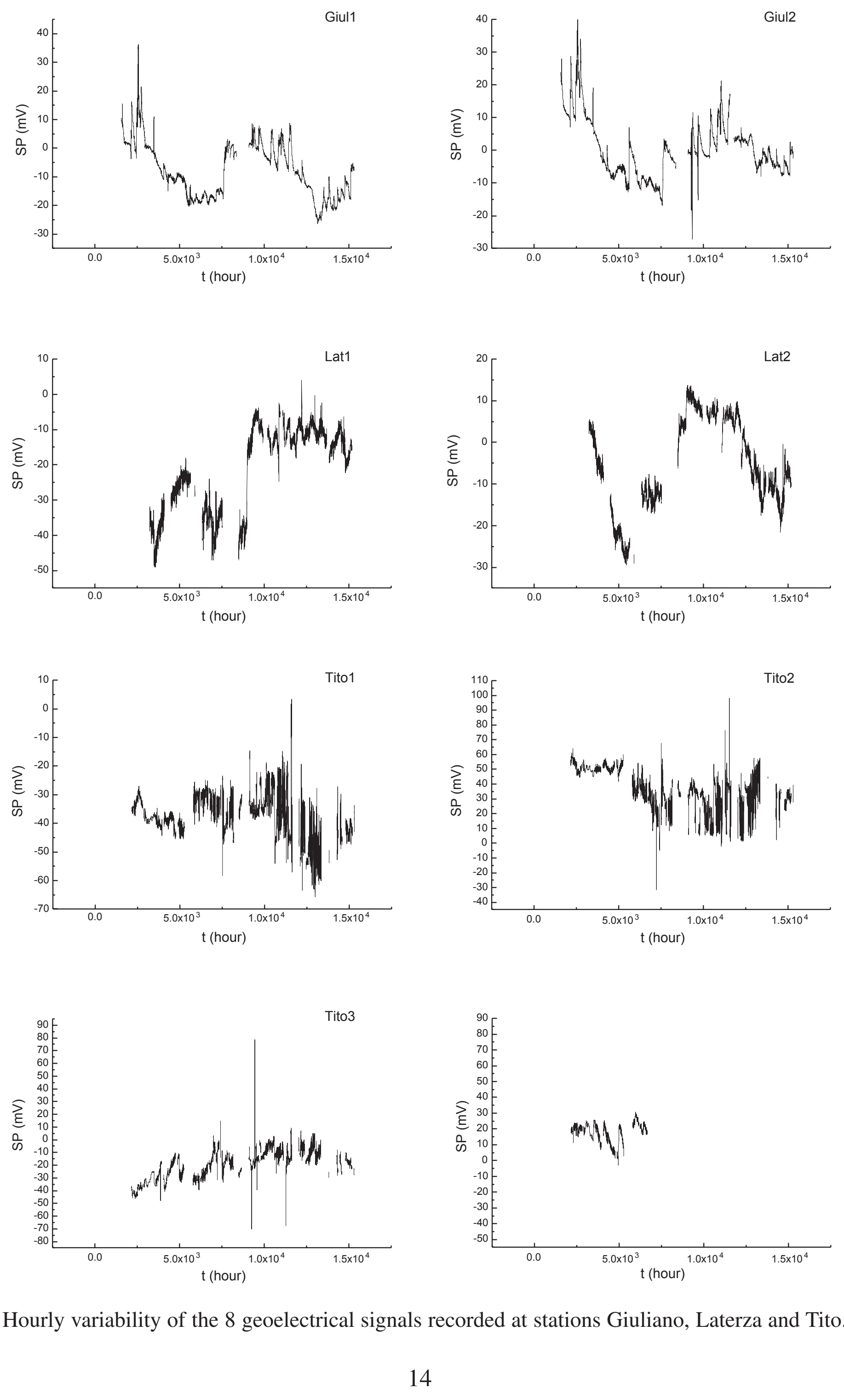

Fig. 2. Hourly variability of the 8 geoelectrical signals recorded at stations Giuliano, Laterza and Tito.

