Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics and Chemistry of the Earth 29 (2004) 295–303
PHYSICS and CHEMISTRY of the EARTH
www.elsevier.com/locate/pce

Investigating the multifractal properties of geoelectrical signals measured in southern Italy

Luciano Telesca [a,*], Marianna Balasco [b], Gerardo Colangelo [a], Vincenzo Lapenna [a], Maria Macchiato [c]

[a] *IMAA-CNR, Institute of Methodologies for Environmental Analysis, National Research Council, C.da S.Loja, Z.I., 85050 Tito Scalo (PZ), Italy*
[b] *Dipartimento di Geologia e Geofisica, Università degli studi di Bari, Bari, Italy*
[c] *Dipartimento di Scienze Fisiche, Università "Federico II", Naples, Italy*

Received 6 May 2003; received in revised form 18 August 2003; accepted 12 September 2003
Available online 12 April 2004

Abstract

Multifractal investigation of the time fluctuations in the hourly time variability of geoelectrical signals, measured from January 2001 to September 2002 by four stations installed in Basilicata region (southern Italy) has been performed. Three stations (Giuliano, Marsico and Tito) are located in a seismic area and one (Laterza) in an aseismic area. The set of the multifractal parameters reveals the larger "complexity" of data measured in seismic areas respect to those recorded in aseismic areas, suggesting their potential in discriminating the two types of signals.

Keywords: Geoelectrical signals; Multifractal spectrum

1. Introduction

The dynamics underlying tectonic processes could be directly revealed by the investigation of the temporal fluctuations of self-potential signals, which may be useful to monitor and understand many seemingly complex phenomena linked to seismic activity (Johnston, 1997; Park, 1995). Self-potential field variability may be induced by stress and fluid flow field variability (Scholz, 1990), therefore, the analysis of these induced fluctuations could contribute to gain information on the governing geophysical mechanisms characterizing normal as well as intense seismic activity. In this context, in this work we investigate the dynamical properties of geoelectrical signals, as they can be detected from observational time series.

Self-potential signals are the result of the interaction among very heterogeneous and not well known mechanisms which can be influenced by the particular structure of the monitored zone (Patella et al., 1997). This means that local features can be mixed to the general ones, thereby increasing the difficulty of rightly characterising and interpreting the signal time variations. In addition, as occurs for many environmental signals, observational data are made even more erratic by the presence of anthropic phenomena: electrical signals coming from anthropic sources may be added, e.g., to the natural signal thereby making its dynamical characterization harder (Cuomo et al., 1997; Pham et al., 1998).

In a previous paper, Cuomo et al. (1998) analyzed the geoelectrical daily means in order to give information about the statistical features of the geoelectrical background noise and the inner dynamics of geophysical processes producing the electrical phenomena observed on earth surface in seismic areas. They discussed the statistical analysis of dynamical systems based on the estimation of their degree of predictability, distinguishing randomness from chaos and providing a parsimonious representation in terms of autoregressive models of observations, by means of the only information coming from the time series itself.

In the study of seemingly complex phenomena, as those generating self-potential signals, methodologies

* Corresponding author. Tel.: +39-0971-427206; fax: +39-0971-427271.
E-mail address: ltelesca@imaa.cnr.it (L. Telesca).

1474-7065/$ - see front matter
doi:10.1016/j.pce.2003.09.015

able to capture the dynamical peculiarities in observational time series are particularly useful tools to obtain information on the features and on the causes of signal time variability. In particular, fractal techniques, developed to extract qualitative and quantitative information from time series, have been applied recently to the study of a large variety of irregular, erratic signals and by now have demonstrated to be very useful to reveal deep dynamical features. Cuomo et al. (2001) detected scaling behaviour in the power spectra of geoelectrical time series, revealing the antipersistent character of the self-potential fluctuations. Telesca et al. (2001) proposed a new approach to investigate correlation between geoelectrical signals and earthquakes, analyzing the time variations of the fractal parameters, characterizing their dynamics. Balasco et al. (2002) found that self-potential measurements seem to be featured by long-range correlations with scaling exponents which indicate that the underlying geophysical process is characterized by stabilizing mechanisms.

In all the previous works, monofractal analyses have been performed, leading to the estimation of only one scaling exponent. Monofractals are homogeneous objects, in the sense that they have the same scaling

