Adaptive methods for fully nonlinear PDE

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Abstract

Hamilton–Jacobi–Bellman and Isaacs equations are important classes of fully nonlinear PDE with applications from stochastic optimal control and two player stochastic differential games. In this talk, we present our recent proof of the convergence of a broad family of adaptive nonconforming DG and C^0-interior penalty methods for the class of these equations that satisfy the Cordes condition in two or three space dimensions. The adaptive mesh refinement is driven by reliable and efficient a posteriori error estimators, and convergence is proven in H^2-type norms without higher regularity assumptions of the solution. A foundational ingredient in the proof of convergence is the concept of the limit space used to describe the limiting behaviour of the finite element spaces under the adaptive mesh refinement algorithm. We develop a novel approach to the construction and analysis of these nonstandard function spaces via intrinsic characterizations in terms of the distributional derivatives of functions of bounded variation. We provide a detailed theory for the limit spaces, and also some original auxiliary function spaces, that resolves some foundational challenges and that is of independent interest to adaptive nonconforming methods for more general problems. These include Poincaré and trace inequalities, a proof of the density of functions with nonvanishing jumps on only finitely many faces of the limit skeleton, symmetry of the Hessians, approximation results by finite element functions and weak convergence results.

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