$$D(q) = \lim_{\varepsilon \to 0} \frac{1}{q-1} \frac{\ln Z(q,\varepsilon)}{\ln \varepsilon}. \quad (3.2)$$

$D(0)$ is the capacity dimension; $D(1)$ is the information dimension, and $D(2)$ is the correlation dimension. An object is called monofractal if $D(q)$ is constant for all values of q, otherwise is called multifractal. In most practical applications the limit in eq. (3.2) cannot be calculated, because we do not have information at small scales, or because below a minimum physical length no scaling can exist at all (Diego *et al.*, 1999). Generally, a scaling region is found, where a power-law can be fitted to the partition function, which in that scaling range behaves as

$$Z(q,\varepsilon) \propto \varepsilon^{\tau(q)}. \quad (3.3)$$

The slope $\tau(q)$ is related to the generalized dimension by the following equation:

$$\tau(q) = (q-1) D(q). \quad (3.4)$$

A usual measure in characterizing multifractals is given by the singularity spectrum or Legendre spectrum $f(\alpha)$ that is defined as follows. If for a certain box j the measure scales as

$$\mu_j(\varepsilon) \propto \varepsilon^{\alpha_j} \quad (3.5)$$

the exponent α, which depends on the box j, is called Hölder exponent. If all boxes have the same scaling with the same exponent α, the sample is monofractal. The multifractal is given if different boxes scale with different exponents α, corresponding to different strength of the measure. Denoting as S_α the subset formed by the boxes with the same value of α, and indicating as $N_\alpha(\varepsilon)$ the cardinality of S_α, for a multifractal the following relation holds:

$$N_\alpha(\varepsilon) \propto \varepsilon^{-f(\alpha)}. \quad (3.6)$$

By means of the Legendre transform the quantities α and $f(\alpha)$ can be related to q and $\tau(q)$

$$\alpha(q) = \frac{d\tau(q)}{dq} \quad (3.7)$$

$$f(\alpha) = q\alpha(q) - \tau(q). \quad (3.8)$$

The curve $f(a)$ is a single-humped function for a multifractal, while it reduces to a point for a monofractal.

In order to quantitatively recognize possible differences in Legendre spectra stemming from different signals, it is possible to fit the spectra to a quadratic function around the position of their maxima at α_0 by a least square method (Shimizu *et al.*, 2002)

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C. \quad (3.9)$$

Parameter B measures the asymmetry of the curve, which is zero for symmetric shapes, positive or negative for left-skewed or right-skewed shapes respectively.

Another parameter is the width of the spectrum, that estimates the range of α where $f(\alpha) > 0$, obtained extrapolating the fitted curve to zero; thus the width is defined as

$$W = \alpha_{\max} - \alpha_{\min} \quad (3.10)$$

where $f(\alpha_{\max}) = f(\alpha_{\min}) = 0$.

These three parameters serve to describe the «complexity» of the signal. If α_0 is low, the signal is correlated and the underlying process «loses fine structure», becoming more regular in appearance (Shimizu *et al.*, 2002). The width W measures the length of the range of fractal exponents in the signal. Therefore, the wider the range, the «richer» the signal in structure. The asymmetry parameter B informs us about the dominance of low or high fractal exponents with respect to the other. A right-skewed spectrum denotes relatively strongly weighted high fractal exponents, corresponding to fine structures, and low ones (more smooth-looking) for left-skewed spectra.