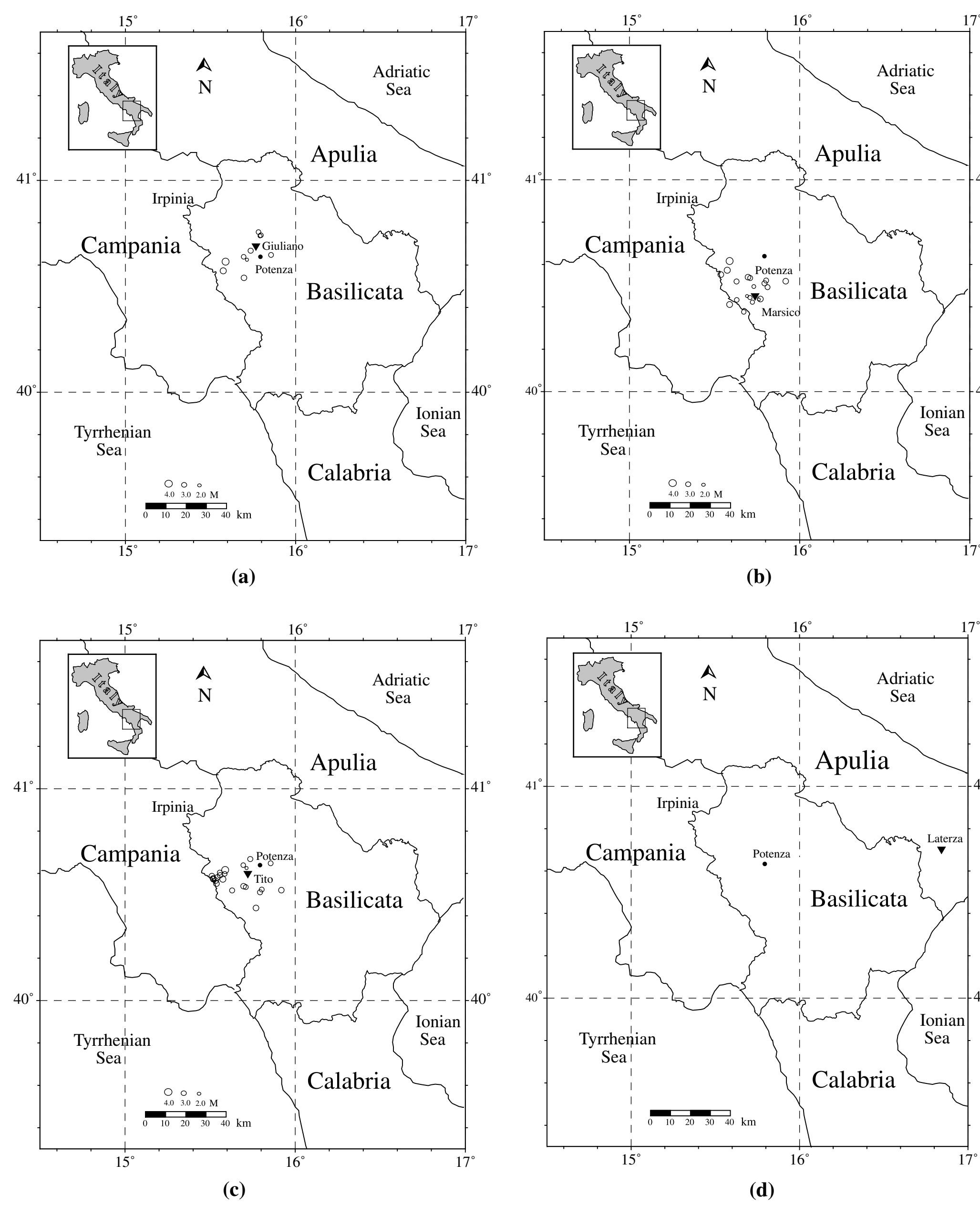

Fig. 1. Ubication of the geoelectrical monitoring stations and epicenters of the earthquakes satisfying the Dobrovol'skiy's rule in relation with the location of the measuring stations: (a) Giuliano, (b) Marsico, (c) Tito and (d) Laterza. The seismic data are extracted from the INGV (National Institute of Geophysics and Volcanology) seismic catalogue (http://www.ingv.it).

properties, characterized by a single singularity exponent (Stanley et al., 1999, and references therein). The need for more than one scaling exponent to describe the scaling properties of the process uniquely, indicates that the process is not a monofractal but could be a multifractal. A multifractal object requires many indices to characterize its scaling properties. Multifractals can be decomposed into many—possibly infinitely many—subsets characterized by different scaling exponents. Thus, multifractals are intrinsically more complex and inhomogeneous than monofractals (Stanley et al., 1999), and characterize systems featured by very irregular dynamics, with sudden and intense bursts of high frequency fluctuations (Davis et al., 1994).

In the present work, we investigate the temporal fluctuations of self-potential data, measured in southern Italy from January 2001 to September 2002, by using the multifractal formalism, in order to evidence typical dynamical features.

2. Data

Our data consist in nine geoelectrical time series recorded at four monitoring stations: Tito (Tito1, Tito2, Tito3 and Tito4), Giuliano (Giul1 and Giul2), Marsico and Laterza (Lat1 and Lat2). As far as the technical features of the experimental equipment is concerned, we refer the reader to Cuomo et al. (1997) and for the results of mono- and multi-parametric preliminary statistical analysis of the monitored variables to Di Bello et al. (1994). The Tito SP acquisition system consists of an array with four electrodes put at the corners of a 100 m side square. Station Giuliano is located just on a strike fault. It measures two SP signals. The dipoles are located parallel and perpendicular to the strike-fault and the distance between the probes is 100 m for the dipole oriented along the strike-fault direction (EW), and 80 m for the other dipole (NS) perpendicular to the fault. The dipole at station Marsico is 100 m long and oriented along NS direction. The Laterza dipole configuration is NW–SE 100 m and NE–SW 80 m. The stations Tito, Giuliano and Marsico are located in seismic sites, while Laterza in an aseismic one.

