On a posteriori error bounds for Galerkin discretisations of the Allen-Cahn problem

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Abstract

We are concerned with the proof of a posteriori error (upper) bounds for fully-discrete Galerkin approximations of the Allen-Cahn equation in two and three spatial dimensions. We begin from discussing the case of the backward Euler method combined with conforming finite elements on standard meshes in space, before continuing with the case of space-time discontinuous Galerkin methods on very general, polytopic, prismatic meshes. For both methods, we prove conditional type a posteriori error estimates in the $L_4(L_4)$-norm that depend polynomially upon the inverse of the interface length ϵ. The derivation relies on the availability of a spectral estimate for the linearized Allen-Cahn operator about the approximating solution in conjunction with a continuation argument and a variant of the elliptic reconstruction. The new analysis also appears to improve variants of known *a posteriori* error bounds in $L_2(H^1)$, $L_\infty(L_2)$-norms in certain regimes. The presentation is based on joint work with K. Chrysafinos (NTU Athens), Zhaonan Dong (INRIA, Paris) and D. Plaka (NTU Athens)

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