4. Results and discussion

We performed multifractal analysis, calculating the Legendre spectra by means of the software FRACLAB, developed at INRIA and available at the Internet site http://www.rocq.inria.fr. Since the data present gaps, we considered for each signal the longest segments with-

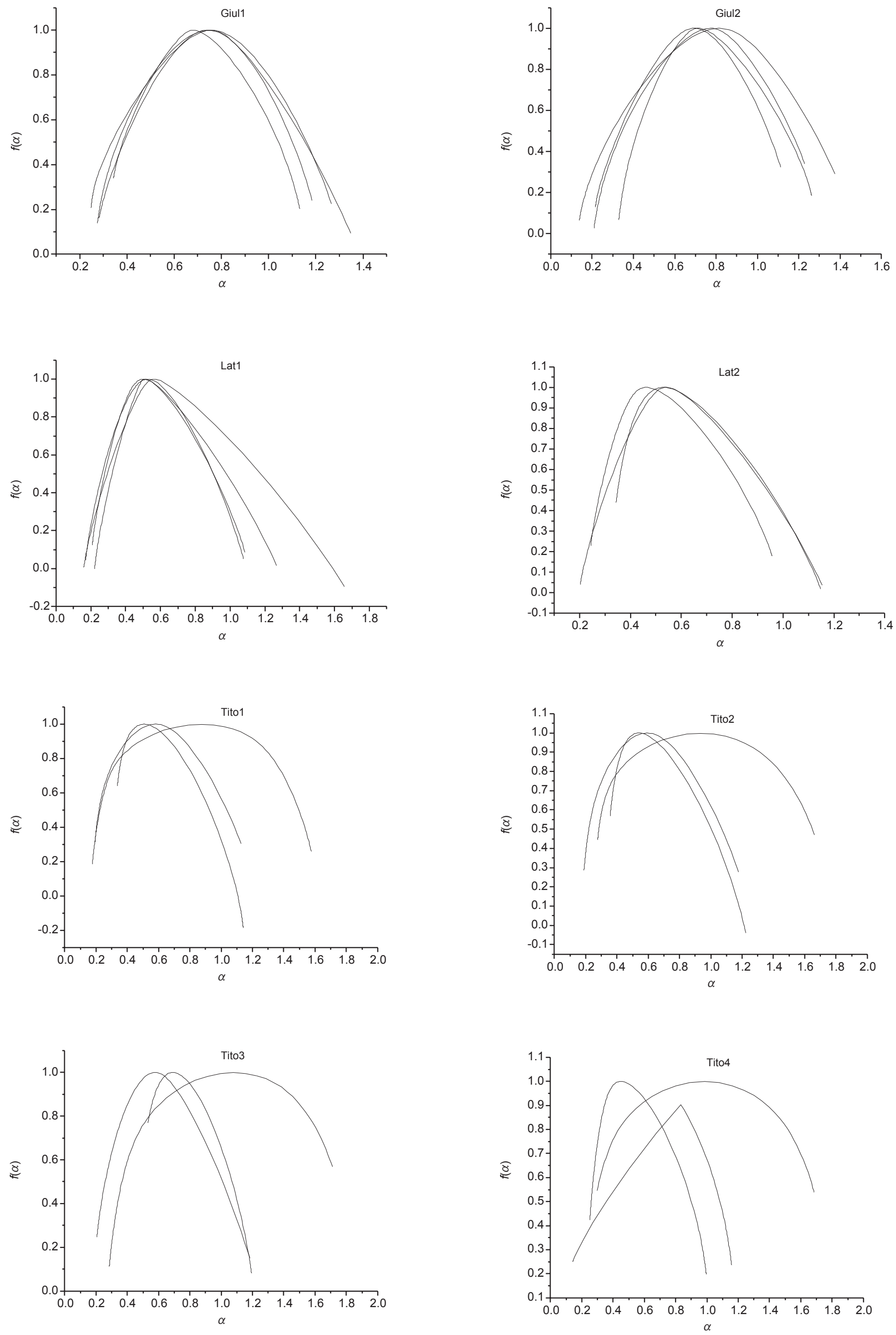

Fig. 3. Legendre spectra of the signals measured in Southern Italy. For each signal, we selected the 3-4 longest segments without gaps, whose order of magnitude of length is 10^3. All the spectra show a single-humped shape, typical of multifractal signals.