The stress field produces cracks on the rock volumes triggering fluid pressure variations. As a result of this process, we have an underground charge motion and, subsequently, we observe anomalies in the electrical field on the surface only if the preparation region is near the measuring station. It is necessary to discriminate the useful events (i.e., earthquakes responsible for significant geophysical variations in a rock volume of the investigated area) from all the seismicity that occurred in the area surrounding the measuring station. Therefore, from the whole seismicity, we selected only earthquakes that could be responsible for strain effects in the areas around the monitoring stations. We used the empirical formula introduced by Dobrovol'skiy (Dobrovol'skiy, 1993; Dobrovol'skiy et al., 1979): $r = 10^{0.43\,M}$, where M is the magnitude and r (km) the radius of the area in which the effects of the earthquake are detectable. We considered only the earthquakes with r greater than the distance between the epicenter and the measuring station. Fig. 1 shows for each station the earthquakes, that according to the Dobrovol'skiy's rule can affect the time variations of the measured signals. As we can clearly observe, the area monitored by station Laterza is characterized by a substantial absence of earthquakes.

Fig. 2 shows the time variations of the self-potential signals along with the earthquakes occurred during the observation period. A striking feature is visible concerning especially the graphs of station Tito: an increased seismic activity, also with events with a relatively high magnitude ($M \geqslant 4.0$), characterizes the SP time fluctuations in the temporal range $10^4 < t < 1.5 \times 10^4$. In this temporal range, the Tito SP fluctuations present the largest variability and irregularity. Giuliano signals are characterized by similar behavior, with an increased number of spikes in the same temporal range as Tito. Marsico signal presents a long gap during the same time range, but its dynamics vary significantly between 1.3×10^4 h and 1.5×10^4 h.

3. Multifractal formalism

The concept of multifractal object has been developed by Mandelbrot (1974) to investigate several features in the intermittency of turbulence (Meneveau and Sreenivasan, 1991). Many authors have applied the multifractality to several fields of the scientific research.

The multifractal formalism is based on the definition of the so-called partition function $Z(q,\varepsilon)$,

$$Z(q,\varepsilon) = \sum_{i=1}^{N_{\text{boxes}}(\varepsilon)} \mu_i(\varepsilon), \tag{1}$$

where the quantity $\mu_i(\varepsilon)$ is a measure and it depends on ε, the size or scale of the boxes used to cover the sample. The boxes are labeled by the index i and $N_{\text{boxes}}(\varepsilon)$ indicates the number of boxes of size ε needed to cover the sample. The exponent q is a real parameter, giving the order of the moment of the measure. The choice of the functional form of the measure $\mu_i(\varepsilon)$ is arbitrary, provided that the most restrictive condition $\mu_i(\varepsilon) \geqslant 0$ is satisfied.

The parameter q can be considered as a powerful microscope, able to enhance the smallest differences of two very similar maps (Diego et al., 1999). Furthermore, q represents a selective parameter: high values of q enhance boxes with relatively high values for $\mu_i(\varepsilon)$; while low values of q favor boxes with relatively low values of

