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MONOTONE COURANT FINITE ELEMENT METHODS FOR THE TRANSPORT EQUATION

In this talk we consider the transport equation with a given incompressible vector field β and a constant $\alpha \geq 0$

$$\alpha u + \frac{\partial u}{\partial \beta} = f \quad \text{in } \Omega, \qquad \beta_n^-(u - u^{\mathrm{D}}) = 0 \quad \text{on } \partial\Omega, \qquad (\frac{\partial u}{\partial \beta} = \beta \cdot \nabla u), \tag{1}$$

where f and u^{D} represent (smooth) data. This equation is prototypical for many problems in continuum mechanics, since, with the zero-order term replaced by the time-derivative, it described the material derivative.

For a Lipschitz-continuous transport field, (1) can be formally solved by the method of characteristics, which reveals the maximum principle: positive data give positive solutions. This property is of fundamental importance in many applications, the most prominant being mass conservation, where u represents the density function. A notorious difficulty in the context of finite element methods, which has attracted a lot of attention in the litterature, is to respect the maximum principle on the discrete level. The first positive result by Brurman and Ern [1] is based on the shock-capturing SUPG-method of Johnson, Szepessy and Hansbo [2]. However, an important remaining diificulty is the solution nonlinear equations. Recently, several other methods have been developped [3, 4, 5].

References

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