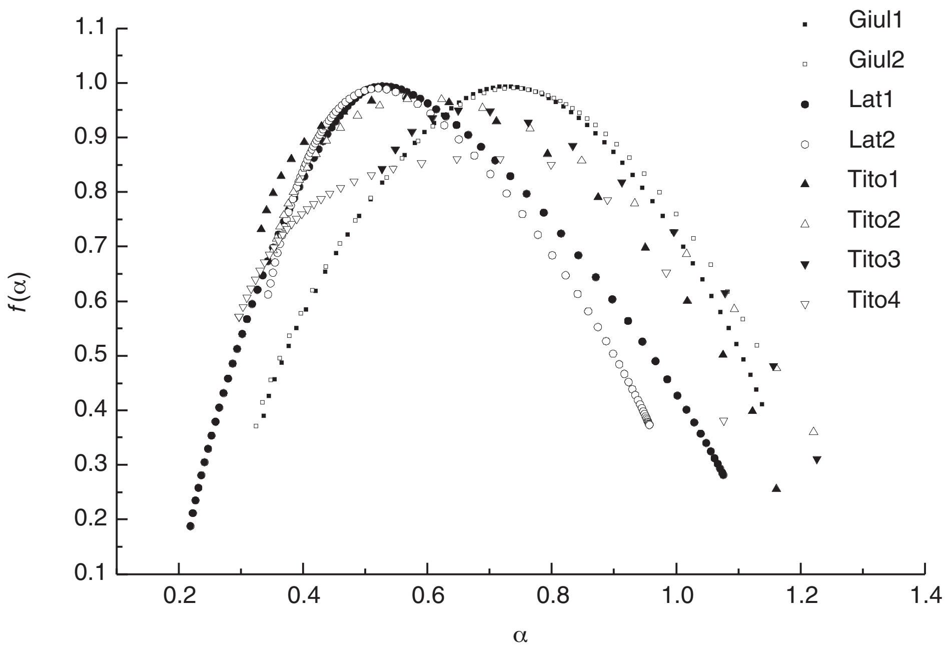

Fig. 4. Average Legendre spectra, calculated, for each signal, averaging the segment multifractal spectra.

out data missing. The order of the magnitude of the length of each segment is about 10^3, thus yielding reliable estimates of the singularity spectrum and multifractal parameters. Figure 3 shows the Legendre spectra for the selected segments for each signal. All the spectra present the typical single-humped shape, which characterizes multifractal signals. Figure 4 shows the average spectra. The differences among the spectra are very clear. Thus, after the determination of the maxima α_0, we fitted each average spectrum by eq. (3.9), thus estimating the asymmetry B and the width W. The results are summarized in fig. 5a-c. The time series Lat1 and Lat2 measured in station Laterza located in an aseismic area present the lowest value for the maximum α_0 (fig. 5a), indicating a more regular process governing the time variability of such signals that seem to be characterized by more «coarse» structures. The smooth-looking behaviour displayed by Lat1 and Lat2 is also recognized in fig. 5b, where the asymmetry B assumes the highest positive value for such signals. Furthermore, the lowest values for the width W indicate that a narrower range of fractal exponents characterizes Lat1 and Lat2 signals. The signals measured by station Tito (Tito1, Tito2, Tito3 and Tito4) and Giuliano (Giul1 and Giul2) located in a seismic area appear more complex because they present large maximum α_0 values, large width W values and relatively low value for the asymmetry B. In fig. 6 the geoelectric signals are represented as points in the space (α_0, B, W) of the multifractal parameters: the signals Lat1 and Lat2, measured in aseismic sites, are well discriminated from the other signals (Tito and Giuliano), recorded in seismic areas.