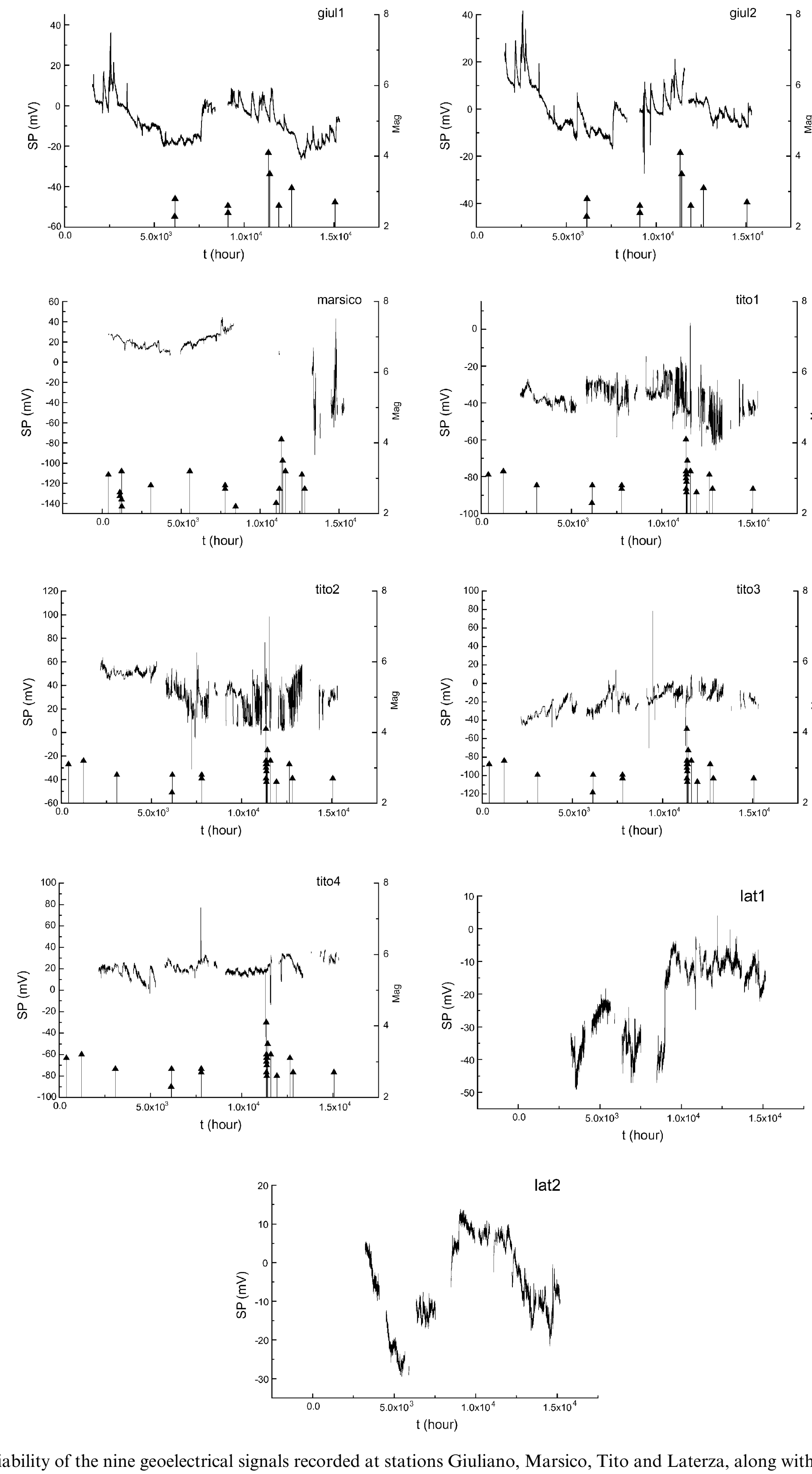

Fig. 2. Hourly variability of the nine geoelectrical signals recorded at stations Giuliano, Marsico, Tito and Laterza, along with the occurrence of the earthquakes selected by means of the Dobrovol'skiy's rule.

$\mu_i(\varepsilon)$. The box size ε can be considered as a filter, so that big values of the size are equivalent to apply a large scale filter to the map. Changing the size ε, one explores the sample at different scales. Therefore, the partition function $Z(q,\varepsilon)$ furnishes information at different scales and moments.

The generalized dimensions are defined by the following equation:

$$D(q) = \lim_{\varepsilon\to 0} \frac{1}{q-1}\frac{\ln Z(q,\varepsilon)}{\ln \varepsilon}, \tag{2}$$

where $D(0)$ is the capacity dimension; $D(1)$ is the information dimension, and $D(2)$ is the correlation dimension. An object is called monofractal if $D(q)$ is constant for all values of q, otherwise is called multifractal. In most practical applications, the limit in Eq. (2) cannot be calculated, because we do not have information at small scales, or because below a minimum physical length no scaling can exist at all (Diego et al., 1999). Generally, a scaling region is found, where a power-law can be fitted to the partition function, which in that scaling range behaves as

$$Z(q,\varepsilon) \propto \varepsilon^{\tau(q)}, \tag{3}$$

where the slope $\tau(q)$ is related to the generalized dimension by the following equation:

$$\tau(q) = (q-1)D(q). \tag{4}$$

An usual measure in characterizing multifractals is given by the singularity spectrum or Legendre spectrum $f(\alpha)$, that is defined as follows. If for a certain box j the measure scales as

$$\mu_j(\varepsilon) \propto \varepsilon^{\alpha_j}, \tag{5}$$

the exponent α, which depends upon the box j, is called Hölder exponent. If all boxes have the same scaling with the same exponent α, the sample is monofractal. The multifractal is given if different boxes scale with different exponents α, corresponding to different strength of the measure. Denoting as S_α the subset formed by the boxes with the same value of α, and indicating as $N_\alpha(\varepsilon)$ the cardinality of S_α, for a multifractal the following relation holds:

$$N_\alpha(\varepsilon) \propto \varepsilon^{-f(\alpha)}. \tag{6}$$

By means of the Legendre transform the quantities α and $f(\alpha)$ can be related with q and $\tau(q)$:

$$\alpha(q) = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q}, \tag{7}$$

$$f(\alpha) = q\,\alpha(q) - \tau(q), \tag{8}$$

where the curve $f(\alpha)$ is a single-humped function for a multifractal, while reduces to a point for a monofractal.

