Regular Decompositions: Discrete, Boundary-Aware, and \( p \)-Stable

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Abstract
A regular decomposition of \( H(\text{curl}) \) on a three-dimensional domain is a representation of that space as a sum of a space of vector fields with components in \( H^1 \) and a space of gradients. In addition, the induced projection mappings must be continuous. The factors of the regular decomposition also inherit local homogeneous boundary conditions.

Similar decompositions of finite element subspaces of \( H(\text{curl}) \) can be found: a finite-element vectorfield can be decomposed into a piecewise polynomial continuous vectorfield plus the gradient of a scalar-valued finite-element function. These discrete regular decompositions can be shown to enjoy (almost) the same stability properties as their continuous counterparts. Most importantly, the stability properties are uniform in the meshwidth (for \( h \)-FEM) or in the polynomial degree (for \( p \)-FEM).

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