The complex geophysical phenomenon underlying the geoelectrical variations is influenced by the geological and tectonic environment outlined in the previous sections. The use of multifractal methods to investigate the temporal fluctuations of geoelectrical signals can

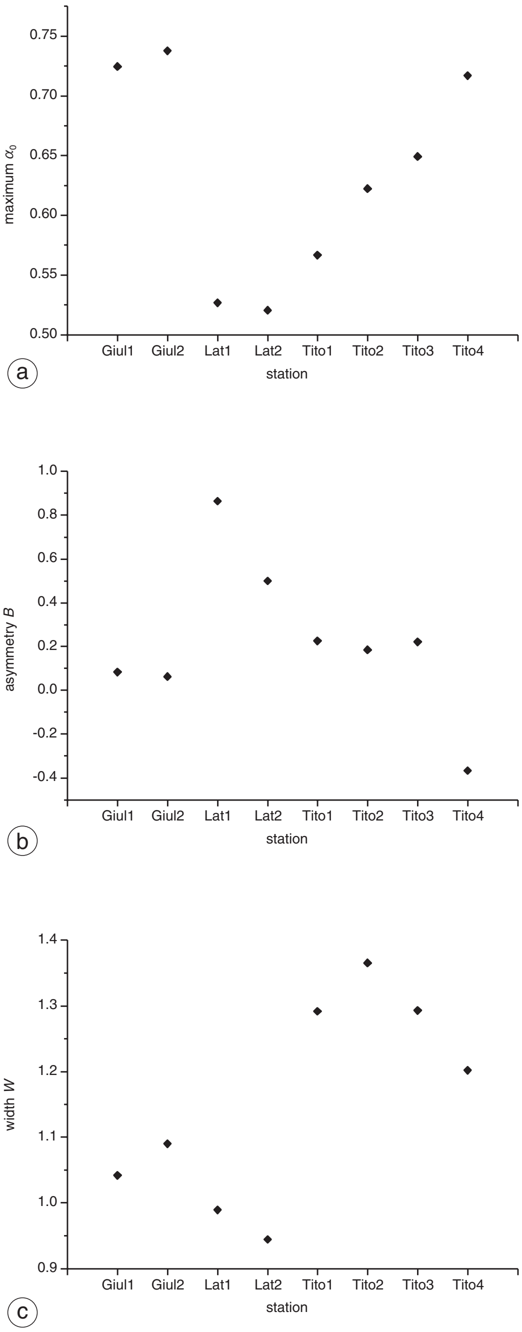

Fig. 5a-c. Variation with the station of the multifractal parameters: a) maximum α_0, b) asymmetry *B* and (c) width *W*.

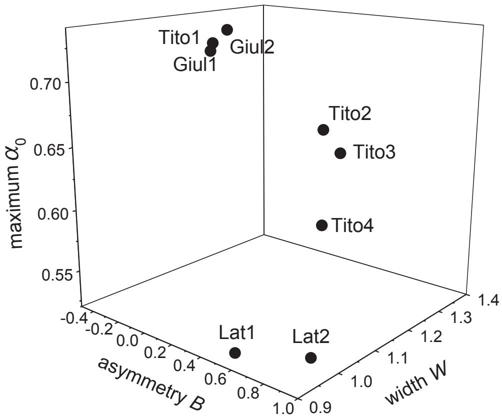

Fig. 6. 3D plot of the multifractal parameters: the signals Lat1 and Lat2, recorded in an aseismic area, are well discriminated from the signals (Tito and Giul) measured in seismic areas.

lead to a better understanding of such complexity. The development of the Southern Apennines fold-and-thrust belt has been attributed to the subduction of the Ionian lithosphere beneath the Adriatic plate (Doglioni *et al.*, 1999; Bonini *et al.*, 2000). This part of the Apennines comprises a stack of east-verging tectonic units representing an accretionary wedge composed of Mesozoic-Cenozoic sediments originally deposited in different paleogeographic domains, both basinal (Lagonegro) and shallow-water which were thrust onto the Apulian foreland to the east (Mostardini and Merlini, 1986; Pescatore, 1988; Schiattarella 1998) (fig. 7).