In order to quantitatively recognize possible differences in Legendre spectra stemming from different signals, it is possible to fit, by a least square method, the spectra to a quadratic function around the position of their maxima at α_0 (Shimizu et al., 2002):

$$f(\alpha) = A(\alpha-\alpha_0)^2 + B(\alpha-\alpha_0) + C, \tag{9}$$

where parameter B measures the asymmetry of the curve, which is zero for symmetric shapes, positive or negative for left-skewed or right-skewed shapes, respectively.

Another parameter is the width of the spectrum, that estimates the range of α where $f(\alpha) > 0$, obtained extrapolating the fitted curve to zero; thus the width is defined as

$$W = \alpha_{\max} - \alpha_{\min}, \tag{10}$$

where $f(\alpha_{\max}) = f(\alpha_{\min}) = 0$.

These three parameters serve to describe the "complexity" of the signal. If α_0 is low, the signal is correlated and the underlying process "loses fine structure", becoming more regular in appearance (Shimizu et al., 2002). The width W measures the length of the range of fractal exponents in the signal; therefore, the wider the range, the "richer" the signal in structure. The asymmetry parameter B informs about the dominance of low or high fractal exponents respect to the other. A right-skewed spectrum denotes relatively strongly weighted high fractal exponents, corresponding to fine structures, and low ones (more smooth-looking) for left-skewed spectra.

4. Results

We performed the multifractal analysis, calculating the Legendre spectra by means of the software FRACLAB, developed at INRIA and available at the internet site http://www-rocq.inria.fr. Since the data present gaps, we considered for each signal the longest segments without data missings. The order of the magnitude of the length of each segment is about 10^3, thus permitting to obtain reliable estimates of the singularity spectrum and multifractal parameters. Fig. 3 shows the Legendre spectra for the selected segments for each signal. All the spectra present the typical single-humped shape, that characterizes multifractal signals. The spectra of the segments for each signal, calculated for different time intervals, are not identical, although they are very similar to each other. Only each Tito signal presents a couple of curves very different from each other, the wider one corresponding to the segment of data extracted in the temporal range in which an increase of the seismic activity has been recorded (see Fig. 2). For each signal and each segment spectrum, we calculated the three parameters, maximum α_0, asymmetry B and width W; then we averaged them, obtaining a set of three multifractal parameters for each original signal. In order to evaluate the significance of the results, we generated

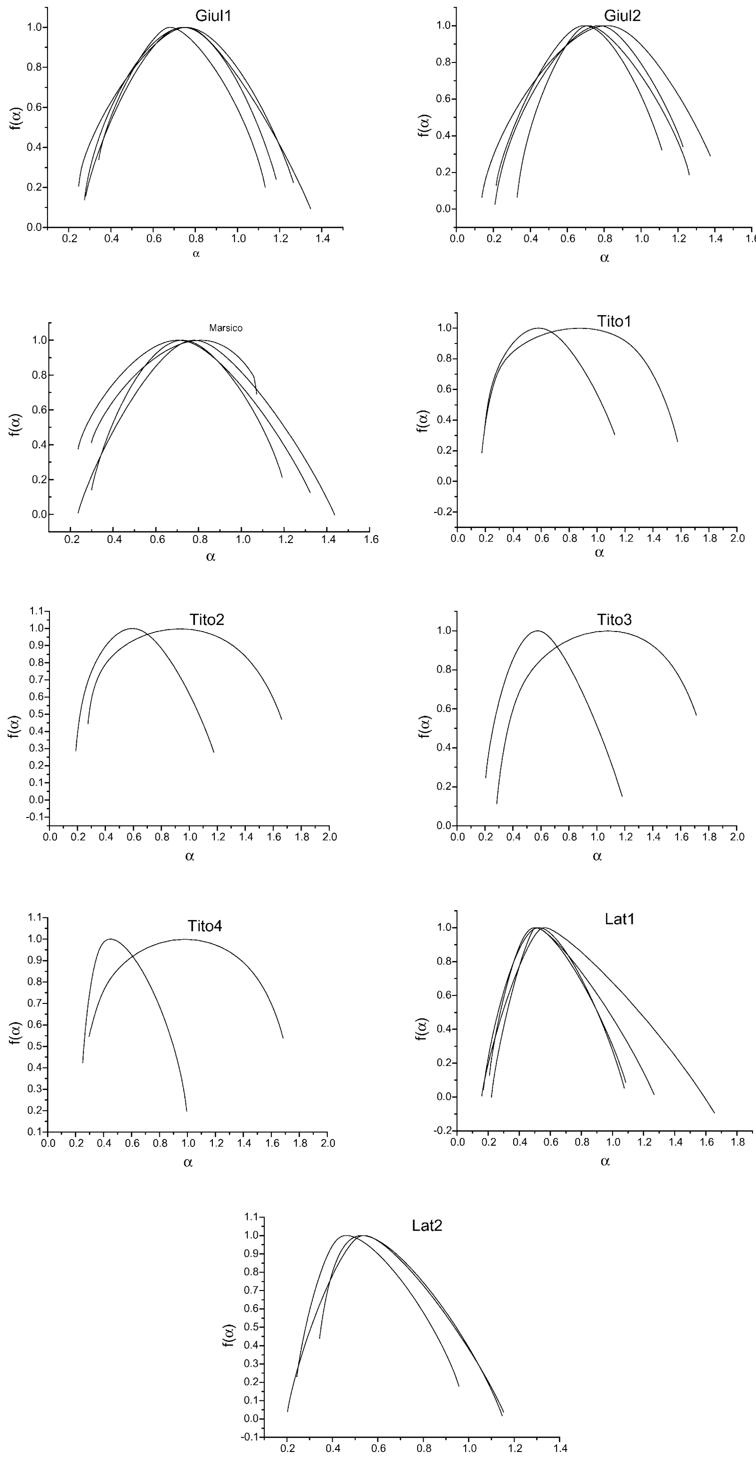

Fig. 3. Legendre spectra of the signals measured in southern Italy. For each signal, we selected two to four longest segments without gaps, whose length is approximately 10^3 points. All the spectra evidence the single-humped shape, typical of multifractal signals.

for each segment 10 surrogate series. Each surrogate series is obtained by the following procedure: (1) Fourier transforming the original series, (2) preserving the amplitudes of the Fourier transform of the series (this implies that the power spectral density of the surrogate series is the same as the original one), (3) randomizing the phases of the Fourier transform, i.e., attributing to the phase a random number between 0 and 2π, and finally (4) inverse-Fourier transforming. The surrogate series, generated by means of this procedure, have the

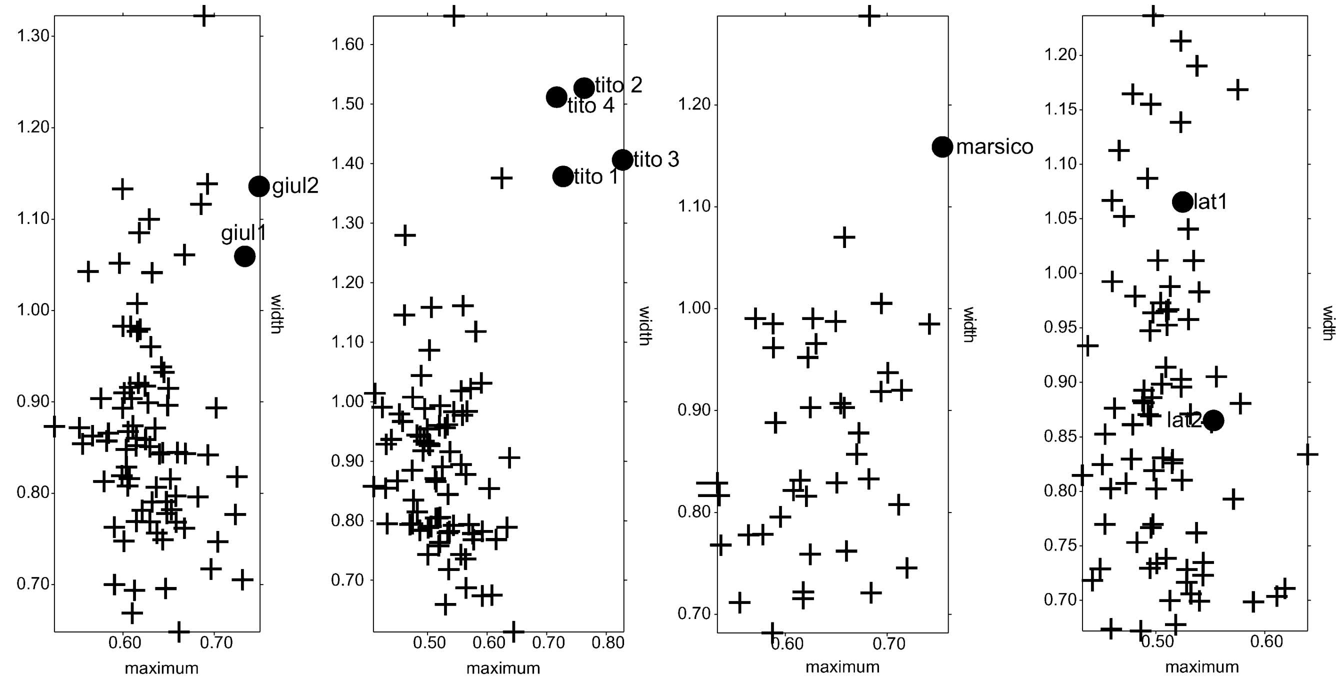

Fig. 4. Maximum α_0–width W relation.