The monitoring network area extends along the Southern Apennine Chain where only Laterza station is out of an active seismic zone. In fact Laterza station is located in Puglia region, this area is part of the Apulian foreland which represents the Plio-Pleistocene foreland of the Southern Apennines orogenic system («Avampaese Apulo» of Selli, 1962). The Apenninic foreland basins not yet incorporated into the orogenic wedge is characterized by a 6 km thick Mesozoic carbonate succession (Apulia carbonate platform – D'Argenio, 1974; Richetti, 1980). The uplift of the Apulian foreland (Middle-Late Pleistocene) has induced a low to

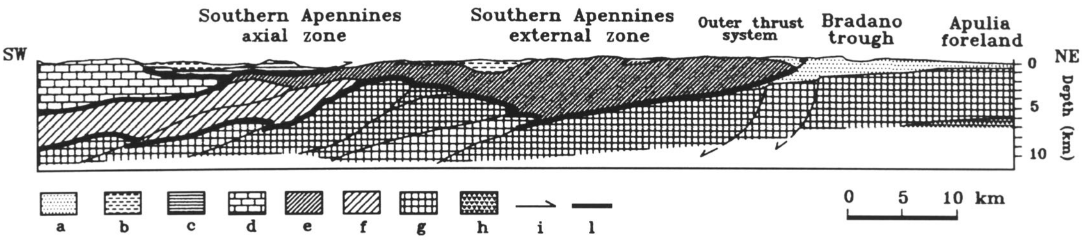

Fig. 7. Schematic section of the shallow structure of the Southern Apennines across the Irpinia region: a – Plio-Pleistocene deposits of the Bradano trough; b – thrust sheet-top succession (Upper Miocene-Early Pleistocene); c – Sicilide and Sannio nappes (Paleogene-Lower Miocene); d – Western Carbonate platform (Mesozoic-Paleogene) and Upper Miocene flysh deposits associated with the foredeep phase; e – Lagonegro Basin upper sequance (Upper Cretaceus-Upper Miocene); f – Lagonegro Basin lower sequance (Lower Triassic-Lower Cretaceus); g – Apulia Carbonate; h – Verrucano Fm. (Permian Lower Triassic); i – thrusts and normal faults; l – boundary of the main tectono-stratigraphic units (Improta *et al.*, 2003).

moderate-energy seismicity documented by paleoliquefaction features (Moretti, 2000) and historical and instrumental recorded seismic events (Pieri *et al.*, 1997): it is considered a poorly tectonized area in the Apenninic foreland (Doglioni *et al.*, 1995).

On the other hand, Giuliano and Tito stations are located in the Campania-Basilicata area, one of the most active seismic zones of the Southern Apennines. Large destructive earthquakes occurred both in historical and recent times in this region, which was struck in 1980 by the strongest event ($m_s = 6.9$) of the past century in the Southern Apennines (Improta *et al.*, 2003).

The highest number of disastrous events with $I \geq$ X MCS took place in the Irpinia area in historical times. Such events (1964, 1930) appear to be concentrated in the same zone; only one strong event, the December 16, 1857 earthquake affected the southern sector of this area (Agri Valley – Basilicata) and 1550 and 1561 earthquakes have been located in the west of the area, in the Vallo di Diano (Campania) (Alessio *et al.*, 1995). Ten year after the devasting Irpinia earthquake, a moderate event on May 1990 ($M_L = 5.2$) and May 1991 ($m_b = 5.1$) occurred approximately 40 km east of the southern end of the 1980 aftershock zone, causing damage in the nearby town of Potenza (Ekström, 1994). The normal faults affecting the internal side of the fold-and-thrust belt are commonly related to this extensional regime, which followed the progressive migration of the compressive thrust-front toward the Apulian foreland (Bonini *et al.*, 2000).

5. Conclusions

The determination of the multifractal parameters of geoelectrical time series recorded in Southern Italy was performed by means of the calculation of the Legendre spectrum. We derived three parameters, the maximum α_0 of the spectrum, the asymmetry B and the width W of the curve. The time series Lat1 and Lat2, recorded in an aseismic site, are characterized by low maximum α_0, high positive asymmetry B and low width W. These features qualify these time series as less complex than the others (Tito1-4 and Giul1-2), which were measured in seismic sites. Therefore, this set of multifractal parameters seem to well discriminate signals measured in seismic from signal recorded in aseismic sites.

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