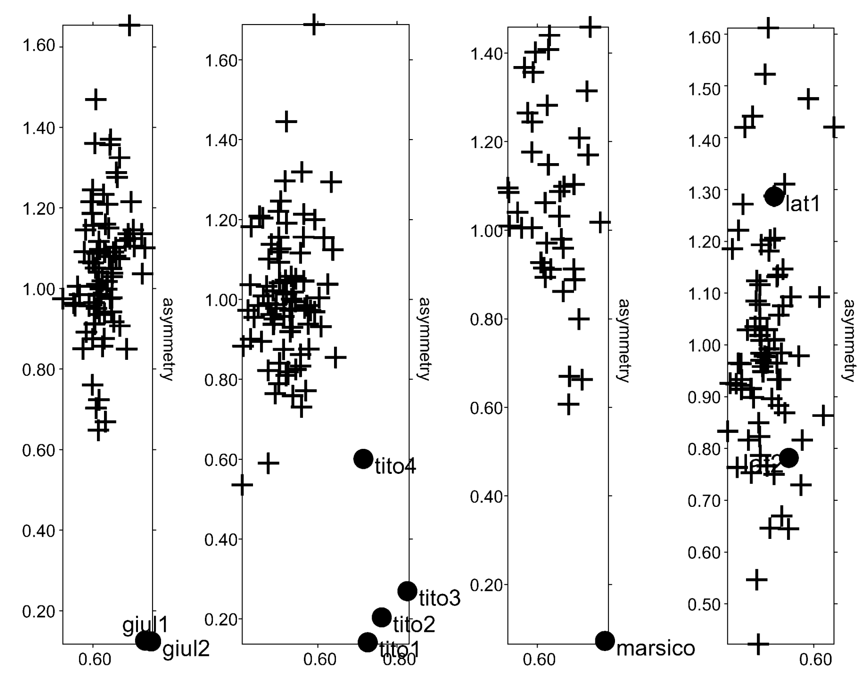

Fig. 5. Maximum α_0–asymmetry B relation.

same linear properties as the original ones, like the power spectrum, while the nonlinear properties, stored in the Fourier phases, are destroyed. For each surrogate series, we calculated the multifractal spectrum, and estimated the three multifractal parameters. Figs. 4–6 show the relation maximum–width, maximum–asymmetry and asymmetry–width, respectively, for the original (circle) and the surrogate series (cross): we observe that the geoelectrical signals measured in seismic areas (Giuliano, Tito and Marsico) are discriminated from the

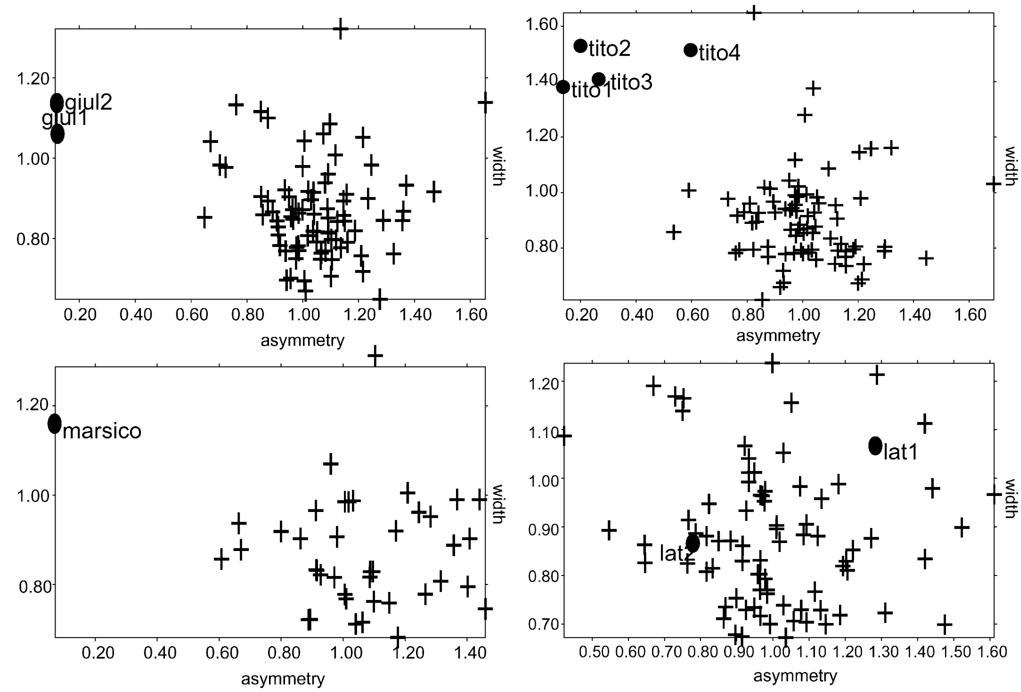

Fig. 6. Asymmetry B–width W relation.

surrogate series; while Laterza signals do not show any separation between the points representing the original signals and the those representing the surrogate.

Since the surrogate procedure eliminates the non-linear features in a time series, we can deduce that Laterza signals are characterized by linear dynamics, while Giuliano, Tito and Marsico signals by nonlinear dynamics.

5. Conclusions

The geophysical phenomenon underlying the geoelectrical variations is complex and the physical laws that govern the process are not completely known. The use of multifractal methods in investigating the temporal fluctuations of geoelectrical signals can lead to a better understanding of such complexity. The determination of the multifractal parameters has been performed by means of the calculation of the Legendre spectrum, from which we estimated the maximum α_0, the asymmetry B and the width W of the spectrum; this set of parameters seem to well discriminate between signals measured in seismic areas and those recorded in aseismic areas. The surrogate procedure, which has led to the generation of time series with the same linear properties (second-order statistics) as the original signals, suggests that Laterza time series, recorded in an aseismic site, are characterized by linear dynamics, while Giuliano, Tito and Marsico signals by nonlinear dynamics.

References

Balasco, M., Lapenna, V., Telesca, L., 2002. $1/f^{\alpha}$ Fluctuations in geoelectrical signals observed in a seismic area of Southern Italy. Tectonophysics 347, 253–268.

Cuomo, V., Lapenna, V., Macchiato, M., Serio, C., 1997. Autoregressive models as a tool to discriminate chaos from randomness in geoelectrical time series: an application to earthquake prediction. Ann. Geofis. XL, 385–400.

Cuomo, V., Lapenna, V., Macchiato, M., Serio, C., Telesca, L., 1998. Linear and non-linear dynamics in electrical precursory time series: implications with earthquake prediction. Tectonophysics 287, 279–298.

Cuomo, V., Di Bello, G., Heinicke, J., Lapenna, V., Martinelli, G., Piscitelli, S., Telesca, L., 2001. Investigating the temporal fluctuations in geoelectrical and geochemical signals jointly measured in a seismic area of Southern Apennine Chain (Italy). Ann. Geofis. 44, 179–191.

Davis, A., Marshak, A., Wiscombe, W., Cahalan, R., 1994. Multifractal characterization of non-stationarity and intermittency in geophysical fields: observed, retrieved or simulated. J. Geophys. Res. 99, 8055–8072.

Di Bello, G., Lapenna, V., Satriano, C., Tramutoli, V., 1994. Self-potential time series analysis in a seismic area of the Southern Apennines: preliminary results. Ann. Geofis. XXXVII (Suppl. N. 5), 1137–1148.

Diego, J.M., Martinez-Gonzales, E., Sanz, J.L., Mollerach, S., Mart, V.J., 1999. Partition function based analysis of cosmic microwave background maps. Mon. Not. R. Astron. Soc. 306, 427–436.

Dobrovol'skiy, I.P., 1993. Analysis of preparation of a strong tectonic earthquake. Phys. Solid Earth 28, 481–492.

Dobrovol'skiy, I.P., Zubkov, S.I., Miachkin, V.I., 1979. Estimation of the size of earthquake preparation zones. Pageoph 117, 1025–1044.

Johnston, M.J.S., 1997. Review of electric and magnetic fields accompanying seismic and volcanic activity. Surv. Geophys. 18, 441–475.

Mandelbrot, B.B., 1974. Intermittent turbulence in self-similar cascades: divergence of high moments and dimensions of the carrier. J. Fluid Mech. 62, 331–358.

Meneveau, C., Sreenivasan, K.R., 1991. The multifractal nature of turbulent energy dissipation. J. Fluid Mech. 224, 429–484.

Park, S.K., 1995. Monitoring resistivity change in Parkfield, California: 1988–1995. J. Geophys. Res. 102, 24545–24559.

Patella, D., Tramacere, A., Di Maio, R., 1997. Modelling earth current precursors in earthquake prediction. Ann. Geofis. 40, 495–517.

Pham, V.N., Boyer, D., Choularias, G., Le Mouel, J.L., Rossignol, J.C., Stavrakakis, G.N., 1998. Characteristics of electromagnetic noise in the Ioannina region (Greece); a possible origin for so called seismic electric signal (SES). Geophys. Res. Lett. 25, 2229–2232.

Scholz, C.H., 1990. The mechanics of earthquakes and faulting. Cambridge University Press, Cambridge. pp. 439.

Shimizu, Y., Thurner, S., Ehrenberger, K., 2002. Multifractal spectra as a measure of complexity in human posture. Fractals 10, 103–116.

Stanley, H.E., Amaral, L.A.N., Goldberger, A.L., Havlin, S., Ivanov, P.Ch., Peng, C.-K., 1999. Statistical physics and physiology: monofractal and multifractal approaches. Physica A 270, 309–324.

Telesca, L., Cuomo, V., Lapenna, V., Macchiato, M., 2001. A new approach to investigate the correlations between geoelectrical time fluctuations and earthquakes in a seismic area of southern Italy. Geophys. Res. Lett. 28, 4375–4378.